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Quantum metrology with Dicke squeezed states

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Abstract
We introduce a new class of quantum many-particle entangled states, called the Dicke squeezed (or DS) states, which can be used to improve the precision in quantum metrology beyond the standard quantum limit. We show that the enhancement in measurement precision is characterized by a single experimentally detectable parameter, called the Dicke squeezing ξ_D, which also bounds the entanglement depth for this class of states. The measurement precision approaches the ultimate Heisenberg limit as ξ_D attains the minimum in an ideal Dicke state. Compared with other entangled states, we show that the DS states are more robust to decoherence and give better measurement precision under typical experimental noise.

Keywords: quantum metrology, Dicke squeezed state, entanglement

Precision measurement plays an important role for scientific and technological applications. In many circumstances, precision measurement can be reduced to detection of a small phase shift by use of optical or atomic interferometry [1–4]. The precision of the phase measurement improves with increase of the number of particles (photons or atoms) in the interferometer. For N particles in non-entangled (classical) states, the phase sensitivity $\Delta\theta$ is constrained by the standard quantum limit $\Delta\theta \sim 1/\sqrt{N}$ from the shot noise [1, 3, 4]. Schemes have been proposed to improve the measurement precision beyond the standard quantum limit by use of quantum entangled states [1–7]. Two classes of states are particularly important for this scenario: one is the GHZ state [6], also called the NOON state in the second quantization representation [7]; and the other is the spin squeezed state [1, 3, 4], which may include the squeezed state of light as a special limit. A number of intriguing experiments have been reported to prepare these states and use them for quantum metrology [8–14]. These states are typically sensitive to decohereand experimental noise [15, 16]. As a result, the number of particles that one can prepare into the GHZ state, or the maximal spin squeezing that one can achieve, are both severely limited by noise in experiment.

In this paper, we introduce a new class of many-particle entangled states for quantum metrology, which we name the Dicke squeezed (DS) states. The DS states have the following interesting features: (i) They represent a wide class of entangled states with possibly many different forms but can be characterized by a single parameter called the Dicke squeezing ξ_D, with $\xi_D < 1$. The Dicke squeezing parameter ξ_D can be conveniently measured in experiments from detection of the collective spin operator of N particles. It provides the figure-of-merit for application of the DS states in quantum metrology in the following sense: for states with ξ_D, the phase sensitivity $\Delta\theta$ and the phase measurement precision $d\theta$ both improve from the standard quantum limit $1/\sqrt{N}$ to the new scaling $\sqrt{\xi_D/N}$. The phase shift can be read out through the Bayesian inference for the DS states. Under a fixed particle number N, the parameter ξ_D attains the minimum $1/(N+2)$ under the ideal Dicke state, and the phase sensitivity correspondingly approaches the Heisenberg limit $\Delta\theta \sim 1/N$, in agreement with the previous result on the Dicke state [17, 18]. (ii) The entanglement of the DS states can be also characterized by the squeezing parameter ξ_D. For a many-body system with a large particle number N, we would like to know how many particles among them have been prepared into genuinely entangled states. This number of particles with genuine entanglement is called the entanglement depth for this system [19, 20]. A criterion proved in [20] indicates that $\xi_D^{-1} - 2$ gives a lower bound of the entanglement depth for any DS states with the squeezing parameter ξ_D. (iii) Compared with the GHZ state or the spin squeezed states, we show that the DS states characterized by ξ_D are much more robust to decoherence and experimental noise such as particle loss. Substantial Dicke squeezing ξ_D remains under a significant amount of noise under which spin squeezing would not be able to survive at all.

For a system of N particles, each of two internal states a, b (with effective spin-1/2), we can define a Pauli matrix $\vec{\sigma}_i$ for each particle i and the collective spin operator $\vec{J}$ as the summation $\vec{J} = \sum_i \vec{\sigma}_i/2$. Note that the components of $\vec{J}$ can be measured globally without the requirement of separate addressing of individual particles. If the particles are indistinguishable like photons or ultracold bosonic atoms, we can use the number of particles n_a, n_b in each mode a, b to denote the states. In this notation (second quantization representation), the GHZ state of N spins $|aa\cdots a\rangle + |bb\cdots b\rangle$ (unnormalized) is represented by $|N0\rangle_{ab} + |0N\rangle_{ab}$, the so called NOON state [7]. The collective spin operators can be expressed in term of the mode operators a, b using the Schwinger representation $J_z = (a^\dagger a - b^\dagger b)/2$, $J_x = (a^\dagger b + b^\dagger a)/2$, $J_y = (a^\dagger b - b^\dagger a)/2i$ [2]. Asmall phase shift θ can be measured through the Mach–Zehnder (MZ) type of interferometer illustrated in figure 1 by inputting a state of two modes a, b and measuring the number difference of the output modes (the output J_z operator). The two beam splitters in the interferometer exchange the J_z and J_y operators and the phase shifter is represented by a unitary operator $e^{i\theta J_z}$, which transforms J_y to $\cos\theta J_y - \sin\theta J_x$. Assume the input state has mean $\langle \vec{J} \rangle = \langle J_x \rangle$ and minimum variance $\langle (\Delta J_y)^2 \rangle$ along the y -direction. By measuring $\langle J_y' \rangle = \cos\theta \langle J_y \rangle - \sin\theta \langle J_x \rangle \approx -\theta \langle J_x \rangle$, the phase sensitivity $\Delta\theta$ is characterized by $\sqrt{\langle (\Delta J_y)^2 \rangle} / \langle J_x \rangle$. This motivates definition of the spin squeezing parameter [3, 4]

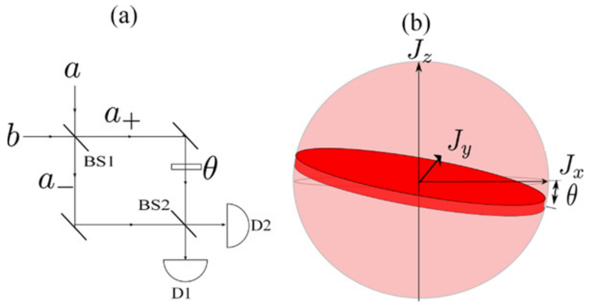

Figure 1. (a) The MZ interferometer setup to measure the relative phase shift θ with input modes a, b in DS states. The detectors D1 and D2 measure the J_z operator by recording the particle number difference in the two output modes. (b) In the Bloch sphere for the collective spin operator $\vec{J}$, a measurement of the phase shift by the MZ interferometer is represented by rotation of a thin disk (its size in x, y, z directions corresponds to the variance of $\vec{J}$ under the DS state) by an angle θ.

$$\xi_S = \frac{N\left\langle \left(\Delta J_y\right)^2 \right\rangle}{\left\langle J_x \right\rangle^2} \tag{1}$$

as the figure-of-merit for precision measurement. The phase sensitivity is estimated by $\sqrt{\xi_S/N}$ for this measurement scheme.

Not all states useful for quantum metrology can be characterized by the spin squeezing ξ_S. An example is the Dicke state $|N/2, N/2\rangle_{ab}$, which has been shown to give the Heisenberg limited phase sensitivity in [17]. However, for this state, $\langle \vec{J} \rangle = 0$ in all the directions, and the spin squeezing ξ_S is not a good measure to characterize states of this kind with $\langle \vec{J} \rangle = 0$. To characterize a broad class of states that are useful for quantum metrology, we introduce the following Dicke squeezing parameter, defined as

$$\xi_D = \frac{N\left(\left\langle \left(\Delta J_z\right)^2 \right\rangle + \frac{1}{4}\right)}{\left\langle J_x^2 + J_y^2 \right\rangle}. \tag{2}$$

One can easily check that $\xi_D = 1$ for the benchmark spin-coherent states. We call any states with $\xi_D < 1$ as the DS states and a major result of this paper is to show that such states are useful for quantum metrology, where the phase sensitivity is improved from $\sqrt{1/N}$ for the benchmark spin coherent state to about $\sqrt{\xi_D/N}$ for the DS states. The parameter ξ_D attains the minimum $1/(N+2)$ under the ideal Dicke state $|N/2, N/2\rangle_{ab}$, and the phase sensitivity $\sqrt{\xi_D/N}$ correspondingly approaches the Heisenberg limit $\sim 1/N$, in agreement with the results in [17, 18]. The definition of the parameter ξ_D is motivated by a similar quantity first introduced in the work [20] for detection of many-particle entanglement. Another related parameter is $\xi_{os} = (N-1)\langle(\Delta J_z)^2\rangle \big/ [\langle J_x^2 + J_y^2 \rangle - N\big/2]$, introduced in [21] for entanglement detection. Albeit similar in form, for metrological applications ξ_G makes more sense as it recovers the correct Heisenberg limit ($\xi_D \to 1/N$, in contrast with $\xi_{os} \to 0$) as one approaches the Dicke states.

The Dicke squeezing parameter ξ_D also characterizes the entanglement depth E_d for many-particle systems. For an N-qubit system, the entanglement depth E_d measures how many qubits have been prepared into genuinely entangled states [19, 20]. A theorem proven in Ref. shows that $\lceil \xi_D^{-1} \rceil - 2$, where $\lceil \xi_D^{-1} \rceil$ denotes the minimum integer no less than ξ_D^{-1}, gives a lower bound of the entanglement depth E_d. For the ideal Dicke state, $|N/2, N/2\rangle_{ab}$, $\xi_D^{-1} = N + 2$ and its entanglement depth is N [20]. Note that the entanglement depth characterizes the particle (qubit) entanglement when we express the state $|N/2, N/2\rangle_{ab}$ in the first quantization representation [19, 20], where one can easily see all the N qubits are genuinely entangled, so its entanglement depth is N. This should not be confused with the mode entanglement between the bosonic operators a and b , which is zero for the Dicke state $|N/2, N/2\rangle_{ab}$. So the defined Dicke squeezing parameter ξ_D provides a figure-of-merit both for entanglement characterization and its application in quantum metrology, and this parameter can be conveniently measured in experiments through detection of the collective spin operator $\vec{J}$.

To show that ξ_D is the figure-of-merit for quantum metrology, we use two complementary methods to verify that the phase measurement precision is improved to $\sqrt{\xi_D/N}$ for a variety of states of different forms. First, in the MZ interferometer shown in figure 1(a), the phase sensitivity is estimated by the intrinsic uncertainty $\Delta\theta$ of the relative phase operator defined between the two arms (modes $a_\pm$). We calculate this phase uncertainty and find that it scales as $\sqrt{\xi_D/N}$ for various input states with widely different ξ_D and N. Second, we directly estimate the phase shift θ by the Bayesian inference through detection of the spin operator J_z, and find that the measurement precision, quantified by the variance $d\theta$ of the posterior phase distribution, is well estimated by $\beta\sqrt{\xi_D/N}$, where $\beta \approx 1.7$ is a dimensionless prefactor. We perform numerical simulation of experiments with randomly chosen phase shift θ and find that the difference between the actual θ and the the measured value of θ obtained through the Bayesian inference is well bounded by the variance $d\theta$, so $d\theta$ is indeed a good measure of the measurement precision.

The Dicke state $|N/2, N/2\rangle_{ab}$ represents an ideal limit, and it is hard to obtain a perfect Dicke state in experiments in particular when the particle number N is large. Here, we consider two classes of more practical states as examples to show that ξ_D is the figure-of-merit for application in quantum metrology when the ideal Dicke state is distorted by unavoidable experimental imperfection. For the first class, we consider pure states of the form $|\Psi(\sigma)\rangle_{ab} = \sum_{n=0}^{N} a_n(\sigma)|n, N-n\rangle$, where the total number of particles is fixed to be N but the number difference between the modes a, b follows a Gaussian distribution $a_n(\sigma) = exp\left\{\frac{-(n-\frac{N}{2})^2}{\sigma^2} + i\frac{\pi}{4}(n-\frac{N}{2})\right\}$ with different widths characterized by the parameter σ. The phase of $a_n(\sigma)$ is chosen for convenience so that the variance of the state is symmetric along the x, y axes. For the second class, we consider mixed states $\rho_{ab}(\eta)$, which come from noise distortion of the Dicke state $|N/2, N/2\rangle_{ab}$ after a particle loss channel with varying loss rate η. To calculate $\rho_{ab}(\eta)$, we note that a loss channel with loss rate η can be conveniently modeled by the transformation $a = \sqrt{1-\eta}\, a_{in} + \sqrt{\eta}\, a_\nu$ and $b = \sqrt{1-\eta}\, b_{in} + \sqrt{\eta}\, b_\nu$, where a_{in}, $b_{\rm in}$ denote the annihilation operators of the input modes that are in the ideal Dicke state $|N/2, N/2\rangle = ((N/2)!)^{-1}(a_{in}^\dagger b_{in}^\dagger)^{N/2}|0, 0\rangle$ and a_ν,b_ν represent the corresponding vacuum modes. By substituting $a_{in}^\dagger$, $b_{in}^\dagger$ with $a^\dagger$, $b^\dagger$ through the channel transformation and tracing over the vacuum modes $a_\nu^\dagger$,$b_\nu^\dagger$, we get the matrix form of $\rho_{\rm ab}(\eta)$ in the Fock basis of the modes a, b. The two classes of states $|\Psi(\sigma)\rangle_{ab}$ and $\rho_{ab}(\eta)$ approach the ideal Dicke state when the parameters σ, η tend to zero.

In the MZ interferometer shown in figure 1(a), the modes $a_\pm$ of the two arms are connected with the input modes a, b by the relation $a_\pm = (\pm a + b)/\sqrt{2}$. The phase eigenstates $|\theta_l\rangle_\pm$ of the modes $a_\pm$ are superpositions of the corresponding Fock states $|n\rangle_\pm$ with

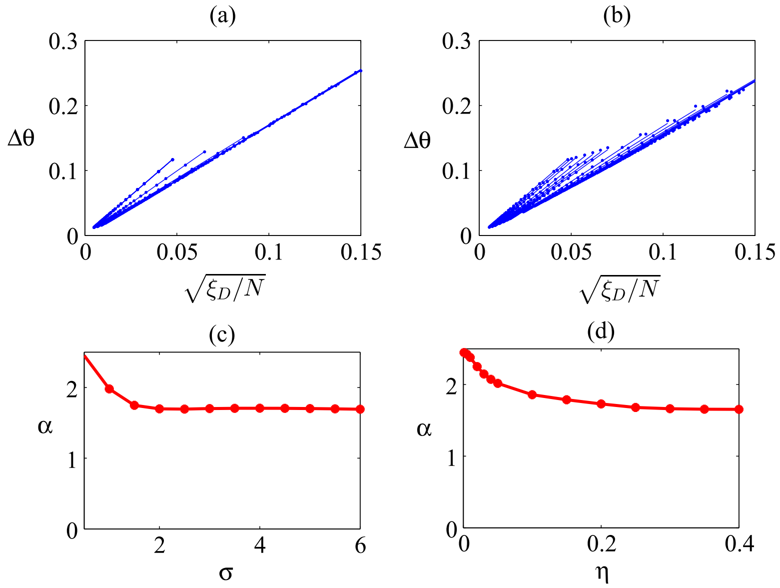

Figure 2. The phase sensitivity $\Delta\theta$ versus the normalized Dicke squeezing $\sqrt{\xi_D/N}$ for two classes of input states: (a) States $|\Psi(\sigma)\rangle_{ab}$ with Gaussian superposition coefficients. (b) Dissipative states $\rho_{ab}(\eta)$ after a loss channel. The resulting points are on a straight line when we vary the particle number N from 20 to 200 (ξ_D changes correspondingly) and the slope of the line changes slightly as we vary the parameter σ (from 0 to 6) or η (from 0 to 0.4). (c) and (d) show the variation of the slope α as a function of the parameter σ or η.

$|\theta_l\rangle_\pm = (s+1)^{-1/2}\sum_{n=0}^{s} e^{in\theta_l}|n\rangle_\pm$, where $\theta_l = 2\pi l/(s+1)$ $(l = 0, 1, \ldots, s)$ and $s+1$ denotes the Hilbert space dimension that eventually takes the infinity limit [22]. For modes $a_\pm$ in a composite state denoted by its density matrix $\rho_\pm$, the probability distribution $P(\theta_r)$ of the relative phase θ_r between the two interferometer arms can be expressed as

$$P(\theta_r) = \sum_{l=0}^{s} {}_\pm\left\langle \theta_l\theta_{l-\delta l}\right| \rho_\pm \left|\theta_l\theta_{l-\delta l}\right\rangle_\pm, \tag{3}$$

where $\delta l = \theta_r(s+1)/(2\pi)$. The phase distribution $P(\theta_r)$ becomes independent of the Hilbert space dimension $s+1$ when s goes to infinity, and the half width $\Delta\theta$ of $P(\theta_r)$ gives an indicator of the intrinsic interferometer sensitivity to measure the relative phase shift for the given input state [17, 18]. We use $\Delta\theta$ to quantify the phase sensitivity for our input states.

In figure 2, we show the calculated phase sensitivity $\Delta\theta$ for the two classes of input states $|\Psi(\sigma)\rangle_{ab}$ and $\rho_{ab}(\eta)$, by varying the parameters σ, η and the particle number N. With fixed parameters σ, η, when we vary the particle number N (typically from 20 to 200 in our calculation), the phase sensitivity $\Delta\theta$ follows a linear dependence with the parameter $\sqrt{\xi_D/N}$ by $\Delta\theta = \alpha\sqrt{\xi_D/N}$ (note that the Dicke squeezing parameter ξ_D changes widely as we vary N and σ, η). The slope α depends very weakly on the parameters σ, η, as shown in figure 2(c) and (d), and roughly we have $\alpha \approx 2$. This shows that, for different types of input states, the phase sensitivity $\Delta\theta$ is always determined by the parameter $\sqrt{\xi_D/N}$ up to an almost constant prefactor α.

A good phase sensitivity $\Delta\theta$ is an indicator of possibility of high-precision measurement of the relative phase shift θ, however, the sensitivity by itself does not give the information of θ. In particular, for the DS states, we typically have $\langle \vec{J} \rangle = 0$ and therefore cannot read out the information of θ by measuring rotation of the mean value of $\vec{J}$. A powerful way to read out the information of θ is through the Bayesian inference [17, 18]. Here, we show that with the Bayesian inference, we can faithfully extract the information of θ with a measurement precision $d\theta = \beta\sqrt{\xi_D/N}$ for the DS states, where the prefactor $\beta \approx 1.7$. We note that each instance of measurement by the MZ interferometer setup shown in figure 1 records one particular eigenvalue j_z of the J_z operator, which occurs with a probability distribution $P(j_z|\theta)$ (called the likelihood) that depends on the relative phase shift θ. With a given input state ρ_{ab} for the modes a, b, the likelihood $P(j_z|\theta)$ is given by

$$P\left(j_z\middle|\theta\right) = \left\langle j, j_z \middle| e^{i\theta J_y} \rho_{ab} e^{-i\theta J_y} \middle| j, j_z \right\rangle, \tag{4}$$

where $|j, j_z\rangle$ denotes the momentum eigenstate with $j = N/2$. The Bayesian inference is a way to use the Bayes' rule to infer the posterior distribution $P_m(\theta|\{j_z\}_m)$ of the phase shift θ after m instances of measurements of the J_z operator with the measurement outcomes $\{j_z\}_m = j_{z1}, j_{z2}, \cdots, j_{zm}$, respectively. After the mth measurement with outcome j_{zm}, the phase distribution $P_m(\theta|\{j_z\}_m)$ is updated by the Bayes' rule

$$P_m(\theta\,|\{j_z\}_m) = \frac{P(j_{zm}|\theta)P_{m-1}(\theta|\{j_z\}_{m-1})}{P(j_{zm}|\{j_z\}_{m-1})}, \tag{5}$$

where $P(j_{zm}|\{j_z\}_{m-1}) = \int d\theta P(j_{zm}|\theta)P_{m-1}(\theta|\{j_z\}_{m-1})$ is the probability to get the outcome j_{zm} conditional on the sequence $\{j_z\}_{m-1}$ for the previous $m-1$ measurement outcomes. Before the first measurement, the prior distribution $P_0(\theta)$ is assumed to be a uniform distribution between 0 and 2π. When the instances of measurements $m \gg 1$, the posterior distribution $P_m(\theta|\{j_z\}_m)$ is typically sharply peaked around the actual phase shift, and we use the half width $d\theta$ of $P_m(\theta|\{j_z\}_m)$ to quantify the measurement precision.

To show that the measurement precision $d\theta$ is indeed determined by $\sqrt{\xi_D/N}$ for the DS states, we numerically simulate the MZ experiment with a randomly chosen actual phase shift θ_{r} in the interferometer. We take input states of the forms of $|\Psi(\sigma)\rangle_{ab}$ or $\rho_{ab}(\eta)$, as specified previously, with the corresponding likelihood $P(j_z|\theta)$ given by equation (4). With this likelihood, we get a sequence of measurement outcomes $j_{z1}, j_{z2}, \cdots, j_{zm}$, which are sampled in our numerically simulated experiments using the corresponding probability distributions $P(j_{zk}|\{j_z\}_{k-1})$ with $k = 1, 2, \cdots, m$, respectively. For this sequence of outcomes, we obtain the corresponding sequence of posterior phase distributions $P_m(\theta|\{j_z\}_m)$, with an example shown in figure 3(a). One can see that the distribution $P_m(\theta\,|\{j_z\}_m)$ indeed gets increasingly sharper with m and its peak approaches the actual phase shift θ_{r}. We use the the central peak position θ_{p} of the distribution $P_m(\theta|\{j_z\}_m)$ as an estimator of the measured phase shift, and the difference $\theta_{pr} = |\theta_p - \theta_r|$ therefore quantifies the measurement error. This error θ_{pr} is typically bounded by $d\theta$, indicating there is no systematic bias by this inference method.

In figure 3(b) and (c), we show the measurement precision $d\theta$ and the estimation error θ_{pr} as functions of the scaled parameter $\sqrt{\xi_D/(Nm)}$, as we vary the types of input states (the parameters σ, η in states $|\Psi(\sigma)\rangle_{ab}$ and $\rho_{ab}(\eta)$), the particle number N, and the number of measurement instances m. All the points for the measurement precision $d\theta$ can be well fit with a linear function $d\theta \approx \beta\sqrt{\xi_D/(Nm)}$ with $\beta \approx 1.7$. The estimation error θ_{pr} from the simulated

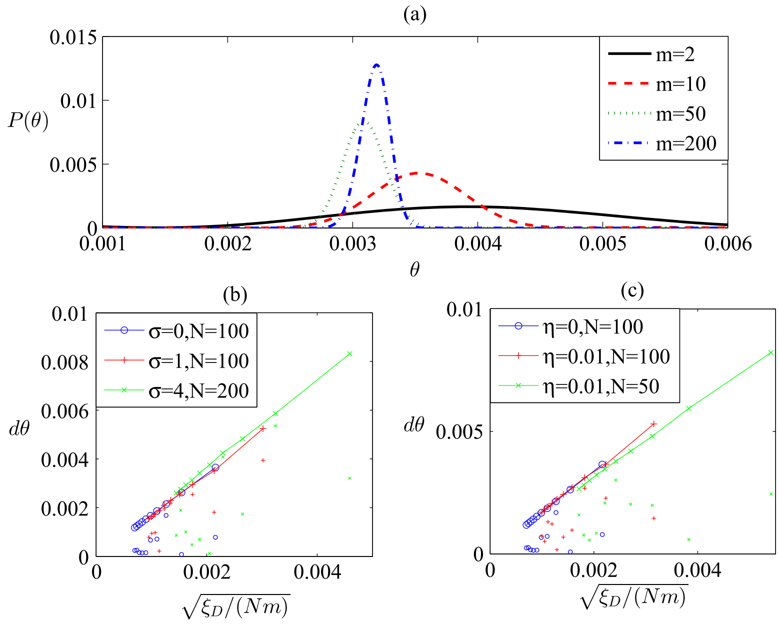

Figure 3. (a) The posterior phase distributions $P_m(\theta|\{j_z\}_m)$ obtained from the Bayesian inference after the mth measurement with $m = 2$, 10, 50, and 200 from our numerically simulated experiments. In the simulation, the actual phase shift $\theta_r = 0.003$ and the input state is $|\Psi(\sigma)\rangle_{ab}$ with $N = 1000$, $\xi_D = 0.0019$, and $\sigma = 1$. (b) and (c): The measurement precision $d\theta$ (the dots along a line fit by $d\theta \approx 1.7\sqrt{\xi_D/(Nm)}$) and the estimation error $\theta_{\rm pr}$ (the scattered points below the line) as functions of the scaled parameter $\sqrt{\xi_D/(Nm)}$ for the Gaussian input states $|\Psi(\sigma)\rangle_{ab}$ (b) and the dissipative input states $\rho_{ab}(\eta)$ (c) with m varying from 20 to 200. The other parameters (σ, N for $|\Psi(\sigma)\rangle_{ab}$ and η, N for $\rho_{ab}(\eta)$) are specified by the inserts of the figure.

experiments (the scattered points) is typically below the corresponding $d\theta$. This supports our central claim: the defined Dicke squeezing parameter ξ_D characterizes the improvement of measurement precision for the DS states compared with the standard quantum limit.

Compared with other entangled states used in quantum metrology, a remarkable advantage of the DS states characterized by the squeezing parameter ξ_D is its noise robustness. For instance, if the noise in experiments is dominated by the dephasing error that does not change the mode population, the numerator does not change in the definition equation (2) for the Dicke squeezing ξ_D and only the denominator drops slowly. With a dephasing rate p (p is the probability for each qubit to become completely decoherent), the squeezing parameter reduces to $\xi_D = 1/[N(1-p) + 2 - p^2]$ if we start with a Dicke state for N particles [20]. We still have substantial squeezing when $N \gg 1$ even if the dephasing error rate $p \gtrsim 50\%$. More generic noise such as particle loss has a larger influence on the Dicke squeezing, however, the DS states are still more robust compared with other forms of entangled states such as the spin squeezed states. In figure 4(a), we show the influence of the particle loss to the Dicke squeezing ξ_D and the spin squeezing ξ_S, starting with comparable values of ξ_S and ξ_D at the loss rate $\eta = 0$ under the same particle number N. The spin squeezed state was determined by minimizing $(\Delta J_z)^2$ with $J_x = 0.1J$ [19]. One can see that that the spin squeezing ξ_S is quickly blown up by very small particle loss, but substantial Dicke squeezing ξ_D remains even under a significant loss rate. In

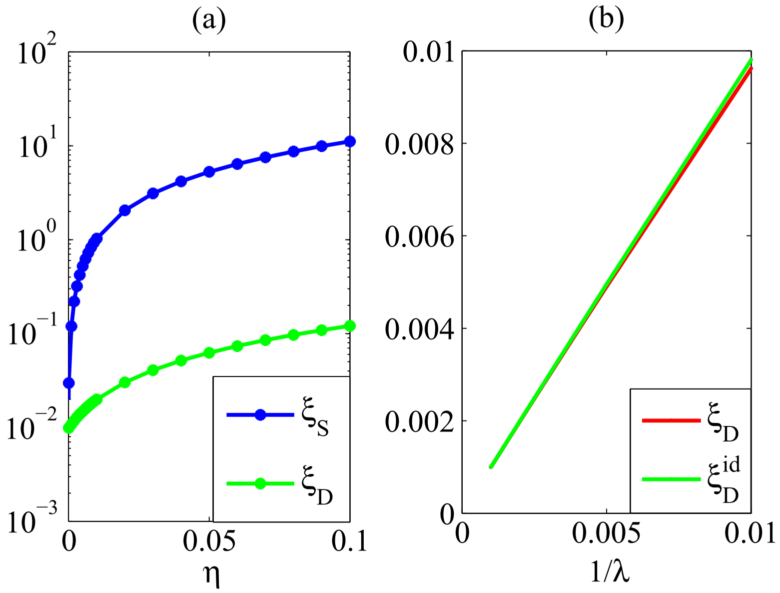

Figure 4. (a) Comparison of the spin squeezing ξ_S and the Dicke squeezing ξ_D under influence of the particle loss with a loss rate η. We take the particle number $N = 100$ and the amounts of squeezing for ξ_S and ξ_D comparable initially at $\eta = 0$. (b) Comparison of the Dicke squeezing ξ_D for a mixture of Dicke states Poissonian distributed in the total particle number according to $P_\lambda(N)$ and ξ_D^{id} for a single Dicke state with a fixed particle number $N = \lambda$.

the asymptotic limit with $N \gg 1$, $\xi_D \approx \eta/(1 - \eta)$ under a loss rate η. Therefore, compared with the standard quantum limit, the measurement precision improves by a constant factor of $\sqrt{\xi_D} \approx \sqrt{\eta/(1 - \eta)}$ for the DS state under loss. This has saturated the bound derived in [23], which proves that under noise the measurement procession can be improved at most by a constant factor for any quantum entangled states (the factor is exactly $\sqrt{\eta/(1 - \eta)}$ under a loss rate η as proven in [23]). The saturation of the improvement bound shows that the DS states characterized by the parameter ξ_D belong to the optimal class of states for improving the measurement precision under noise (note that the conventional spin squeezed states measured by the squeezing parameter ξ_S are not optimal for improving measurement precision under noise as ξ_S is quickly blown up to be larger than 1 (see figure 4 (a)), yielding no improvement compared with the standard quantum limit). Another source of noise important for experiments is the fluctuation in the total particle number N. The squeezing ξ_D is robust to this fluctuation. To show this, we consider ξ_D under an initial states, which is an ensemble of Dicke states with various particle numbers N mixed together according to the Poissonian distribution $P_\lambda(N) = \frac{\lambda^N e^{-\lambda}}{N!}$ with $\langle N \rangle = \lambda$. In figure 4(b), we compare ξ_D under this state and ξ_D^{id} under a single Dicke state with a fixed particle number $N = \lambda$. One can see that the difference is small, indicating that the Dicke squeezing ξ_D is insensitive to the total number fluctuation in the initial state.

In summary, we have proposed a new class of many-particle entangled states characterized by the introduced Dicke squeezing parameter ξ_D to improve the measurement precision in quantum metrology. We show that the phase information can be read out through the Bayesian inference and the measurement precision is improved by a factor of $\sqrt{\xi_D}$ compared with the standard quantum limit. A distinctive advantage of the DS states is its noise robustness and we show that the Dicke squeezing ξ_D is much more robust compared with other forms of entangled states used in quantum metrology. Substantial Dicke squeezing can be generated in experiments, for instance, through the atomic collision interaction in spinor condensates [24, 25]. With the characterization and measurement method proposed in this paper, the Dicke squeezing may lead to a fruitful approach for precision quantum metrology using entangled quantum states.

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