Energy Efficient Broadcast in Multiradio Multichannel Wireless Networks

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***Abstract*—The broadcast is a fundamental operation in computer and communication networks. We study broadcast in multiradio multichannel multi-hop wireless networks. Suppose through configuration, each node is already assigned with a transmission power level and a set of radio channels for receiving and forwarding data. Our problem is to select a forward scheme for broadcasting from a given source node and to minimize total energy consumption. This is a known NP-hard minimization problem. In this paper, we construct a polynomial-time $(1.35+\epsilon)(1+\ln(n-1))$-approximation algorithm where n is the number of nodes in given network and ϵ is any positive constant. We also show that there is no polynomial-time $(\rho \ln n)$-approximation for $0<\rho<1$ unless $NP \subseteq DTIME(n^{O(\log\log n)})$.**

I. Introduction

During the last decade, wireless communication has been growing rapidly. Its applications have been found in military operation, traffic control, healthcare, weather forecasting, etc. Especially, it has changed people's life style. Nowaday, almost everybody carries a wireless communication device, such as mobile phone, iPad and iPhone, etc. Those devices are usually equipped with batteries and hence their energy supply is limited. This fact makes energy efficiency become an important issue in study of wireless networks.

There are two types of energy saving techniques in the literature. The first type is to adjust transmission power level to minimize the total energy consumption [1], [2], [3]. The second type is to keep transmission power at a certain level at each node and to schedule wireless nodes into sleep or active state in order to minimize the total energy consumption [4], [5], [6] or to maximize the lifetime of network system [7], [8]. Our work in this paper belongs to the second type.

Consider a multi-radio multi-channel multi-hop wireless network (MR-MC network). We assume that there are totally C non-overlapping orthogonal frequency channels in the network and denote the C channels by $1, 2, ..., C$. Each node v is equipped with some omni-directional radio interfaces. Suppose there is already a channel assignment on the node set V, i.e., each node v is assigned with a channel subset $B(v)$. This means that each node v can receive messages through any channel in subset $B(v)$ and can also forward messages using any channel in subset $B(v)$.

Suppose that at each node v, the transmission power is fixed at a certain level $p(v)$. Compared with transmission power, energy spending for receiving data at each node can be ignored. Thus, energy consumption at each node v is $p(v)$ multiplying the number of forward channels, i.e., channels used for sending messages.

Our problem is to find a forward scheme, i.e., for each node v, find a channel subset $F(v) \subseteq B(v)$ such that data can be broadcasted from a given source node to all wireless nodes and the total energy consumption is minimized.

This problem has been proved to be NP-hard [9]. We are going to design a polynomial-time algorithm which produces an approximation solution within a factor of $(1.35+\epsilon)(1+\ln(n-1))$ from optimal, where n is the number of wireless nodes in the input network and ϵ is any positive number. We will also show that this is almost the best possible result and indeed, for any $0<\rho<1$, there is no polynomial-time $(\rho \ln n)$-approximation for this problem.

II. Related Work

Broadcast is a fundamental operation in communication networks. Many research efforts have been made on study of broadcast in various networks. Wan *et al.* [3] minimized the total energy consumption of broadcast by adjusting transmission power at each node in static wireless ad hoc networks. Before their work, there are several broadcast routing algorithms already proposed in the literature, such as the Broadcast Incremental Power (BIP) algorithm, the Shortest Path Tree (SPT) and the Minimum Spanning Tree (MST) algorithm [1], [2]. However, those algorithms were evaluated through

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simulations. Wan *et al.* [3] provided the first theoretical analysis for those algorithms and showed that all of them produce approximation solutions within a factor of 12 from optimal. So far, a sequence of following-up efforts [10], [11], [12], [13], [14] have improved this factor from 12 to 6.

Li *et al.* [6], Cagalj *et al.*[5] and Liang [4] also minimized the total energy consumption of broadcast with a different approach in static wireless ad hoc networks. They noted that in real systems, after network is configured, each node is assigned with a transmission power level which would not be changed for any broadcast request. Therefore, their minimization of total energy consumption is under the assumption that the transmission power level at each node is given. Liang [4] presented an algorithm evaluated through simulations. Cagalj *et al.*[5] gave a $O(\log^3 n)$-approximation. Li *et al.* [6] obtained an approximation algorithm with performance ratio $(1 + 2\ln(n-1))$.

Agarwal *et al.* [15] studied energy efficient broadcast in wireless ad hoc network with hitch-hiking. Nava [16] considered the broadcast problem in 3-dimensional networks. Dai and Wu [17], Li *et al.* [18], [19], and Guo and Yang [20] minimized the total energy consumption of broadcast in wireless networks with directional antennas or adaptive antennas. Ghosh [21] designed distributed algorithm and Wu and Dai [22] designed an broadcast algorithm with self-pruning technique. Ni *et al.* [23] considered the broadcast storm problem.

In this paper, we study energy efficient broadcast in multi-radio multichannel wireless networks. We also assume that at each node transmission power has been configured at a certain level. Therefore, our work is closely related to that in [6]. Indeed, the wireless ad hoc network is a special case of multiradio multichannel wireless networks when only one radio (and one channel) is available. We will design an $(1.35 + \epsilon)(1 + \ln(n-1))$-approximation algorithm for the latter problem, which is an improvement of result in [6]. The technique used in this design is initiated in [24].

III. Network Model and Problem Formulation

Denote by $r(v)$ the communication radius determined by $p(v)$ with relation

$$p(v) = c \cdot r(v)^{\alpha}$$

where c and α are two constants with $2 \leq \alpha \leq 6$. We use a directed graph $G = (V, E)$ to model the MR-MC network topology, where V represents the wireless nodes set in the network. An arc $(u, v) \in E$ if and only if $B(u) \cap B(v) \neq \emptyset$ and $d(u, v) \leq r(u)$, where $d(u, v)$ is the Euclidean distance between u and v.

In the MR-MC network, some nodes are selected as forward nodes to relay the packet. We define the forward scheme, F, as a function on V, where $F(v)$ is the set of forward channels at node v, i. e., those channels that node v uses to relay broadcast packets. For any two nodes $u, v \in V$, we say v is reachable directly from u under forward scheme F, if $u = v$ or $(u, v) \in E$ and $F(u) \cap B(v) \neq \emptyset$. The energy cost of the forward scheme is defined as follows:

$$W(F) = \sum_{v \in F} |F(v)| \cdot p(v).$$

Given a source node s, we need to find a forward scheme such that there is a broadcast tree under the forward scheme and the energy cost of the forward scheme is minimized. Note that in a broadcast tree, the forward set of any leaf node must be empty set. Therefore, our problem can be formally stated as the follows:

> Broadcast in MR-MC Network: Consider a graph $G = (V, E)$ and a source node s. Each node v is associated with a set $B(v)$ of radio channels and a weight $p(v)$. The problem is to assign each node v with a subset $F(v)$ of $B(v)$ such that there exists a broadcast tree T from source s satisfying condition that for each edge $(u, v) \in T, F(u) \cap B(v) \neq \emptyset$, and $W(T) = \sum_{v \in NL(T)} |F(v)| \cdot p(v)$ is minimized, where $NL(T)$ is the set of internal nodes of T.

IV. Transformation

Our problem can be transformed into the following weighted arborescence problem.

> Min Node-Weight Arborescence: Given a node-weighted directed graph and a source node, find an arborescence rooted at the source node and to minimize the total weight of internal nodes other than the source node.

Let us describe the transformation. Construct an auxiliary graph $G_{aux} = (V_{aux}, E_{aux})$ based on the input directed graph $G = (V, E)$ of Broadcast in MR-MC Network as follows.

(1) Create a source node s^* with weight $p(s^*) = 0$.

(2) For each node v, create $|B(v)|$ nodes (v, i) with weight $p(v, i) = p(v)$ for $i \in B(v)$.

(3) Connect s^* to every (s, i) where s is the input source node of Broadcast in MR-MC Network.

(4) For each $(u, v) \in E$, add arcs $((u, i), (v, j))$ for $i \in B(u)$, $j \in B(v)$ and $i \in B(v)$, i.e., for any $(u, v) \in E$ and $i \in B(u) \cap B(v)$, we add arcs from (u, i) to any (v, j) for $j \in B(v)$.

That is:

$V_{aux} = \{s^*\} \cup \cup_{v \in V} \{(v, j) \mid j \in B(v)\}$ with $p(s^*) = 0$, and $p((v, j)) = p(v)$ for $j \in B(v)$.

$E_{aux} = \{(s^*, (s, i)) | i \in B(s)\} \cup \cup_{(u,v) \in E}$ $\{((u, i), (v, j)) | (u, v) \in E, i \in B(u) \cap B(v), j \in B(v)\}$.

There is an example for the transformation shown in Fig. 1. Fig. 1(a) shows input graph of Broadcast in MR-MC Network. Fig. 1(b) shows the corresponding transformed graph of Min Node-Weight Arborescence.

In the following, we will prove that Broadcast in MR-MC Network is equivalent to Min Node-Weight Arborescence with input directed graph $G_{aux} = (V_{aux}, E_{aux})$ with weight $p(v, i) = p(v)$ and $p(s^*) = 0$.

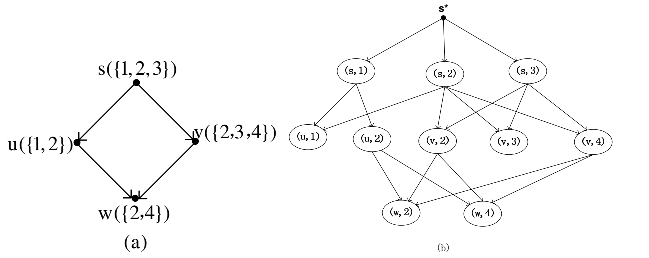


Fig. 1. **An example of transformation**

Lemma 4.1: There exists a forward scheme F with cost at most W containing a broadcast tree from node s if and only if G_{aux} contains an arborescence with weight at most W from node s^*.

Proof: First, suppose forward scheme F contains a broadcast tree T. We construct an arborescence T_{aux} for G_{aux} as follows:

Initially, Set $T_{aux} = \emptyset$.

(1) For each $i \in B(s)$, add arc $(s^*, (s, i))$ to T_{aux}.

(2) For each arc (u, v) in T, choose a channel i from $F(u) \cap B(v)$ and add arcs $((u, i), (v, j))$ to T_{aux} for every $j \in B(v)$.

From the construction above, it is easy to see that for any path from s to a node v, T_{aux} contains a path from s^* to node (v, j) for any $j \in B(v)$. Therefore, T_{aux} being an arborescence follows from T being a broadcast tree. Moreover, only in the case that arc (u, v) exists (i.e., u is an internal node of T) and $i \in F(u)$, (u, i) can be an internal node of T_{aux}. Therefore, the total weight of internal nodes other than s^* in T_{aux} is at most $\sum_{u \in NL(T)} |F(u)| \cdot p(u) \leq W$.

Next, suppose G_{aux} contains an arborescence T_{aux} with weight at most W. For each $u \in V$, define

$$F(u) = \{i \mid ((u, i), (v, j)) \text{ in } T_{aux}\}.$$

We show that there exists a broadcast tree T such that for any arc (u, v) in T, $F(u) \cap B(v) \neq \emptyset$. To do so, let T consist of arcs

$$\{(u, v) \mid ((u, i), (v, j)) \text{ in } T_{aux}\}.$$

By definition of G_{aux}, we know $i \in F(u) \cap B(v)$ and hence $F(u) \cap B(v) \neq \emptyset$. We claim that T is a broadcast tree. In fact, for each node $v \in V$, T_{aux} contains a path from s^* to (v, j) for any $j \in B(v)$ since T_{aux} is an arborescence. This path would induce a path from s to v. Finally, we note that $i \in F(u)$ if and only if (u, i) is an internal nodes of T_{aux}. Therefore, $W(F) \leq \sum_{(u,i) \in NL(T_{aux})} p(u) \leq W$. ■

Lemma 4.2: The minimum weight for a forward scheme F to contain a broadcast tree is *opt* if and only if the minimum cost of an arborescence in G_{aux} is *opt*.

Proof: It follows immediately from Lemma 4.1. ■

Lemma 4.3: There is a polynomial-time ρ-approximation for BROADCAST IN MR-MC NETWORK if and only if there is a polynomial-time ρ-approximation for MIN NODE-WEIGHT ARBORESCENCE on input G_{aux} with weight $p(u, i) = p(u)$.

Proof: Let *opt* be the minimum cost of forward scheme to contain a broadcast tree. By Lemma 4.2, *opt* is also the the minimum cost of an arborescence in G_{aux}.

Suppose an arborescence T_{aux} with cost at most $\rho \cdot opt$ for G_{aux} can be computed in polynomial-time. Then from the Proof of Lemma 4.1, we know that a forward scheme F, which contains a broadcast tree and has cost at most $\rho \cdot opt$, can be constructed in polynomial-time. This means that if there is a polynomial-time ρ-approximation for MIN NODE-WEIGHT ARBORESCENCE on input G_{aux} with weight $p(u, i) = p(u)$, then there is a polynomial-time ρ-approximation for BROADCAST IN MR-MC NETWORK.

Conversely, suppose a forward scheme F contains a broadcast tree T, then we claim that such a T can be constructed in polynomial-time from F. After T is constructed, we can construct T_{aux} in the way as shown in the proof of Lemma 4.1. This means that if there is a polynomial-time ρ-approximation for BROADCAST IN MR-MC NETWORKon input instance G, then there is a polynomial-time ρ-approximation for MIN NODE-WEIGHT ARBORESCENCE on input G_{aux}.

Now, we show how to construct a broadcast tree T from forward scheme F. Consider the following algorithm:

$Y \leftarrow \{s\}$;
$Z \leftarrow V - \{s\}$;
$T \leftarrow \emptyset$;
while $Z \neq \emptyset$ **do begin**
  choose $u \in Y$;
  $Y \leftarrow Y - \{u\}$;
  for every $v \in Z$ **do**
    if $F(u) \cap B(v) \neq \emptyset$
    then $Y \leftarrow Y \cup \{v\}$
      $T \leftarrow T \cup (u, v)$ and
      $Z \leftarrow Z - \{v\}$;
end-while;
output T.

Since F contains a broadcast tree, for each node v there is a path from s to v such that every arc (u, w) on the path satisfies condition $F(u) \cap B(w) \neq \emptyset$. Therefore, every node v in this algorithm would be eventually removed from Z, that is, the algorithm will end in polynomial-time with $Z = \emptyset$, and output a broadcast tree T. ■

V. APPROXIMATION ALGORITHM

Let $G = (V, E)$ be a directed graph with nonnegative weight function p on V. Node $v \in V$ is called a sink node if its out-degree is zero, and an internal node otherwise. Suppose H is a subgraph of G. The set of internal nodes in H is denoted by $I(H)$. H is said to be an arborescence rooted at a node s(or a s-arborescence) if the in-degree of s is zero, and the in-degree of other node is exactly one. H is said to be a spanning subgraph of G if the node set of H is exactly V. For any subset $U \subset V$, G_U denotes the subgraph induced by U.

Li *et al.* [25] gave a $1.5 \ln(n)$-approximation algorithm for unweighted case of MIN NODE-WEIGHT ARBORESCENCE. In this section we are going to present a $(1.35 \ln n)$-

approximation algorithm for MIN NODE-WEIGHT ARBORESCENCE. First we introduce some notations and terminologies.

Let s be the source node. For any subset $S \subset V$ containing source node s, let $\tilde{G}_S = (V, \tilde{E}_S)$ be the spanning subgraph of G, where an arc $(u, v) \in E$ is an arc in $\tilde{E}_S$ if and only if u is in S. A strongly connected component O of $\tilde{G}_S$ is said to be an *orphan* with respect to S if the following two conditions are satisfied: O does not contain the source node s; and for any arc (u, v) in $\tilde{G}_S$, if v is in O, then u is also in O, i.e. $\tilde{G}_S$ does not contain incoming arc toward O. We use $O(S)$ to denote the set of orphans with respect to S. It is easy to check that $\tilde{G}_S$ contains a spanning s-arborescence if and only if $O(S) = \emptyset$.

Note that in the following discussion when we say "orphan", it is always related to some node set S.

Definition 5.1: For some node set S, an arborescence T rooted at a node $u \neq s$ is said to be an $(m-1)$-arborescence w.r.t. S, if it intersects with exactly $m-1$ orphans. An arborescence T is called a legal $m+$ arborescence w.r.t. S if it only contains i-arborescence with $i < m$ as its subtree, and either it is rooted at s or it is intersecting with at least m orphans w.r.t. S.

For a legal $m+$ arborescence T w.r.t. S, define the quotient cost w.r.t. S as follows:

$$c_S(T) = \frac{\sum_{v \in \{I(T) \setminus \{S\}\}} p(v)}{Norphans},$$

where $Norphans$ represents the number of orphans w.r.t. S that T intersects with.

Lemma 5.1: For any B, suppose $|O(B)| \geq m$ at some point, then any spanning s-arborescence can be decomposed into node disjoint legal $m+$ arborescences w.r.t B.

Proof: This lemma can be proved by induction on the number of orphans.

Consider a spanning s-arborescence T. T is a tree intersecting with all the orphans. The depth of a node is the distance of the node from s. Choose a node v of maximum depth, such that the subtree rooted at v intersects with at least m orphans in $O(B)$. Note that no subtree rooted at a proper descendant of v intersects with m orphans, hence the subtree rooted at v is indeed a legal $m+$ arborescence w.r.t B. Delete the subtree rooted at v, and delete the orphans intersecting with it from $O(B)$. The remaining part is also a s-arborescence. If it still intersects with at least m orphans, then by the induction hypothesis we can find a decomposition of the remaining arborescence into legal $m+$ arborescence w.r.t. B and we are done. If there are at most $m-1$ orphans left in $O(B)$, then the remaining part which contains s is the last arborescence rooted at s. This concludes the proof. ■

According to Lemma 5.1, the optimal solution OPT can be decomposed into node-disjoint arborescences. Thus we can get an approximate solution by adding arborescence of low weight iteratively. This process is informally described as in Algorithm 1.

We have the following result.

Algorithm 1 Greedy Algorithm

$B \leftarrow \{s\}$;
$O(B) \leftarrow \{\{v\} | v \in V \setminus \{s\}\}$;
while $O(B) \neq$ **do**
  Selecting a legal $m+$ arborescence T w.r.t. B with minimum quotient cost $c_B(T)$;
  $B = B \cup I(T)$;
  Update $O(B)$;
end while
Return an s-arborescence in $\tilde{G}_B$.

Lemma 5.2: Let OPT be the optimal solution, i.e. OPT is an arborescence with minimum weight $\omega(OPT)$. Suppose that $|O(B)| > 0$, then there exists a legal $m+$ arborescence T w.r.t. B such that

$$c_B(T) \leq \frac{\omega(OPT)}{|O(B)|}.$$

Proof: Since OPT is a spanning s-arborescence, it can be decomposed into legal $m+$ arborescences according to Lemma 5.1. Denote the arborescences in this decomposition by $\{T_1^*, T_2^*, ..., T_k^*\}$, and denote the number of orphans intersecting T_i^* by n_i. Clearly we have that $\sum_{i=1}^k n_i = |O(B)|$. Let T be the legal $m+$ arborescence with minimum quotient cost, i.e. T is what the algorithm selects at this step then

$$c_B(T) \leq c_B(T_i^*) \leq \frac{\sum_{v \in \{I(T_i^*) \setminus \{B\}\}} p(v)}{n_i}, i = 1, 2, ..., k,$$

so

$$n_i c_B(T) \leq \sum_{v \in \{I(T_i^*) \setminus \{B\}\}} p(v), i = 1, 2, ..., k.$$

Summing these inequalities up we get

$$\begin{aligned} (\textstyle\sum_{i=1}^k n_i) c_B(T) &\leq \textstyle\sum_{i=1}^k \sum_{v \in \{I(T_i^*) \setminus \{B\}\}} p(v) \\ &\leq \textstyle\sum_{v \in \{I(\bigcup_{j=1}^k T_j^*) \setminus \{B\}\}} p(v) \\ &\leq \omega(OPT). \end{aligned}$$

Thus,

$$c_B(T) \leq \frac{\omega(OPT)}{\sum_{i=1}^k n_i} \leq \frac{\omega(OPT)}{|O(B)|}.$$

■

Theorem 5.3: If the best ratio legal $4+$ arborescence can be computed in polynomial time, then Algorithm 1 has approximation ratio $\frac{m}{m-1} H(n-1)$.

Proof: Suppose the algorithm runs for k iterations. At the beginning of iteration i, the set of internal nodes is denoted by B_i, and the legal $m+$ arborescence selected in that iteration is denoted by T_i. Also for convenience sake let B_0 be the empty set and $O(B_0) = V \setminus \{s\}$. Let l_i be the number of orphans w.r.t B_i intersecting with T_i in iteration i, and $\omega_{B_i}(T_i)$ be $\sum_{v \in \{I(T_i) \setminus \{B\}\}} p(v)$. By Lemma 5.2, we have for each $1 \leq i \leq k$,

$$\frac{\omega_{B_i}(T_i)}{l_i} \leq \frac{\omega(OPT)}{|O(B_i)|}.$$

Since the iteration k is the last iteration, $|O(B_k)| = 0$, $|O(B_{k-1})| = l_{k-1}$, so we have,

$$\omega_{B_i}(T_i) \leq \omega(OPT).$$

At iteration i, the arborescence T_i rooted at u_i is selected. Any orphan intersecting with T_i either still survives as a connected component in $\tilde{G}_{B_i \bigcup I(T_i)}$ but is not an orphan anymore, or get merged into some new connected components of $\tilde{G}_{B_i \bigcup I(T_i)}$. Furthermore, each new component of $\tilde{G}_{B_i \bigcup I(T_i)}$ which does not contain u_i can not be an orphan.

So, if $u \neq s$, we have

$$|O(B_{i+1})| \leq |O(B_i)| - (l_i - 1),$$

consequently

$$|O(B_i)| - |O(B_{i+1})| \geq (l_i - 1) \geq \frac{m-1}{m} l_i.$$

If $u = s$, then the component of $\tilde{G}_{B_i \bigcup I(T_i)}$ containing u is not an orphan, and

$$|O(B_i)| - |O(B_{i+1})| = l_i \geq \frac{m-1}{m} l_i.$$

Combine the previous discussions together, we have,

$$\begin{array}{rcl} \omega_{B_i}(T_i) & \leq & \frac{l_i}{|O(B_i)|}\omega(OPT) \\ & \leq & \frac{m/(m-1)(|O(B_i)|-|O(B_{i+1})|)}{|O(B_i)|}\omega(OPT). \end{array}$$

Sum these inequalities up over all i we get the cost of the solution outputted by the algorithm is:

$$\begin{array}{rcl} \sum_{i=0}^{k-1} \omega_{B_i}(T_i) & \leq & \frac{m}{m-1}\omega(OPT)\sum_{i=0}^{k} \frac{|O(B_i)|-|O(B_{i+1}|}{|O(B_i)|} \\ & \leq & \frac{m}{m-1}\omega(OPT)\sum_{j=1}^{|V|-1}\frac{1}{j} \\ & = & \frac{m}{m-1}H(n-1)|OPT|. \end{array}$$

■

So the problem is how to find the optimal m+ arborescence at each step. For $m = 2$, Li *et al.* [25] present an algorithm to find such a structure and they show that for $m = 3$ there also exists a polynomial time algorithm [6]. However we do not know how to find a m+ arborescence optimally when $m \geq 4$. For the case where $m = 4$, we can have an algorithm to approximate this structure, which gives an algorithm with ratio $(1.35 + \epsilon)\ln(n)$ using the greedy scheme described above.

Thus the complete algorithm is presented in Algorithm 2. In this algorithm to work, $\omega(OPT)$ is the weight of an optimal solution. However we do not know the exact value, but only an upper bound instead, so we have to guess the optimal weight approximately and run Algorithm 2 for each possible value.

Suppose B_i is the set of internal nodes at the start of iteration i and T_i is the legal 4+ arborescence selected at iteration i w.r.t. B_i. For convenience let $B_0 = \emptyset$, which means that $O(B_0) = V \setminus \{s\}$. Let $\omega_{B_i}(T_i) = \sum_{v \in \{I(T_i) \setminus \{B\}\}} p(v)$, and l_i be the number of orphans intersecting with T_i. Then we can prove the following result.

Theorem 5.4: At iteration i,

$$\omega_{B_i}(T_i) \leq \delta \frac{1+\epsilon}{1-\epsilon} \cdot \frac{|O(B_i)| - |O(B_{i+1})|}{|O(B_i)|} \cdot \omega(OPT),$$

Algorithm 2 $(1.35 + \epsilon'')(1 + \ln(n-1))$-Algorithm

INPUT: $\epsilon > 0$;
OUTPUT: An arborescence with approximation ratio $(1.35 + \epsilon'')(1 + \ln(n-1))$.
$\delta = 1.35$, $Q = 1/\epsilon$;
$B \leftarrow \{s\}$;
$O(B) \leftarrow \{\{v\}|v \in V \setminus \{s\}\}$;
while There are more than $3Q$ orphans left **do**
  $\omega = \frac{\omega(OPT)}{|O(B)|}$;
  (Step 1)Compute the 2+ arborescence $T^{(1)}$ with minimum quotient cost ρ_1;
  (Step 2)Compute the 3+ arborescence $T^{(2)}$ with minimum quotient cost ρ_2;
  for $j = 0$ to Q **do**
    Compute the best ratio legal 4+ arborescence $T_j^{(3)}$ with exactly j 3-arborescence attached to it; {This can be done by enumerating all possible j sets of 3 orphans because Q is a fixed value.}
  end for
  (Step 3)Denote the arborescence with the smallest quotient cost among all $T_j^{(3)}$ by $T^{(3)}$, and let the quotient cost be ρ_3;
  Compute the approximation of the best ratio legal 4+ arborescence that intersects with at least Q orphans as follows:
  Set $A = \emptyset$; {A stores all the legal 4+ arborescence that intersects with at least Q orphans to be constructed}
  for each vertex v **do**
    fix v as the root;
    while The number of marked orphan is less than Q **do**
      Pick the lightest 3-arborescence with no marked orphans, and mark the three orphans;
    end while
    Add the arborescence constructed to set A;
  end for
  (Step 4)Denote best ratio arborescence in A by $T^{(4)}$, and let the quotient cost be ρ_4;
  (Step 5)If $2\rho_1 \leq \delta\omega(1.5\rho_2 \leq \delta\omega)$ then let $T_0 = T^{(1)}(T_0 = T^{(2)})$. Else let T_0 be $T^{(3)}$ or $T^{(4)}$, whichever achieves the minimum in $\min(\delta\rho_3, \rho_4)$;
  $B = B \cup I(T_0)$, calculate $O(B)$;
end while
Connect the remaining orphans optimally;
Let B be the set of internal nodes selected by this algorithm and T be an s-arborescence.
Return T;

for any $\epsilon > 0$.

Proof: We divide the proof into two parts.

Lemma 5.5: If T_i is selected in Step 1 or Step 2, then

$$\omega_{B_i}(T_i) \leq \delta \cdot \frac{|O(B_i)| - |O(B_{i+1})|}{|O(B_i)|} \cdot \omega(OPT).$$

Proof: Assume T_i is generated in Step 1. In this case, $l_i \geq 2$, so we have $|O(B_i)| - |O(B_{i+1})| \geq l_i/2$. From the condition in step 5 we know that

$$2\rho_1 \leq \delta\omega,$$

which is

$$\frac{\omega_{B_i}(T_i)}{|O(B_i)| - |O(B_{i+1})|} \leq 2\frac{\omega_{B_i}(T_i)}{l_i} \leq \delta\frac{\omega(OPT)}{|O(B_i)|}.$$

Simplifying we have

$$\omega_{B_i}(T_i) \leq \delta \cdot \frac{|O(B_i)| - |O(B_{i+1})|}{|O(B_i)|} \cdot \omega(OPT).$$

Now suppose T_i is generated in Step 2. In this case, $l_i \geq 3$, so we have $|O(B_i)| - |O(B_{i+1})| \geq 2/3l_i$. From the condition in step 5 we know that

$$1.5\rho_2 \leq \delta\omega,$$

which is

$$\frac{\omega_{B_i}(T_i)}{|O(B_i)| - |O(B_{i+1})|} \leq 1.5\frac{\omega_{B_i}(T_i)}{l_i} \leq \delta\frac{\omega(OPT)}{|O(B_i)|}.$$

Simplifying we have

$$\omega_{B_i}(T_i) \leq \delta \cdot \frac{|O(B_i)| - |O(B_{i+1})|}{|O(B_i)|} \cdot \omega(OPT).$$

■

Lemma 5.6: If the first two conditions in Step 5 are not met, i.e., T_i is generated in Step 3 or Step 4, and the best ratio legal 4+ arborescence has more than Q 3-arborescences, then

$$\rho_4 \leq \delta(1+\epsilon)\omega.$$

Proof: Suppose that T_i^* is a best ratio legal 4+ arborescence at iteration i, and T_i^* contains more than Q 3-arborescences. Let $\omega_3(n_3)$ be the total weight of internal nodes of the 3-arborescences (number of 3-arborescence), and $\omega_2(n_2)$ be the total weight of internal nodes of the 2-arborescences (number of 2-arborescence). Note that single branch can be paired up and treated as a 2-arborescence. Let ω_1 be the total weight of internal nodes of the single branch (if one exists). Since the quotient cost is at most ω, we have

$$\omega_3 + \omega_2 + \omega_1 \leq (3n_3 + 2n_2 + 1)\omega. \tag{1}$$

We can view a 3-arborescence as a 2-arborescence by dropping an orphan. This makes it a legal 3+ arborescence. If the quotient of the legal 3+ arborescence is at most $\delta\omega$, then we can simply use this legal 3+ arborescence with quotient cost $\rho_4 \leq \delta\omega$. The difficult case is when after converting to a legal 3+ arborescence, the quotient cost is at least $\delta\omega$. Thus we can assume that

$$\omega_3 + \omega_2 + \omega_1 \geq (2n_3 + 2n_2 + 1)\delta\omega. \tag{2}$$

Combine these two inequalities together, we get

$$(3 - 2\delta)n_3 - 2(\delta - 1)n_2 - (\delta - 1) \geq 0. \tag{3}$$

By taking three 2-arborescences of T_i^* and duplicating the paths to the orphans in the cheapest 2-arborescence, we can convert them into two 3-arborescences. This increases the total weight by a factor of at most 4/3. Now we have at least $n_3 + 2/3n_2$ 3-arborescences.

Now the algorithm greedily picks 3-arborescence. We can pick at least $n_3 + 2/3n_2$ orphans in this manner, and this can be a candidate. Since each time we greedily pick a 3-arborescence that belong to three distinct 3-arborescence in the best ratio legal 4+ arborescence, therefore we might pick one orphan from each of the $n_3 + 2/3n_2$ 3-arborescences(the $n_3 + 2/3n_2$ 3-arborescences contains $3(n_3 + 2/3n_2)$ and have weight at most $\omega_3 + \frac{4\omega_2}{3}$). Because $n_3 + 2/3n_2 > Q$, we derive an upper bound on the weight of 4+ arborescence generated at this step.

Thus the legal 4+ arborescence T_i has weight at most

$$\frac{1}{3}(\omega_3 + \frac{4\omega_2}{3}) + \omega_1 \leq \frac{1}{3}(\omega_3 + \omega_2 + \omega_1) + \frac{1}{9}(\omega_2 + \omega_1) + \frac{5}{9}\omega_1. \tag{4}$$

And since the first two conditions of Step 5 are not satisfied, which means that $\omega_2/(2n_2) \geq \delta\omega/2$, and $\omega_3/(3n_3) \geq 2\delta\omega/3$. Substituting ω_2 and ω_3 in Equation 4 we get

$$\begin{aligned} \tfrac{1}{3}(\omega_3 &+ \tfrac{4\omega_2}{3}) + \omega_1 \\ &\leq \tfrac{1}{3}(3n_3 + 2n_2 + 1)\omega \\ &\quad + \tfrac{1}{9}((3n_3 + 2n_2 + 1)\omega - 2n_3\delta\omega) \\ &\quad + \tfrac{5}{9}((3n_3 + 2n_2 + 1)\omega - 2n_3\delta\omega - n_2\delta\omega) \\ &\leq \omega(3n_3 + 2n_2 - \tfrac{4\delta}{3}n_3 - \tfrac{5\delta}{9}n_2 + 1). \end{aligned} \tag{5}$$

Multiplying Equation 3 by $\omega/2$ and adding to Equation 5, we get that the total weight is at most

$$\omega((\frac{9}{2} - \frac{7\delta}{3})(n_3 + \frac{2}{3}n_2) + \frac{3}{2} - \delta). \tag{6}$$

Also since $n_3 + \frac{2}{3}n_2 \leq l_i$ and $n_3 > Q = 1/\epsilon$, we have

$$\begin{aligned} \rho_4 &\leq \frac{\omega((\frac{9}{2} - \frac{7\delta}{3})(n_3 + \frac{2}{3}n_2) + \frac{3}{2} - \delta)}{l_i} \\ &\leq \frac{\omega((\frac{9}{2} - \frac{7\delta}{3})(n_3 + \frac{2}{3}n_2) + \frac{3}{2} - \delta)}{n_3 + \frac{2}{3}n_2} \\ &\leq \omega((\frac{9}{2} - \frac{7\delta}{3}) + \frac{(\frac{3}{2} - \delta)}{n_3}) \\ &\leq \omega((\frac{9}{2} - \frac{7\delta}{3}) + (\frac{3}{2} - \delta)\epsilon) \end{aligned} \tag{7}$$

Set $\delta = 1.35$, we have the following result

$$\rho_4 \leq \delta(1+\epsilon)\omega.$$

■

Now we come back to the proof of Theorem 5.4. If T_i is selected from Step 1 or Step 2, we can use Lemma 5.5 to prove this claim. Otherwise, we consider the following two cases.

1) Suppose T_i^* contains no more than Q 3-arborescences,

- $\delta\rho_3 \le \rho_4$, then T_i is from Step 3 which is a legal 4+ arborescence with minimum quotient cost. Therefore $\rho_3 \le \omega$. Since $l_i \ge 4$, we can get that

$$\begin{aligned}\omega_{B_i}(T_i) &\le l_i \cdot \frac{\omega(OPT)}{|O(B_i)|} \\ &\le \frac{4}{3} \cdot \frac{|O(B_i)|-|O(B_{i+1})|}{|O(B_i)|} \cdot \omega(OPT).\end{aligned} \tag{8}$$

- $\rho_4 \le \delta\rho_3 (\le \delta\omega)$, then T_i is from Step 4 in which case $l_i \ge Q$. So we can get that

$$\begin{aligned}\omega_{B_i}(T_i) &\le \delta \cdot l_i \cdot \frac{\omega(OPT)}{|O(B_i)|} \\ &\le \frac{\delta}{1-\epsilon} \cdot \frac{|O(B_i)|-|O(B_{i+1})|}{|O(B_i)|} \cdot \omega(OPT).\end{aligned} \tag{9}$$

2) Suppose T_i^* contains more than Q 3-arborescences,

- $\delta\rho_3 \le \rho_4$. By Lemma 5.6 we know that $\delta\rho_3 \le \rho_4 \le \delta(1+\epsilon)\omega$. So we have

$$\begin{aligned}\omega_{B_i}(T_i) &\le (1+\epsilon) \cdot l_i \cdot \frac{\omega(OPT)}{|O(B_i)|} \\ &\le \frac{4}{3}(1+\epsilon) \cdot \frac{|O(B_i)|-|O(B_{i+1})|}{|O(B_i)|} \cdot \omega(OPT).\end{aligned} \tag{10}$$

- $\rho_4 \le \delta\rho_3$, then T_i is from Step 4 in which case $l_i \ge Q$. By Lemma 5.6 we know that $\rho_4 \le \delta(1+\epsilon)\omega$. So we have

$$\begin{aligned}\omega_{B_i}(T_i) &\le \delta(1+\epsilon) \cdot l_i \cdot \frac{\omega(OPT)}{|O(B_i)|} \\ &\le \delta\frac{1+\epsilon}{1-\epsilon} \cdot \frac{|O(B_i)|-|O(B_{i+1})|}{|O(B_i)|} \cdot \omega(OPT).\end{aligned} \tag{11}$$

This concludes the proof of Theorem 5.4. ■

Theorem 5.7: BROADCAST IN MR-MC NETWORK can be approximated within $(1.35+\epsilon'')(1+\ln(n-1))$, for $\epsilon'' > 0$.

Proof: The total number of internal nodes output by the algorithm is to sum up all the $\omega_{B_i}(T_i)$, which is $\delta\frac{1+\epsilon}{1-\epsilon} \cdot H(|V|-1) \cdot \omega(OPT)$. Set $\epsilon = \frac{\epsilon'}{2+\epsilon'}$, then $\frac{1+\epsilon}{1-\epsilon} = 1+\epsilon'$. Also note that $\delta = 1.35$, so $\delta\frac{1+\epsilon}{1-\epsilon} \cdot H(|V|-1) = 1.35(1+\epsilon')H(n-1)$. Since $1.35(1+\epsilon')H(n-1) \le (1.35+\epsilon'')(1+\ln(n-1))$, so we obtain an approximation algorithm with approximation ratio $(1.35+\epsilon'')(1+\ln(n-1))$, where $\epsilon'' > 0$. ■

VI. IMAPPROXIMABILITY

Let us first consider the following problem:

> SET-COVER: Given a finite set X and a collection $\mathcal{C}$ of subsets of X, find a minimum set cover where a set cover is a subcollection $\mathcal{A} \subseteq \mathcal{C}$ such that $\cup_{A\in\mathcal{A}} A = X$.

About SET-COVER, Feige [26] showed the following result on imapproximability.

Lemma 6.1: SET-COVER has no polynomial-time $\rho \ln n$-approximation for $0 < \rho < 1$ unless $NP \subseteq DTIME(n^{O(\log\log n)})$ where $n = |X|$.

In this section, we will show the following result for BROADCAST IN MR-MC NETWORK.

Theorem 6.2: BROADCAST IN MR-MC NETWORK has no polynomial-time $\rho \ln n$-approximation for $0 < \rho < 1$ unless $NP \subseteq DTIME(n^{O(\log\log n)})$.

Proof. Consider an instance of SET-COVER, a finite set X and a collection $\mathcal{C}$ of subsets of X. We construct an instance of

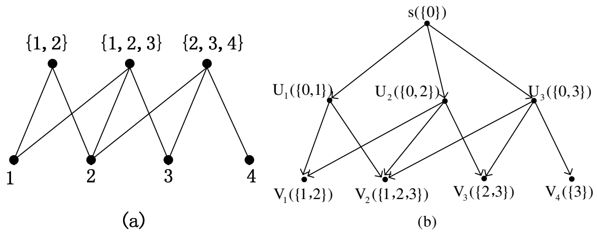


Fig. 2. **An example of transformation**

BROADCAST IN MR-MC NETWORK with $p(v) = 1$ for every node v as follows:

Let $V = \{s\} \cup \{U_i \mid S_i \in \mathcal{C}\} \cup \{V_x \mid x \in X\}$ and $E = \{(s, U_i) \mid S_i \in \mathcal{C}\} \cup \{(U_i, V_x) \mid x \in A\}$ (Fig. 2). Suppose $\mathcal{C} = \{S_1, S_2, ..., S_m\}$. Define a node label assignment $A : V \to 2^{Z^+}$ by setting

$$\begin{aligned} A(s) &= \{0\}, \\ A(U_i) &= \{0, i\} \text{ for } S_i \in \mathcal{C}, \\ A(V_x) &= \{i \mid x \in S_i\} \text{ for } x \in X. \end{aligned}$$

Suppose BROADCAST IN MR-MC NETWORK has a polynomial-time $(\rho \ln n)$-approximation for some $0 < \rho < 1$. Let $F : V \to 2^{Z^+}$ be such an approximation solution on the instance constructed as above. Define $\mathcal{A} = \{S_i \mid i \in F(U_i)\}$. Since each V_x can only be reached from some U_i with $x \in S_i$, there must exist $S_i \ni x$ such that $i \in F(U_i)$. This means that $\mathcal{A}$ is a set cover. Suppose $\mathcal{A}^*$ is a minimum set cover. Define $F' : V \to 2^{Z^+}$ by setting

$$\begin{aligned} F'(s) &= \{0\}, \\ F'(U_i) &= \{i\} \text{ if } S_i \in \mathcal{A}^*, \\ F'(U_i) &= \emptyset \text{ if } S_i \notin \mathcal{A}^*, \\ F'(V_x) &= \emptyset \text{ for all } x \in X. \end{aligned}$$

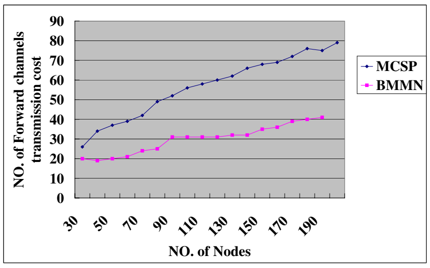
Clearly, F' is a solution of B-DM and hence $|F| \le (\rho \ln n)|F'|$. Therefore,

$$\begin{aligned} |\mathcal{A}| &\le |F| - 1 \\ &\le (\rho \ln n)|F'| - 1 \\ &= (\rho \ln n)(|\mathcal{A}^*| + 1) - 1 \\ &\le (\rho' \ln n)|\mathcal{A}^*|. \end{aligned}$$

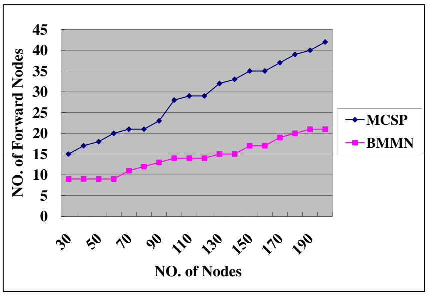
for any $0 < \rho' < \rho < 1$ and sufficiently large $|\mathcal{A}^*|$. Therefore, SET-COVER has polynomial-time $(\rho' \ln n)$-approximation. Hence, $NP \subseteq DTIME(n^{O(\log\log n)})$. □

VII. SIMULATIONS

In this section, we compare our Algorithm 2(denote by BMMN for short) with the Multi-channel Self-pruning (MCSP) [9]. We want to know the relationship between the number of forward channels and the number of nodes in the network(N), as well as the transmission radius(R). The multiradio multichannel wireless network in the simulation is generated by randomly deploying $N(30 \le N \le 60)$ nodes in

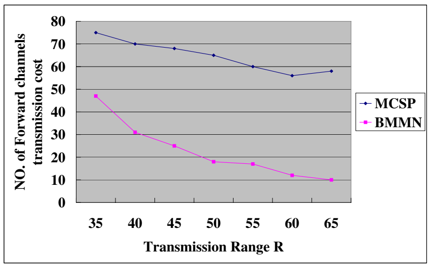


(a) Number of Forward Channels vs. N

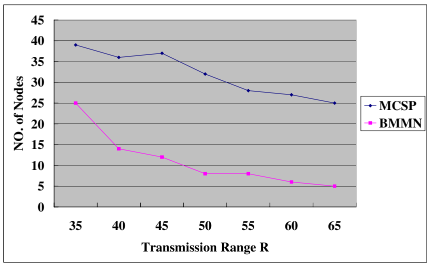


(b) Number of Forward Nodes vs. N

Fig. 3. Varying the number of nodes



(a) Number of Forward Channels vs. R



(b) Number of Forward Nodes vs. R

Fig. 4. Varying R

a 100×100 area. Each node is equipped with m radios and p channels, where m and p are random numbers between 1 and 5 respectively. The transmission radius of each node varies from 35 to 65. We run the algorithms for different N and R.

In our simulation, we set the transmission power level of each node to be some fixed value. Thus, the transmission range of each node is the same, which is a disk with radius R. In this case, total energy consumption in BROADCAST IN MR-MC NETWORKS can be measured by the number of forward channels over all forward nodes (for simplicity, we will call it *the number of forward channels*).

Fig. 3 presents the comparison of BMMN and MCSP in generated number of forward channels and generated number of forward nodes when varying the number of sensors N. The transmission range R is set to be 40, We can see that as the number of sensors increases, the number of forward channels and the number of forward nodes increase, and BMMN requires much less number of forward channels and the number of forward nodes than MCSP respectively.

To examine the influence of transmission range R, we first fix the number of sensors N to be 100. The results are shown in Fig. 4. We can see that as the transmission range increases, the number of forward channels and the number of forward nodes decrease, and BMMN requires much less number of forward channels and the number of forward nodes than MCSP respectively.

The above simulations show our algorithm is better than the algorithm in [9].

VIII. CONCLUSION

We showed that two problems BROADCAST IN MR-MC NETWORKS and MIN NODE-WEIGHT ARBORESCENCE are equivalent, and we also designed polynomial-time $(1.35 + \epsilon)(1 + \ln(n - 1))$-approximation for them. Using the approach in [25], it is easy to see that above result also implies a polynomial-time $(2.7 + \epsilon)(1 + \ln(n - 1))$-approximation for minimum strongly connected dominating set in directed graphs, which is better than $3(1 + \ln n)$-approximation in [25]. We also showed that for $0 < \rho < 1$, there is no polynomial-time $(\rho \ln n)$-approximation for BROADCAST IN MR-MC NETWORKS.

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