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AN IMPROVED COURNOT COMPETITION MODEL: CONSIDERATION OF MARKET SHARE OBJECTIVE

TONGYANG LI

Institute for Interdisciplinary Information Sciences
Tsinghua University
Beijing, P. R. China
ltywjl@gmail.com

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Cournot competition model calculates profit by company's product quantity, but it does not take market share objective into consideration. It has obstacles of considering nonlinear aspects because the partial derivative equations are very hard to solve. Our algorithm both improves Cournot's model with consideration of market share ratio, and avoids the difficulty of solving several partial derivative equations directly. In our algorithm, we give a parameter σ to describe a company's consideration of market share objective, formulate the competition between companies, and calculate new market share ratio using parameter σ. We consider two network structures in our model: complete graph network and star graph network. For complete graph network, our calculation and numerical analysis is based on the case of 3 companies, but the result for general number of companies, n, is also discussed. We give a series of numerical results of new market share ratio after given the initial market share ratio and the parameter σ, and show that the initial market share ratio and new market share ratio are highly linear. Another important property in this network is the convergence of market share ratio when n goes to infinity. For star graph network, we give calculation for general cases and show that $n = 3, 4$ are the only numbers of companies that have positive increase of the market share ratio of the company.

Keywords: Algorithm; Cournot competition model; market share; partial derivative equation.

Mathematics Subject Classification: 68R99

1. Introduction

In traditional analysis in economics, we often use a highly simplified assumption that the aim of companies is to maximize their own profits. But for many cases, this is not the real case. However, when a product is ready to put on the market, the sales strategy will be discussed by managers with different considerations and the factors they consider are very complicated. In 1958, Baumol already realized that managers may be driven by other motives than pure profit-maximization, and he suggested a sales-maximization model as a more realistic alternative [1]. After this, researchers notice that market share may provide a crucial motive for managers. In 1988, Peck reported the empirical findings of a survey in corporate objectives among 1000 American and 1031 Japanese top managers [8]. Increasing market share ranks third in the American and second in the Japanese sub-sample, whereas return on investment is first among American and third among Japanese top managers. Many studies reveal that the consideration of non-profit factor actually does exist in real big companies.

In the paper "A modified Cournot model of the natural gas market in the European Union: Mixed-motives delegation in a politicized environment" written by Thijs Jansen, Arie van Lier, Arjen van Witteloostuijn and Tim Boon von Ochssee [5], the authors analyze the attitude of Gazprom (the Russian energy monopolist) in the European gas market. Paillard [7, p. 16] observes that we need "to take into account... the Russian elite's desire to make the West pay for the humiliation of the 1990s after the collapse of the Soviet Union. This view is not that of a West that is suspicious of Russian power, but actually a reasonable conclusion given (then) President Putin's numerous declarations as well as those of his administration, dressed as they may be in diplomatic language. Energy, the one true weapon in the hands of the Russian authorities, may therefore be seen as an instrument of blackmail, used now and then by the Russian elite to regain its old status, at least in regard to Europe."

The paper translated this attitude into a modeling context: this means that the "utility function" of Gazprom is different from that of, say, Algerian Sonatrach or Norwegian Statoil, the other two big companies in Europe gas market. After the translation, they use a modified Cournot model to analyze the Russian government's non-profit objective.

We are inspired by the background and the model in the paper above. However, the paper did not concentrate on specific non-profit factors, and there is not too much mathematics in the paper to give convincing quantitative result. Our work fix up these issues. We build our model based on Cournot competition model. (To understand the classical Cournot model, please read Sec. 2). In our paper, we consider a general oligopoly market with several monopolists. We choose market share ratio as the non-profit incentive of the utility function of one company. The reasons why we choose market share ratio are:

(1) This is basically one of the most important factors considered by big companies.
(2) We can obtain very interesting result in calculation, as in the following sections of the paper.

Denote n to be the number of monopolists. To begin with, we make following two assumptions: (1) The network is a complete graph; (2) We first assume that

$n = 3$. we are given the initial market share ratio r and the market share consideration parameter σ. Using this two variable, we can formulate our modified Cournot model in Sec. 3. After the description of the model, in Sec. 4, we will explain the mathematics we use in our model, together with the detailed calculation. In Sec. 5, we make a conclusion that r and r_{new} is highly linear. Moreover, after giving a fixed σ, we can calculate this linear function and use it to estimate r_{new}. Even with a reversed procedure, given r and r_{new} from statistics σ can be done from the calculation. In Sec. 6, we discuss the situation of general n, where, n is the number of monopolists, and r_{new} has a limit for given r and σ. In Sec. 7, we use the same method and analysis the star graph network.

2. The Classical Cournot Model

Cournot's competition model has to do with companies trying to decide how much of a homogeneous goods to produce. Imagine that three companies, A, B, C, produce the same type of goods. Each company must decide how much goods they will produce. The problem is that the price of each good is dependent on the total amount of goods produced. If a lot of goods is produced, the market will be glutted and the companies will make no money. Thus, the companies want to optimize their expected earnings. This is basically the idea of Cournot model.

Denote q_A, q_B, q_C to be the quantities that company A, B, C can produce, respectively. Let Q be the sum of q_A, q_B and q_C. Therefore

$$Q = q_A + q_B + q_C.$$

We also need a price function to tell us how much each unit of quantity will sell for. In this case, we assume the price function $P(Q)$ to be linear, i.e., $P(Q) = a - bQ$ for $0 \leq Q \leq \frac{a}{b}$. For $Q > \frac{a}{b}$ we assume that $P(Q) = 0$.

Finally, assume that the cost per product for the three company is c_A, c_B, c_C respectively (they are all fixed positive real numbers). Now, we can give the utility function of A, B, C as following:

$$\begin{cases} \pi_A = q_A(a - bQ - c_A) \\ \pi_B = q_B(a - bQ - c_B) \\ \pi_C = q_C(a - bQ - c_C). \end{cases}$$

According to classical result of the Cournot model, we assume that

$$\begin{cases} \dfrac{\partial \pi_A}{\partial q_A} = 0 \\ \dfrac{\partial \pi_B}{\partial q_B} = 0 \\ \dfrac{\partial \pi_C}{\partial q_C} = 0. \end{cases}$$

After solving these equations, we get the equilibrium point

$$\begin{cases} q_A^* = \dfrac{1}{4b}(a - 3c_A + c_B + c_C) \\ q_B^* = \dfrac{1}{4b}(a + c_A - 3c_B + c_C) \\ q_C^* = \dfrac{1}{4b}(a + c_A + c_B - 3c_C). \end{cases}$$

And the profits now are

$$\begin{cases} \pi_A = b(q_A^*)^2 \\ \pi_B = b(q_B^*)^2 \\ \pi_C = b(q_C^*)^2. \end{cases}$$

These are the basic knowledge we need to know for the classical Cournot competition model. In the next section, we will give our own Cournot model.

3. Description of the Model

What we are going to do now: **company A has consideration on market share ratio**. More specifically, his utility function is now $\pi_A + \alpha M$, where M represents the consideration on market share.

In the paper of the natural gas market, the authors consider the situation that Gazprom also considers another aspect, say, geopolitical power. In the paper, M is a function depends on its own production and the total production of its rivals, i.e., $M = M(Q_R, Q_{-R})$, here $Q_{-R} = Q_A + Q_N$, where Q_A, Q_N represents the quantity of Algerian Sonatrach and Norwegian Statoil respectively, the other two main natural gas companies that influences natural gas market in Europe. However, in the paper the authors did not consider the first partial derivative of $\frac{\partial M}{\partial Q_{-R}}$ in the calculation, so their result is a little bit strange. Our calculation fix this problem, as we will show in Sec. 4.

Formal description: Consider Cournot competition with three companies: A, B, and C. Assume that the cost per product for the three company is c_A, c_B, c_C respectively, as in the classical Cournot model. Without loss of generality, we can assume $b = 1$, i.e., $P(Q) = a - Q$ is the price function. The utility function for A, B, C are:

$$\begin{cases} U(A) = \pi_A + l \cdot \dfrac{q_A}{Q} \cdot (Q^*)^2 = q_A(a - Q - c_A) + \dfrac{\sigma}{2} \cdot \dfrac{q_A}{Q}(Q^*)^2 \\ U(B) = \pi_B = q_B(a - Q - c_B) \\ U(C) = \pi_C = q_C(a - Q - c_C). \end{cases}$$

Here $Q = q_A + q_B + q_C$, Q^* is the sum of quantity at the equilibrium point in the initial Cournot model (without consideration of market share objective), and

σ is the market share consideration parameter stated before. For simplicity, denote $l = \frac{\sigma}{2}$.

The reason why $(Q^*)^2$ is multiplied with $\frac{q_A}{Q}$ is to make π_A and $M = l\frac{q_A}{Q}(Q^*)^2$ into the same level. For simplicity, denote $w = l \cdot (Q^*)^2$.

The reason why $\frac{\sigma}{2}$ is multiplied with $\frac{q_A}{Q}$ is to express the extent that company A considers market share ratio. The reason for the $\frac{1}{2}$ is that $q_A(a - Q - c_A)$ is a function of q_A with quadratic coefficient $-\frac{1}{2}$, therefore we need to multiply by $\frac{1}{2}$.

4. Calculation

To calculate the equilibrium, we need to have

$$\begin{cases} \dfrac{\partial U_A}{\partial q_A} = 0 \\ \dfrac{\partial U_B}{\partial q_B} = 0 \\ \dfrac{\partial U_C}{\partial q_C} = 0 \end{cases}$$

which is equivalent to

$$\begin{cases} (a - c_A) - 2q_A - q_B - q_C + w\dfrac{\partial(q_A/Q)}{\partial q_A} = 0 \\ (a - c_B) - q_A - 2q_B - q_C = 0 \\ (a - c_C) - q_A - q_B - 2q_C = 0. \end{cases}$$

<u>The idea of taking first partial derivative</u>: We do not consider $\frac{\partial q_A}{\partial q_B}$ and $\frac{\partial q_A}{\partial q_C}$, because this will make the calculation impossible to be done and ruin the analysis of Cournot equilibrium. But we DO consider $\frac{\partial q_B}{\partial q_A}$ and $\frac{\partial q_C}{\partial q_A}$. Another explanation is that we can regard q_A as the variable and q_B, q_C are the result of the variance of q_A.

From the second and third equations above, we have

$$\begin{cases} q_B = (a - c_B) - Q \\ q_C = (a - c_C) - Q. \end{cases}$$

Adding these two equations together, we obtain

$$Q = \frac{1}{3}(2a - c_B - c_C + q_A).$$

Therefore

$$\frac{q_A}{Q} = \frac{q_A}{\frac{2a - c_B - c_C}{3} + \frac{1}{3}q_A}.$$

Notice that $q_A^* = \frac{1}{4}(a - 3c_A + c_B + c_C)$ and $Q^* = \frac{1}{4}(3a - c_A - c_B - c_C)$, we have $2a - c_B - c_C = 3Q^* - q_A^*$. Therefore

$$\frac{q_A}{Q} = \frac{q_A}{\frac{2a - c_B - c_C}{3} + \frac{1}{3}q_A} = \frac{3q_A}{3Q^* - q_A^* + q_A}.$$

Denote $c = 3Q^* - q_A^*$. Plugging this into $(a - c_A) - 2q_A - q_B - q_C + w\frac{q_A/Q}{\partial q_A} = 0$ and use $q_A^* = \frac{1}{4}(a - 3c_A + c_B + c_C)$, we have

$$4q_A^* - 4q_A + 9w\frac{c}{(c + q_A)^2} = 0. \tag{$*$}$$

Since $9w\frac{c}{(c+q_A)^2} > 0$ for all q_A, the solution q_A of $(*)$ must have $q_A > q_A^*$. To make $(*)$ more clear, denote $q_A = q_A^* + dQ^*$, here $d > 0$. Doing some calculation, we find that $(*)$ is equivalent to

$$9l\left(3 - \frac{q_A^*}{Q^*}\right) = 4d(3 + d)^2.$$

If we use r to represent the **market share ratio** of **initial** Cournot equilibrium, i.e., $r = \frac{q_A^*}{Q^*}$, we have

$$9\sigma(3 - r) = 8d(3 + d)^2.$$

On the other hand, we have

$$\begin{aligned}\frac{q_A}{Q} &= \frac{3q_A}{3Q^* - q_A^* + q_A}\\ &= \frac{3(q_A^* + dQ^*)}{3Q^* - q_A^* + (q_A^* + dQ^*)}\\ &= \frac{3\frac{q_A^*}{Q^*} + 3d}{3 + d}\\ &= \frac{3r + 3d}{3 + d},\end{aligned}$$

which gives the **market share ratio** of **present** Cournot equilibrium. Therefore, by giving the constant σ in the market share objective of A and the market share ratio r of initial Cournot equilibrium, **we can calculate the market share ratio of present Cournot equilibrium**.

For some specific value, $\sigma = 0.5$ and $\sigma = 1.0$, the result is:

$\sigma = 0.5$:

$$r = 0.1 \quad r_{new} = 0.2495$$

$$r = 0.2 \quad r_{new} = 0.3400$$

$$r = 0.3 \quad r_{new} = 0.4308$$

$$r = 0.4 \quad r_{new} = 0.5219$$

$$r = 0.5 \quad r_{new} = 0.6133$$

$$r = 0.6 \quad r_{new} = 0.7049$$

$\sigma = 1.0$:

$$r = 0.1 \quad r_{new} = 0.3634$$

$$r = 0.2 \quad r_{new} = 0.4475$$

$$r = 0.3 \quad r_{new} = 0.5320$$

$$r = 0.4 \quad r_{new} = 0.6169$$

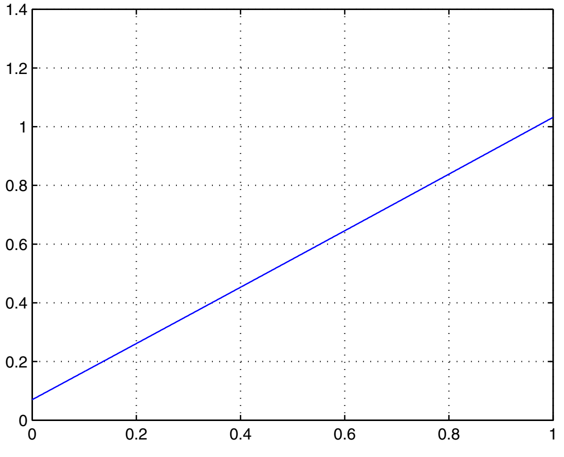
$$r = 0.5 \quad r_{new} = 0.7022$$

$$r = 0.6 \quad r_{new} = 0.7880.$$

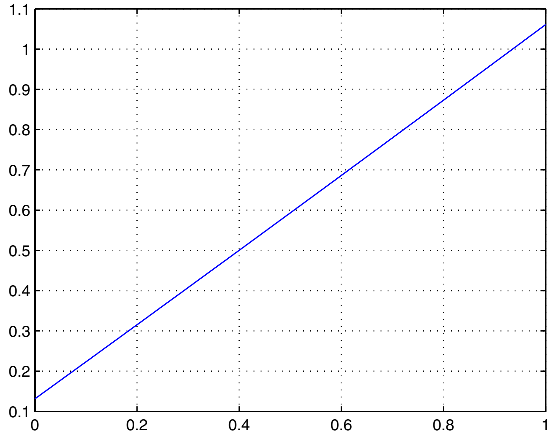
The result is very interesting. For $\sigma = 0.5$, when $r = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6$, the market share ratio of company A is raised by 10–14 percentage points. For $\sigma = 1.0$, when $r = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6$, the market share ratio of company A is raised by 18–26 percentage points. Although company A might sacrifice plenty of profit, the market share ratio is actually raised.

More interesting, the value of r_{new} is highly linear. For some special σ, the relation of r and r_{new} is shown in following graphs:

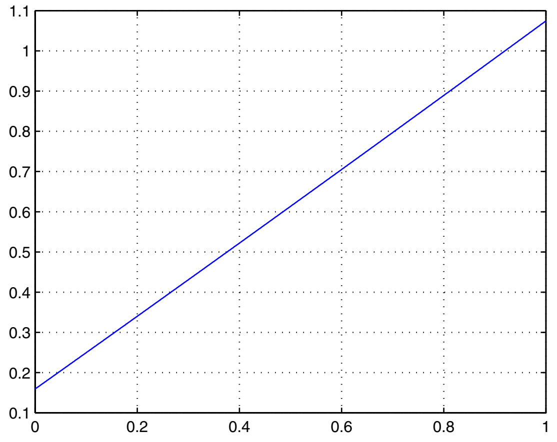
$\sigma = 0.2$:



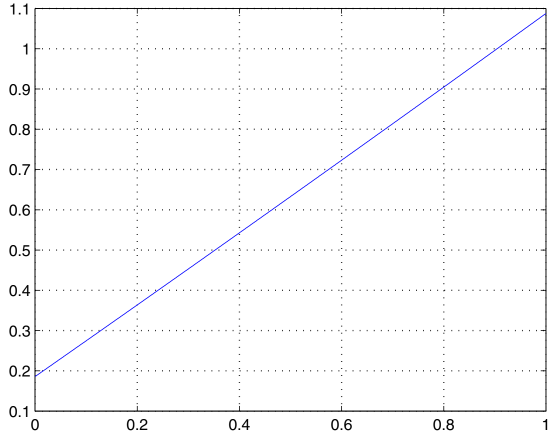
$\sigma = 0.4$:



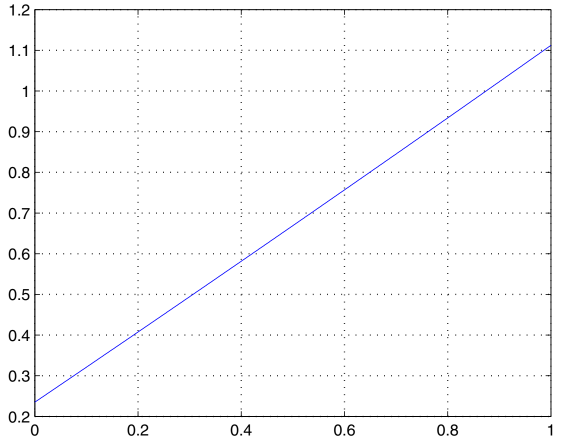
$\sigma = 0.5$:



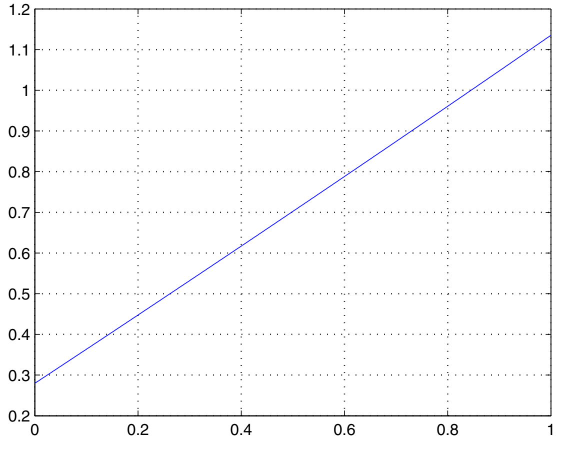
$\sigma = 0.6$:



$\sigma = 0.8$:



$\sigma = 1.0$:



Using matlab to give linear fitting and quadratic fitting for 1001 points ($r = \frac{i}{1000}$ for $i = 0, 1, \ldots, 1000$), we have the following result:

The best linear fitting:

$$\sigma = 0.2 \quad l(x) = 0.9619x + 0.0687$$
$$\sigma = 0.4 \quad l(x) = 0.9296x + 0.1292$$
$$\sigma = 0.5 \quad l(x) = 0.9151x + 0.1569$$
$$\sigma = 0.6 \quad l(x) = 0.9016x + 0.1832$$
$$\sigma = 0.8 \quad l(x) = 0.8771x + 0.2319$$
$$\sigma = 1.0 \quad l(x) = 0.8554x + 0.2762$$

and

$$\text{The best quadratic fitting:}$$
$$\begin{aligned}
\sigma = 0.2 \quad & q(x) = 0.0070x^2 + 0.9549x + 0.0699 \\
\sigma = 0.4 \quad & q(x) = 0.0120x^2 + 0.9176x + 0.1312 \\
\sigma = 0.5 \quad & q(x) = 0.0140x^2 + 0.9011x + 0.1592 \\
\sigma = 0.6 \quad & q(x) = 0.0157x^2 + 0.8859x + 0.1858 \\
\sigma = 0.8 \quad & q(x) = 0.0185x^2 + 0.8586x + 0.2350 \\
\sigma = 1.0 \quad & q(x) = 0.0207x^2 + 0.8347x + 0.2797.
\end{aligned}$$

It can be noticed that for $\sigma = 0.2, 0.4, 0.5, 0.6, 0.8, 1.0$, the quadratic coefficient a_{fit} in quadratic fitting is in $(0, 0.025)$. We can say that for $0 \leq \sigma \leq 1$, we have $|a_{fit}| < 0.025$. Therefore, we can convincingly say that r_{new} is highly linear with r. This phenomenon is very interesting, because it gives a good example of the linear increase in percentage point of market share ratio, rather than linear increase in quantity. This model might be appropriate to analyze some real phenomenon that the market share ratio of a company has increased enormously, with a speed faster than the linear one.

5. Result for $n = 3$

As we have shown in Sec. 4, the relation of r and r_{new} is highly linear, that is, $r_{new} = ar + b$. From the data we obtain above, we have $a \in (0.85, 1)$ and $b \in (0, 0.28)$. After giving a fixed σ, we can calculate this linear function as we have done in Sec. 4 and use this to estimate r_{new}. For instance, when $\sigma = 0.5$ and $r = 30\%$, we get $r_{new} = 43.08\%$ as an accurate result and we get $r_{new} = 0.9151 \times 30\% + 0.1569 \approx 43.14\%$ in the linear fitting. We can see the accurate value and estimation value only differs 0.06%, which is tiny compared with 30%. Therefore, if we are sure about the value σ, we can calculate r_{new} with the linear fitting with high accuracy without the complicated calculation of r_{new} in Sec. 4.

Also, we can calculate the value σ given r and r_{new}. This also makes sense in reality; after obtaining the real statistics of old market share ratio and new market share ratio, we can point out the extent how the company cares about market share ratio. More precisely, from

$$9\sigma(3 - r) = 8d(3 + d)^2$$

and

$$r_{new} = \frac{3r + 3d}{3 + d},$$

if both r and r_{new} are given, we can determine the value d in the second equality:

$$d = \frac{3(r_{new} - r)}{3 - r_{new}}.$$

Moreover, plug this into the first equality above, we can express σ by r and r_{new}:

$$\sigma = 24(r_{new} - r)\frac{3 - r}{(3 - r_{new})^3}.$$

We can give the value of σ by the equation above. The list below gives the value of σ for some special r and r_{new}:

σ	$r_{new} = 0.2$	$r_{new} = 0.3$	$r_{new} = 0.4$	$r_{new} = 0.5$	$r_{new} = 0.6$	$r_{new} = 0.7$
$r = 0.1$	0.317055	0.707209	1.18798	1.78176	2.51736	3.43223
$r = 0.2$		0.341411	0.764679	1.29024	1.94444	2.76157
$r = 0.3$			0.368685	0.82944	1.40625	2.13035
$r = 0.4$				0.399360	0.902778	1.53859
$r = 0.5$					0.434028	0.986274
$r = 0.6$						0.473412

Similarly, if we have got the actual data of r and r_{new}, we can obtain σ using this method. The value σ is in some ways appropriate to represent market share ratio consideration of a company, and we can determine it only by r and r_{new}.

6. Calculation and Result for General n

We assume that the competition network is still a complete graph, but now we have n companies and company 1 has consideration on market share ratio with parameter σ. In this situation, from

$$\begin{cases} \dfrac{\partial U_2}{\partial q_2} = 0 \\ \dfrac{\partial U_3}{\partial q_3} = 0 \\ \vdots \quad \vdots \quad \vdots \\ \dfrac{\partial U_n}{\partial q_n} = 0, \end{cases}$$

we get

$$Q = \frac{1}{n}q_1 + \frac{n-1}{n}a - \frac{\sum_{i=2}^n c_i}{n}$$

and

$$\frac{q_1}{Q} = \frac{q_1}{\frac{1}{n}q_1 + \frac{(n-1)a - \sum_{i=2}^n c_i}{n}}. \tag{$*$}$$

In the Cournot equilibrium without consideration of market share ratio, we have

$$q_1^* = \frac{1}{n+1}\left(a + \sum_{i=1}^n c_i - (n+1)c_1\right), \quad Q^* = \frac{1}{n+1}\left(na - \sum_{i=1}^n c_i\right).$$

From the above two equations we have

$$(n-1)a - \sum_{i=2}^n c_i = nQ^* - q_1^*.$$

Plug this into $(*)$, we have

$$\frac{q_1}{Q} = \frac{nq_1}{q_1 + nQ^* - q_1^*}. \tag{$**$}$$

Plug $(**)$ into

$$\frac{\partial U_1}{\partial q_1} + w\frac{\partial(q_1/Q)}{\partial q_1} = 0,$$

after some calculation, we obtain

$$(n+1)q_1^* - (n+1)q_1 + w\frac{n^2c}{(q_1+c)^2} = 0. \tag{$***$}$$

Similar as the special case $n = 3$, denote $q_1 = q_1^* + d_nQ^*$. Plug this into $(***)$, we find out $(***)$ is equivalent to

$$l \cdot n^2\left(n - \frac{q_1^*}{Q^*}\right) = d_n(n+1)(n+d_n)^2, \tag{$\Box$}$$

and

$$\frac{q_1}{Q} = \frac{n(r+d_n)}{n+d_n}.$$

We have considered how $\frac{q_1}{Q}$ varies when $n = 3$ and σ is fixed. Now we consider another situation: $l = \frac{\sigma}{2}$ and r are fixed, but n varies. This makes sense in reality because we might not be sure about the number of companies, but we know our present market share ratio r and parameter σ.

We discuss r_{new} when $n \to \infty$. $(\Box)$ is equivalent to

$$\frac{l \cdot n^2(n-r)}{n+1} = d_n^3 + 2nd_n^2 + n^2d_n.$$

Since l, r are fixed, when $n \to \infty$, $\frac{l \cdot n^2(n-r)}{n+1} = O(n^2)$. Hence

$$\begin{aligned} d_n^3 + 2nd_n^2 + n^2d_n &= O(n^2) \\ \Rightarrow n^2d_n &= O(n^2) \\ \Rightarrow d_n &= O(1). \end{aligned}$$

$(\Box)$ is also equivalent to

$$\frac{l \cdot n^2(n-r)}{n^2(n+1)} = \frac{d_n^3}{n^2} + 2\frac{d_n^2}{n} + d_n.$$

Taking $n \to \infty$, we have

$$\lim_{n\to\infty} d_n = l = \frac{\sigma}{2}.$$

Therefore

$$\lim_{n\to\infty} r_{new} = \lim_{n\to\infty} \frac{n(r+d_n)}{n+d_n}$$
$$= \lim_{n\to\infty} \frac{r+d_n}{1+\frac{d_n}{n}}$$
$$= r + \frac{\sigma}{2}.$$

This result can be testified by numerical calculation when $\sigma = 0.5$:

r_{new}	$n=3$	$n=5$	$n=10$	$n=100$	$n=1000$	$n=10000$	$n=100000$
$r=0.1$	0.249487	0.278953	0.308942	0.345215	0.349513	0.349951	0.349988
$r=0.2$	0.340033	0.372082	0.404868	0.444726	0.449464	0.449946	0.449987
$r=0.3$	0.430847	0.465332	0.500832	0.544238	0.549414	0.549941	0.549987
$r=0.4$	0.521934	0.558705	0.596833	0.64375	0.649364	0.649936	0.649986
$r=0.5$	0.613296	0.6522	0.692873	0.743263	0.749314	0.749931	0.749986
$r=0.6$	0.704938	0.745819	0.78895	0.842776	0.849264	0.849926	0.849985

From the data above, we can observed that $r_{new} \to r + 0.25$ for $r = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6$ when $n \to \infty$. Moreover, this phenomenon gives an explanation of the highly-linear of r and r_{new} in the special case $n = 3$.

We hope our modified Cournot competition model can be a good tool for people to estimate the new market share ratio r_{new} given market share consideration parameter σ in a complete graph network. On the other hand, given r and r_{new}, our model provides an estimation of the extent how the company take market share objective into consideration, that is, determine parameter σ.

7. Calculation and Result for Star Graph Network

Denote $G = (V, E)$ to be the graph that represents the star graph network. Denote $V = \{v_A, v_2, v_3, \ldots, v_n\}$ to be vertex set and v_A is the center of the star. Assume that company A has consideration on market share ratio with parameter σ. In this situation, for all $i = 2, 3, \ldots, n$, we have

$$\frac{\partial(U_i)}{\partial q_i} = 0 \Rightarrow a - c_i - 2q_i - q_A = 0$$
$$\Rightarrow q_i = \frac{1}{2}(a - c_i - q_A).$$

Summing this from $i = 2$ to $i = n$, we obtain

$$Q - q_A = \frac{1}{2}\left((n-1)a - \sum_{i=2}^{n} c_i - (n-1)q_A\right)$$
$$\Rightarrow Q = \frac{1}{2}\left((n-1)a - \sum_{i=2}^{n} c_i\right) - \frac{n-3}{2}q_A. \qquad (*)$$

Similar as before, we denote $q_A^*, q_2^*, q_3^*, \ldots, q_n^*$ and Q^* to be the quantity of company $A, 2, 3, \ldots, n$, and the sum of quantity at the equilibrium point in the initial Cournot model (without consideration of market share objective), respectively. Also denote $l = \frac{\sigma}{2}$ and $w = l \cdot (Q^*)^2$. We still use r to represent the market share ratio of initial Cournot equilibrium and r_{new} to represent the market share ratio of new Cournot equilibrium, i.e., $r = \frac{q_A^*}{Q^*}$ and $r_{new} = \frac{q_A}{Q}$.

Notice that in $(*)$, we did not use $\frac{\partial(U_A)}{\partial q_A} + w\frac{\partial(q_A/Q)}{\partial q_A} = 0$. Therefore, $(*)$ should also be correct for q_A^* and Q^*. Consequently

$$Q^* = \frac{1}{2}\left((n-1)a - \sum_{i=2}^{n} c_i\right) - \frac{n-3}{2}q_A^*$$
$$\Rightarrow \frac{1}{2}\left((n-1)a - \sum_{i=2}^{n} c_i\right) = Q^* + \frac{n-3}{2}q_A^*.$$

Plug this into

$$\frac{q_A}{Q} = \frac{q_A}{\frac{1}{2}((n-1)a - \sum_{i=2}^{n} c_i) - \frac{n-3}{2}q_A},$$

we have

$$\frac{q_A}{Q} = \frac{q_A}{Q^* + \frac{n-3}{2}q_A^* - \frac{n-3}{2}q_A}. \tag{$**$}$$

Now, plug $(*)$ into $\frac{\partial(U_A)}{\partial q_A} + w\frac{\partial(q_A/Q)}{\partial q_A} = 0$, after some calculation, we get

$$\frac{n-5}{2}(q_A - q_A^*) + w\frac{Q^* + \frac{n-3}{2}q_A^*}{(Q^* + \frac{n-3}{2}q_A^* - \frac{n-3}{2}q_A)^2} = 0. \tag{$\square$}$$

Notice that

$$w\frac{Q^* + \frac{n-3}{2}q_A^*}{(Q^* + \frac{n-3}{2}q_A^* - \frac{n-3}{2}q_A)^2} > 0.$$

Therefore

$$\frac{n-5}{2}(q_A - q_A^*) < 0. \tag{$***$}$$

If $n \geq 5$, from $(***)$ we must have $q_A < q_A^*$. Plug this into $(*)$ we have $Q > Q^*$. Therefore

$$r_{new} = \frac{q_A}{Q} < \frac{q_A^*}{Q^*} = r.$$

But this is counter-intuitive: company A expects the increasing of market share ratio, but we get decreasing result. Therefore, we should not use this model for $n \geq 5$.

For $n = 3, 4$, the model does make sense intuitively. For $n = 4$, ($\Box$) is equivalent to

$$w\frac{2Q^* + q_A^*}{(Q^* + \frac{1}{2}q_A^* - \frac{1}{2}q_A)^2} = q_A - q_A^*.$$

Denote $q_A = q_A^* + dQ^*$. Plug this into the above equation, we get

$$l \cdot \frac{8 + 4r}{(2 - d)^2} = d$$
$$\Leftrightarrow 4l(2 + r) = d(2 - d)^2,$$

and ($**$) is equivalent to

$$r_{new} = \frac{2r + 2d}{2 - d}.$$

Therefore, after given l and r, we can calculate r_{new} as in Sec. 4.

For some specific value, $\sigma = 0.05$ and $\sigma = 0.1$, the result is:

$\sigma = 0.05$:

$$\begin{aligned} r &= 0.1 \quad r_{new} = 0.1600 \\ r &= 0.2 \quad r_{new} = 0.2661 \\ r &= 0.3 \quad r_{new} = 0.3726 \\ r &= 0.4 \quad r_{new} = 0.4794 \\ r &= 0.5 \quad r_{new} = 0.5865 \\ r &= 0.6 \quad r_{new} = 0.6940, \end{aligned}$$

$\sigma = 0.1$:

$$\begin{aligned} r &= 0.1 \quad r_{new} = 0.2325 \\ r &= 0.2 \quad r_{new} = 0.3469 \\ r &= 0.3 \quad r_{new} = 0.4623 \\ r &= 0.4 \quad r_{new} = 0.5786 \\ r &= 0.5 \quad r_{new} = 0.6959 \\ r &= 0.6 \quad r_{new} = 0.8143. \end{aligned}$$

We can observe that our algorithm works very well in these two situations.

This model also works for $n = 2, 3$. For $n = 3$, we can use the same method in calculation. For $n = 2$, we can escape this part, because a star graph with 2 nodes is a complete graph with 2 nodes. In conclusion, the modified Cournot competition model works in star graph network, but only concords with intuition when $n = 3, 4$.

8. Conclusion and Future Work

The above work gives an improved Cournot competition model. Using this model, we can analyze market competition with consideration of market share objective. Two structures of network have been carefully discussed: complete graph and star graph. For complete graph network, the highly-linearity between r and r_{new} is noticed, and the value of r_{new} converges after taking $n \to \infty$ are two very interesting properties in the model. For star graph network, we point out when $n = 3, 4$, the model is useful, and when $n \geq 5$, the model is relatively limited. Complete graph network is appropriate for the situation that the market share ratio of companies does not differ to much; star graph network is appropriate for the situation that an oligopoly has preponderant market share ratio.

This paper is an implementation of Cournot competition model. In the future, other network structures will be chosen for calculation. Also, we will consider the complicated situation that more than 1 company care for market share ratio. This is much more complicated in calculation because the partial derivative equation will be much harder to solve, but we will make effort in this direction.

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