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Distributed quantum information processing in fiber-coupled cavity QED systems

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Abstract: Fiber-coupled cavity QED system is a perfect theoretical model to study the deterministic distributing quantum information processes. Recent works on distributing quantum information processing in fiber-coupled cavity QED systems were reviewed in this paper. In this review, quantum state transfer, entanglement distribution, quantum logic gates and et al. in this system were discussed. The several types of the schemes, such as resonant interaction, adiabatic passage, and virtual-excitation processes were summarized. Finally, the experimental progresses in this direction were discussed.

Key words: quantum computation; cavity QED; entanglement distribution; state transfer
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光纤耦合的腔量子电动力学系统中的分布式量子信息

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摘　要:光纤耦合的腔量子电动力学系统是一个完美的理论模型,可以实现决定性的量子信息过程.该文综述了最近在光纤耦合的腔量子电动力学系统中实现分布式量子信息处理的工作.讨论了如何在该系统中实现量子态传输、纠缠分配和量子逻辑门,并概述了多种不同的方案,如共振耦合、绝热操控、虚激发过程.最后,讨论了这个方向上的实验进展.

关键词:量子计算;腔量子电动力学;纠缠分配;态传输

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0 Introduction

The concept of quantum computation and quantum simulation was firstly discussed by Feynman in 1982[1]. This new method of information processing has shown a lot of advantages. For example, quantum Shor algorithm can be used for large N integer factorization problem in polynomial time[2], which is substantially faster than the most efficient known classical algorithms.

In order to realize practical quantum computation, the scalability is one of the most difficult criteria to be fulfilled. In order to fulfill the scalability criteria, distributed quantum computing was proposed[3-4]. The quantum computation can be divided into subroutines, each of which has a separate quantum processor. The local quantum processor has a few qubits, in which universal quantum logic gates and qubit-specific quantum measurement can be realized with high fidelity and efficiency. Therefore, one of the key problems in distributed quantum computing is how to realize universal quantum logic gates among distant quantum processors[5].

In this review, we will focus on how to realize distributed quantum computing between distant cavity QED systems, which are connected with quantum transmission lines, such as fibers[6-10]. We discuss decoherence effects in the quantum information processing, and how to decrease decoherence by speeding up the evolution[8], or by virtual excitation[9] or adiabatic passage[11-14]. The coupled cavity QED systems can also be used for simulating many-body systems[15-17], and quantum network[18-19]. Here we focus on how to deterministically realize quantum information process in distant cavity QED systems.

The review is organized as follows. In the Sec. 1, we introduce the basic model of fiber-coupled cavity QED systems. In the section 2, we discuss how to realize quantum state transfer between distant nodes. In the section 3 and 4, we will discuss how to realize entanglement and quantum logic gates between distant nodes. We will discuss three types of schemes, respectively through resonant interaction, virtual-excitation processes, and adiabatic passage. In the section 5, we will discuss how to deal with the decoherence processes in the schemes and experimental possibility. In the section 6, we will give the summary.

1 The model of fiber-coupled cavity QED systems

As shown in Fig. 1, the distant cavities QED systems are connected with an optical fiber. The eigenmodes of the fiber are b_k, with $k=0,1,2,\cdots$. The Hamiltonian of the fiber modes is $H_f = \sum_k \omega_k b_k^\dagger b_k$. The interaction Hamiltonian between fiber and cavity modes is $H_{fc} = \sum_k \nu_k [a_1 + (-1)^k a_2] b_k^\dagger + \text{H.C.}$ The factors $(-1)^k$ model the phase difference of π between the modes on both ends of the fiber for every second fibre-mode. ν_k is the coupling strength between the fiber modes b_k and the cavity modes a_1 and a_2. In each cavity, there is one two-level atom trapped in. The fiber modes b_k couple with both cavity modes. The atoms couple with the local cavity mode. The γ and κ are the atomic spontaneous emission rate and the cavity photon loss rate.

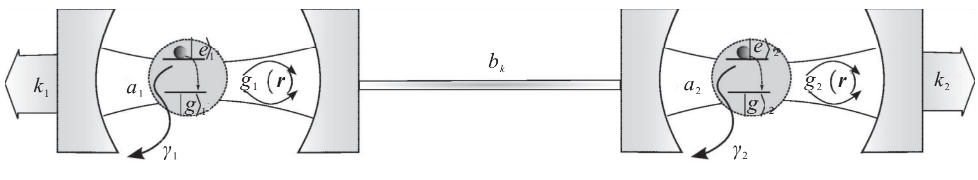

Fig. 1 Fiber coupled the cavity QED systems

If the length of the fiber is short, the fiber modes distance is much larger than the coupling strength between fiber and cavity modes. We can only consider one fiber mode, which couples with cavity modes near resonantly. The short fiber limit requires that $2L\nu_k/(2\pi c) \ll 1$, where c is the light velocity in fiber. In this case, the Hamiltonian H_f can be approximated to[7-8]

$$H_{\mathrm{f}} = \nu[b(a_1 + a_2) + \mathrm{H.C.}], \tag{1}$$

where the phase $(-1)^k$ has been absorbed into the annihilation and creation operators of the mode of the second cavity field.

The interaction Hamiltonian between atoms and cavity mode is

$$H_{\mathrm{ac}} = \sum_{j=1}^{2} (\Delta_j a_j^{\dagger} a_j + g_j \sigma_j^{-} \sigma_j^{\dagger} + \mathrm{H.C.}), \tag{2}$$

where $\Delta_j = \omega_{\mathrm{cj}} - \omega_{\mathrm{e}}$, ω_{cj} is the mode frequency of cavity j, ω_{e} is the atomic transition frequency. In the interaction picture, the total Hamiltonian of the atom-cavity-fiber combined system is

$$H = H_{\mathrm{ac}} + H_{\mathrm{f}} = \sum_{j=1}^{2} (\Delta_j a_j^{\dagger} a_j + g_j \sigma_j^{-} \sigma_j^{\dagger} + \mathrm{H.C.}) + \nu[b(a_1 + a_2) + \mathrm{H.C.}]. \tag{3}$$

2　Quantum state transfer among distant quantum nodes

2.1　Adiabatic passage through large-detuned Raman processes

One way to realize the quantum state transfer between distant cavities is to adiabatically control external parameters, as previously proposed by[6,20]. The system considered is two Λ-type atoms held in two separate cavities that are coupled by an optical fiber, as shown in Fig. 2. Each atom has one excited state $|e\rangle_i$ and two ground states $|g_1\rangle_i$ and $|g_2\rangle_i$, where i denotes the site position. The transition $|e\rangle_i \leftrightarrow |g_1\rangle_i$ is coupled to a laser with frequency ω_1, while the transition $|e\rangle_i \leftrightarrow |g_2\rangle_i$ is coupled to a quantized cavity mode with frequency ω_{c}. The corresponding Rabi frequencies for the two transitions are Ω_i and g_i, respectively. In the interaction picture, the interaction Hamiltonian describes the laser-atom-cavity system can be expressed as ($\hbar = 1$)

$$H_{\mathrm{lac}} = \sum_{j=1}^{2} (g_j a_j^{\dagger} e^{i\Delta_l t} |e\rangle_j \langle g_1|_j + \Omega_j e^{i\Delta_c t} |e\rangle_j \langle g_2|_j + \mathrm{H.C.}), \tag{4}$$

with $\Delta_l(\Delta_{\mathrm{c}})$ being the detuning between the cavity (laser) and the atomic transition $|e\rangle_j \leftrightarrow |g_1\rangle_j (|e\rangle_j \leftrightarrow |g_2\rangle_j)$.

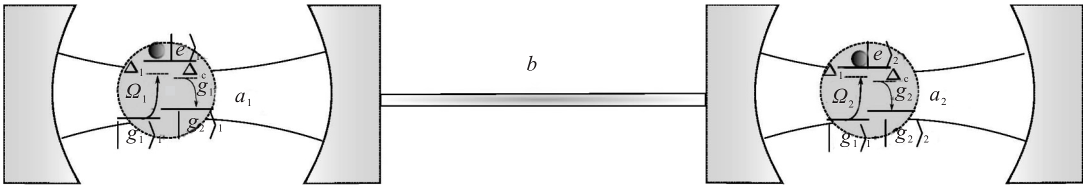

Fig. 2　Two Λ-type atoms are trapped in two spatially separated cavities 1 and 2 respectively

Under the large-detuned condition, i.e., $\Delta_l, \Delta_{\mathrm{c}} \gg g_j, \Omega_j$, the atomic excited states are virtually excited and can be adiabatically eliminated, resulting in an effective Hamiltonian

$$H_{\mathrm{e}} = \sum_{j=1}^{2} (\lambda_{1,j} |g_1\rangle_j \langle g_1|_j + \lambda_{\mathrm{c},j} |g_2\rangle_j \langle g_2|_j a_j^{\dagger} a_j + \lambda_j a |g_1\rangle_j \langle g_2|_j + \mathrm{H.C.}), \tag{5}$$

for the laser-atom-cavity system, where $\lambda_{l,j} = \frac{\Omega_j^2}{\Delta_l}, \lambda_{\mathrm{c},j} = \frac{g_j^2}{\Delta_{\mathrm{c}}}, \lambda_j = \frac{\Omega_j g_j}{2}(\frac{1}{\Delta_l} + \frac{1}{\Delta_{\mathrm{c}}})$. Therefore, taking into account the interaction Hamiltonian between the fiber and cavity modes as given in Eq. (3), the effective Hamiltonian of the total system is now given by

$$H = H_{\mathrm{e}} + H_{\mathrm{f}} = H'_0 + H_{\mathrm{part}}, \tag{6}$$

With

$$H'_0 = \sum_{j=1}^{2} \lambda_{1,j} |g_1\rangle_j \langle g_1|_j, \tag{7}$$

$$H_{\text{part}} = \sum_{j=1}^{2} (\lambda_{c,j} | g_2\rangle_j \langle g_2 |_j a_j^\dagger a_j + \lambda_j a | g_1\rangle_j \langle g_2 |_j + \text{H.C.}) + [\nu b(a_1 + a_2) + \text{H.C.}]. \quad (8)$$

Suppose that at the initial time all modes of both the cavities and fiber are not excited and the atom in the first cavity is in a superposition state of $(\alpha | g_1\rangle_1 + \beta | g_2\rangle_1)$, where α and β are complex numbers and fulfill the normalization condition. The atom in the second cavity is in the ground state $| g_2\rangle_2$. The goal of quantum state transfer is to deterministically accomplish the operation $(\alpha | g_1\rangle_1 + \beta | g_2\rangle_1) \otimes | g_2\rangle_2 \rightarrow | g_2\rangle_1 \otimes (\alpha | g_1\rangle_2 + \beta | g_2\rangle_2)$. The time evolution of the total system is governed by the Schrödinger equation: $\mathrm{i}\frac{\partial}{\partial t} | \Psi(t)\rangle = H | \Psi(t)\rangle$, which will be confined in the single-excitation subspace $\forall \in \{ | \phi_1\rangle = | 000\rangle | g_1\rangle_1 | g_2\rangle_2, | \phi_2\rangle = | 100\rangle | g_2\rangle_1 | g_2\rangle_2, | \phi_3\rangle = | 001\rangle | g_2\rangle_1 | g_2\rangle_2, | \phi_4\rangle = | 010\rangle | g_2\rangle_1 | g_2\rangle_2, | \phi_5\rangle = | 000\rangle | g_2\rangle_1 | g_1\rangle_2 \}$, with $| n_1 n_f n_2\rangle$ denoting the field state with n_1 photons in the cavity 1, n_2 in the cavity 2, and n_f in the fiber.

Note that, for the Hamiltonian H_{part}, dark states with respect to two cavity modes exist. This is not the case for the full Hamiltonian H in (6), as the term (7) that describes Stark shifts induced by laser fields deteriorate the dark state. In order to manipulate the dark state by controlling the system's parameters, it is required to apply another laser field to produce an ac-Stark shifts to neutralize the term (7). The two relevant dark states of H_{part} read $| D_0\rangle \propto \nu\lambda_2 | \phi_1\rangle - \lambda_1\lambda_2 | \phi_3\rangle + \nu\lambda_1 | \phi_5\rangle \propto \nu\lambda_2 | 000\rangle | g_1\rangle_1 | g_2\rangle_2 - \lambda_1\lambda_2 | 001\rangle | g_2\rangle_1 | g_2\rangle_2 + \nu\lambda_1 | 000\rangle | g_2\rangle_1 | g_1\rangle_2$, $| D_1\rangle \propto | 000\rangle | g_2\rangle_1 | g_2\rangle_2$. For the first dark state $| D_0\rangle$ the cavity modes are not populated due to destructive interference whereas the second dark state $| D_1\rangle$ is always decoupled from the atom-field interaction. Provided that the system is initially in a superposition of the two dark states in $\alpha | D_0\rangle + \beta | D_1\rangle$ and the laser intensities are changed slowly in such a way that the initial condition $\lambda_2 \gg \lambda_1$ is adiabatically converted to the final case $\lambda_1 \gg \lambda_2$. So the system is finally in $| 000\rangle | g_2\rangle_1 (\alpha | g_1\rangle_2 + \beta | g_2\rangle_2)$. This thus achieves the quantum state transfer. One important feature of the scheme is that the details of the laser pulses are not important as long as the process is performed adiabatically.

2.2 Resonant interaction

Another way to realize the quantum state transfer between distant cavity is through resonant coherent interaction[7-8]. Compared with the adiabatic passage, the dynamics through resonant interaction is much faster. In order to realize the quantum state transfer with a high fidelity, the coupling between atoms and the cavity should be turned on/off suddenly at certain time.

Suppose that at the initial time all modes of both cavities and fiber are not excited and the atom in the first cavity is in a superposition state of $(\alpha | g\rangle_1 + \beta | e\rangle_1)$, where α and β are complex numbers and fulfill the normalization condition. The atom in the second cavity is in the ground state $| g\rangle$. The goal of quantum state transfer is to deterministically accomplish the operation $(\alpha | g\rangle_1 + \beta | e\rangle_1) \otimes | g\rangle_2 \rightarrow | g\rangle_1 \otimes (\alpha | g\rangle_2 + \beta | e\rangle_2)$.

The time evolution of the total system is governed by the Schrödinger equation: $\mathrm{i}\frac{\partial}{\partial t} | \Psi(t)\rangle = H | \Psi(t)\rangle$, where we set $\hbar = 1$. The basis of the Hamiltonian is $\phi_1 = | 000\rangle | g\rangle_1 | e\rangle_2$, $\phi_2 = | 000\rangle | e\rangle_1 | g\rangle_2$, $\phi_3 = | 001\rangle | g\rangle_1 | g\rangle_2$, $\phi_4 = | 010\rangle | g\rangle_1 | g\rangle_2$, $\phi_5 = | 100\rangle | g\rangle_1 | g\rangle_2$, where $| n_1 n_f n_2\rangle$ denotes the field state with n_1 photons in the cavity 1, n_2 in the cavity 2, and n_f in the fiber. We suppose that the detuning $\Delta_i = 0$, the Hamiltonian (3) in the one excitation basis reads

$$H = \begin{pmatrix} 0 & 0 & g & 0 & 0 \\ 0 & 0 & 0 & 0 & g \\ g & 0 & 0 & \nu & 0 \\ 0 & 0 & \nu & 0 & \nu \\ 0 & g & 0 & \nu & 0 \end{pmatrix}, \quad (9)$$

where $g_1 = g_2 = g$ has been assumed.

The Hamiltonian (9) has five eigenvalues: $E_1 = 0$, $E_{2,3} = \mp g$, $E_{4,5} = \mp \sqrt{g^2 + 2\nu^2}$. The unitary matrix S diagonalizes the Hamiltonian matrix (9). A state of the entire system with one excitation number can be expanded in terms of the basis vectors as

$$|\Psi(t)\rangle = \sum_{i=1}^{5} C_i(t)\,|\phi_i\rangle. \tag{10}$$

By use of the unitary matrix S, Schrödinger equation can be rewritten as the compact form

$$\mathrm{i}\frac{\partial}{\partial t}SC = SHS^{-1}SC, \tag{11}$$

where $C = [C, C, C, C, C]^{\mathrm{T}}$. Since the matrix SHS^{-1} is diagonal, a general solution of Eq. (11) is given by

$$C_j(t) = \sum_{k=1}^{5} [S^{-1}]_{jk}\,[SC(0)]_k \mathrm{e}^{-\mathrm{i}E_k t}. \tag{12}$$

Using this solution, for the initial condition $C(0) = [0,1,0,0,0]^{\mathrm{T}}$, it is easily shown that at $t = \pi/(\sqrt{N}g)$ the initial state $(\alpha|g\rangle_1 + \beta|e\rangle_1) \otimes |g\rangle_2$ evolves in the state $|g\rangle_1 \otimes (\alpha|g\rangle_2 + \beta|e\rangle_2)$ if the parameter r fulfills the condition $r^2 = (4k^2 - 1)/2, k = 1,2,3,\cdots$.

3 Entanglement generation among distant nodes

3.1 Resonant interation

Here we discuss how to create entangled states between distant cavities by resonant interaction[8,21-26]. For the fiber-coupled cavity QED system, we can use the canonical transformations[7-8]

$$a_1 = \frac{1}{2}(c_+ + c_- + \sqrt{2}c),$$

$$a_2 = \frac{1}{2}(c_+ + c_- - \sqrt{2}c),$$

$$b = \frac{1}{\sqrt{2}}(c_+ - c_-), \tag{13}$$

to introduce three normal bosonic modes c and $c_\mp$. The frequencies of the normal mode c and $c_\mp$ are ω and $\omega \mp \sqrt{2}\nu$, respectively. Excitations of the non-resonant modes are highly suppressed, and can be safely neglected. In this way, the system reduces to two qubits resonantly coupled through a single-mode of the cavity field, and the Hamiltonian in the interaction picture becomes

$$H = \frac{1}{\sqrt{2}}(g_1\sigma_1^- c^\dagger - g_2\sigma_2^- c^\dagger + \mathrm{H.C.}). \tag{14}$$

Suppose that the atoms are in the state $|e\rangle_1|g\rangle_2$, and the mode c is in the vacuum state at the initial time. From Eq. (14), the system at the later time is restricted to the subspace spanned by the basis vectors $|\phi_1\rangle = |0\rangle_c|g\rangle_1|e\rangle_2$, $|\phi_2\rangle = |0\rangle_c|e\rangle_1|g\rangle_2$, $|\phi_3\rangle = |1\rangle_c|g\rangle_1|g\rangle_2$, where $|0\rangle_c$ and $|1\rangle_c$ are number states of the normal modes c with zero and one photon, respectively. By solving the Schrödinger equation, one can show that at time t the system evolves in the state $|\Psi(t)\rangle$. In order to generate the Bell state, we find the results:

(i) if $g_2 = (1 + \sqrt{2})g_1$ and $g_1 t = \pi/\sqrt{2 + \sqrt{2}}$, the field comes back to the vacuum state and the atoms are in the entangled state $|\Psi_{\mathrm{E1}}\rangle = (|e\rangle_1|g\rangle_2 + |g\rangle_1|e\rangle_2)/\sqrt{2}$.

(ii) if $g_2 = (-1 + \sqrt{2})g_1$ and $g_1 t = \pi/\sqrt{2 - \sqrt{2}}$, the field comes back to the vacuum state and the atoms are in the entangled state $|\Psi_{\mathrm{E2}}\rangle = (|e\rangle_1|g\rangle_2 - |g\rangle_1|e\rangle_2)/\sqrt{2}$.

We can use numerical method to test the regime that the non-resonant normal modes can be safely neglected. It is observed that the maximum of the fidelity can be larger than 0.99 if r is beyond 20. In practice, in the regime $\nu \gg g$, the fiber mode is adiabatically eliminated. The loss of fiber mode can be neglected.

3.2 Adiabatic passage through resonant Raman processes

We note that entangled states for distant atoms held in separate cavities can also be prepared by adiabatic passage through resonant Raman processes as proposed by Refs. [6, 11, 13, 22, 27–29]. The system considered is also two Λ-type atoms held in two separate cavities that are coupled by an optical fiber, as similarly shown in Fig. 2. The difference is that there is no detuning between the cavity (laser) and the corresponding atomic transition, i. e., $\Delta_l = \Delta_c = 0$. In the interaction picture, the interaction describing the laser-atom-cavity-fiber system can be expressed as ($\hbar = 1$)

$$H = \sum_{j=1}^{2} (g_j a_j^{\dagger} | e\rangle_j \langle g_1 |_j + \Omega_j | e\rangle_j \langle g_2 |_j + \text{H. C.}) + \nu[b(a^{+} + a^{+}) + \text{H. C.}]. \tag{15}$$

Suppose that at the initial time all modes of both cavities and fiber are not excited and the atom in the first cavity is in $| g_1\rangle_1$ while the atom in the other cavity is in $| g_2\rangle_2$. The time evolution of the total system is governed by the Schrödinger equation: $i\frac{\partial}{\partial t}| \Psi(t)\rangle = H| \Psi(t)\rangle$, the corresponding state subspace $\{| \phi_1\rangle = | 000\rangle | g_1\rangle_1 | g_2\rangle_2, | \phi_2\rangle = | 000\rangle | e\rangle_1 | g_2\rangle_2, | \phi_3\rangle = | 100\rangle | g_2\rangle_1 | g_2\rangle_2, | \phi_4\rangle = | 010\rangle | g_2\rangle_1 | g_2\rangle_2, | \phi_5\rangle = | 001\rangle | g_2\rangle_1 | g_2\rangle_2, | \phi_6\rangle = | 000\rangle | g_2\rangle_1 | e\rangle_2, | \phi_7\rangle = | 000\rangle | g_2\rangle_1 | g_1\rangle_2\}$, with $| n_1 n_f n_2\rangle$ denoting the field state with n_1 photons in the cavity 1, n_2 in the cavity 2, and n_f in the fiber.

For the Hamiltonian in Eq. (15), there exist two dark states

$$| D'_0\rangle = | \phi_1\rangle = | 000\rangle | g_1\rangle_1 | g_2\rangle_2;\ | D'_1\rangle \propto g_1\Omega_2 | \phi_1\rangle - \Omega_1\Omega_2(| \phi_3\rangle - | \phi_5\rangle) - g_2\Omega_1 | \phi_7\rangle \propto g_1\Omega_2 | 000\rangle | g_1\rangle_1 | g_2\rangle_2 - \Omega_1\Omega_2(| 100\rangle - | 001\rangle) | g_2\rangle_1 | g_2\rangle_2 - g_2\Omega_1 | 000\rangle | g_2\rangle_1 | g_1\rangle_2. \tag{16}$$

In order to prepare the entangled state between the two atoms, i. e., $\frac{1}{\sqrt{2}}(| g_1\rangle_1 | g_2\rangle_2 - | g_2\rangle_1 | g_1\rangle_2)$, the laser intensities should first be adjusted in such a way so that the condition $\Omega_2 \gg \Omega_1$ is satisfied, then changed slowly (say, adiabatically) to made Ω_2 decrease while Ω_1 increased, and finally approached to the case fulfilling $g_1\Omega_2 = g_2\Omega_1$. Besides, the condition $\Omega_1, \Omega_2 \ll g_1, g_2$ should also be guaranteed such that the two cavity modes are virtually populated during the operation process.

Based on such a mechanism of adiabatic passage of dark sate, the qutrit-qutrit entangled state between two distant atoms[22] and multi-particle W-type entangled state between multiple distant atoms[11] can be prepared.

3.3 Virtual-excitation processes

Another type of method for quantum communication and entanglement between two distant atoms held in separate cavities is that based on virtual-excitation processes[24, 30–32].

The system considered is also two Λ-type atoms held in two separate cavities that are coupled by an optical fiber, the related atomic configuration and atom-field coupling is similarly shown in Fig. 2. In the interaction picture, the interaction Hamiltonian describing the laser-atom-cavity-fiber system can be expressed as ($\hbar = 1$)

$$H = \sum_{j=1}^{2} (g_j a_j^{\dagger} e^{i\Delta_l t} | e\rangle_j \langle g_1 |_j + \Omega_j e^{i\Delta_c t} | e\rangle_j \langle g_2 |_j + \text{H. C.}) + [\nu b(a_1 + a_2) + \text{H. C.}]. \tag{17}$$

Assume $g_1 = g_2 \equiv g$, $\Omega_1 = \Omega_2 \equiv \Omega$, $\delta = \Delta_c - \Delta_l$, $\lambda = \frac{\Omega g}{2}(\frac{1}{\Delta_l} - \frac{1}{\Delta_c})$, $\eta = \frac{\Omega^2}{\Delta_l}$, and $\zeta = \frac{g^2}{\Delta_c}$. Under the condition $\{\Delta_c, \Delta_l\} \gg \{g, \Omega\}$, $| \delta| \gg \lambda$, $| \delta \pm \sqrt{2}\nu| \gg \frac{\lambda}{2}$, and $\sqrt{2}\nu \gg \{\frac{\lambda}{2}, \frac{\zeta}{4}\}$, not only the atomic excited states but also the photons in the cavities and fibers are virtually populated. The off-resonant Raman coupling leads to the Stark shifts and Heisenberg XY coupling between the atoms[30]. If all the modes of the cavities and fibers are initially in the vacuum state, they will remain in the vacuum state during the process. The effective Hamiltonian of the system finally reduces to

$$H_{\mathrm{eff}} = \sum_{j=1}^{2} \mu \mid g_1\rangle_j \langle g_1 \mid_j - \chi(S_1^+ S_2^- + S_1^- S_2^+), \tag{18}$$

Where $\mu = \frac{\lambda^2}{4}[\frac{2}{\delta} + \frac{1}{\delta - \sqrt{2}\nu} + \frac{1}{\delta + \sqrt{2}\nu}] - \eta$, $\chi = \frac{\lambda^2}{4}[\frac{2}{\delta} - \frac{1}{\delta - \sqrt{2}\nu} - \frac{1}{\delta + \sqrt{2}\nu}]$, and $S_j^+ = \mid g_1\rangle_j \langle g_2 \mid_j$.

Assume that the two atoms are initially prepared in $\mid g_1\rangle_1 \mid g_2\rangle_2$, the evolution of the system can be written as

$$e^{-i\mu t}[\cos(\chi t) \mid g_1\rangle_1 \mid g_2\rangle_2 - i\sin(\chi t) \mid g_2\rangle_1 \mid g_1\rangle_2]. \tag{19}$$

Setting the interaction time to satisfy $\chi t = \pi/4$, a maximal entangled state for two distant atoms is obtained: $\frac{1}{\sqrt{2}}(\mid g_1\rangle_1 \mid g_2\rangle_2 - i \mid g_2\rangle_1 \mid g_1\rangle_2)$, where the common phase factor $e^{-i\mu t}$ is discarded.

4 Distant quantum controlled-phase gate among distant nodes

In this section, we investigate how to realize the controlled-phase gate between the two atomic systems: $(\mid 0\rangle_1 + \mid 1\rangle_1) \otimes (\mid 0\rangle_2 + \mid 1\rangle_2)/2 \rightarrow (\mid 0\rangle_1 \mid 0\rangle_2 + \mid 0\rangle_1 \mid 1\rangle_2 + \mid 1\rangle_1 \mid 0\rangle_2 + e^{i\phi} \mid 1\rangle_1 \mid 1\rangle_2)/2$.

4.1 Resonant interaction

For the resonant scheme, there are two types of methods. The first one works in the regime that ν is comparable with g[10]. With the similar idea of Ref. [10], the controlled-Z gate can be easily extended to N-qubit controlled-Z gate[33]. The second one requires $\nu \gg g$[8]. Here we focus on the first type of scheme. The first type of scheme uses three-level atoms, each has an excited state $\mid e\rangle$ and two ground states $\mid g\rangle$ and $\mid s\rangle$. The scheme differs here in that an additional atomic state (not coupled to the atom-field interaction) is used for the controlled-Z gate. Besides, the scheme uses the asymmetric encoding[10,34], qubit 1 is encoded in $\mid g\rangle$ and $\mid e\rangle$, while qubit 2 is encoded in $\mid g\rangle$ and $\mid s\rangle$: $\mid g\rangle_1 \equiv \mid 0\rangle_1$, $\mid e\rangle_1 \equiv \mid 1\rangle_1$, $\mid g\rangle_2 \equiv \mid 0\rangle_2$, $\mid s\rangle_2 \equiv \mid 1\rangle_2$. The important feature of such type of asymmetric encoding is that the evolution for the gate operation is within the null-excitation and single-excitation subspaces.

For the input state $\mid \Psi(0)\rangle = \mid 000\rangle \otimes (\mid g\rangle_1 + \mid e\rangle_1) \otimes (\mid g\rangle_2 + \mid s\rangle_2)/2$. The aim of the controlled-Z gate is to perform the unitary transformation that produces the output sate $\mid \Psi_Z\rangle = \mid 000\rangle \otimes (\mid g\rangle_1 \mid g\rangle_1 + \mid g\rangle_1 \mid s\rangle_2 + \mid e\rangle_1 \mid g\rangle_2 + \mid e\rangle_1 \mid s\rangle_2)/2$. Note that if the input basis state is $\mid 000\rangle \mid g\rangle_1 \mid g\rangle_2$ or $\mid 000\rangle \mid g\rangle_1 \mid s\rangle_2$, the system will not evolve as such bases are completely decoupled. Whereas the system in the basis state $\mid 000\rangle \mid e\rangle_1 \mid g\rangle_2$ will evolve within the single-excitation subspace $\forall_{eg} \in \{\mid 000\rangle \mid e\rangle_1 \mid g\rangle_2, \mid 100\rangle \mid g\rangle_1 \mid g\rangle_2, \mid 001\rangle \mid g\rangle_1 \mid g\rangle_2, \mid 010\rangle \mid g\rangle_1 \mid g\rangle_2, \mid 000\rangle \mid g\rangle_1 \mid e\rangle_2\}$; while if the system is in the basis state $\mid 000\rangle \mid e\rangle_1 \mid s\rangle_2$, it will evolve within another single-excitation subspace $\forall_{es} \in \{\mid 000\rangle \mid e\rangle_1 \mid s\rangle_2, \mid 100\rangle \mid g\rangle_1 \mid s\rangle_2, \mid 001\rangle \mid g\rangle_1 \mid s\rangle_2, \mid 010\rangle \mid g\rangle_1 \mid s\rangle_2\}$. The realization of the gate operation is to figure out the common time T at which the evolutions for the state bases $\mid 000\rangle \mid e\rangle_1 \mid g\rangle_2$ and $\mid 000\rangle \mid e\rangle_1 \mid s\rangle_2$ that are dominated by the Hamiltonian (3) approach to $\mid \Psi_{eg}(T)\rangle = \mid 000\rangle \mid e\rangle_1 \mid g\rangle_2$ and $\mid \Psi_{es}(T)\rangle = e^{i\pi} \mid 000\rangle \mid e\rangle_1 \mid s\rangle_2$, respectively.

The idea[10] has been generalized to the more complex case where two distant qubits interact with the local quantized fields that are coupled through a set of bosonic field modes[35]. Besides, based on such a method of asymmetric encoding of quantum logic operation, the distributed controlled-Z gates for N atomic qubits held in separate cavities coupled by optical fibers can be achieved[33].

4.2 Virtual-excitation processes

In this section, we turn to another scheme proposed by Zheng[9], for which the controlled-Z gate is achieved without excitation of photons in the cavities and fibers. The scheme also considers two atoms trapped in distant cavities coupled by an optical fiber, as shown in Fig. 3. Each atom has one excited state $|e\rangle$ and two ground states $|f\rangle$ and $|g\rangle$, where $|f\rangle$ and $|g\rangle$ are used for quantum information encoding, i. e., $|f\rangle_j \equiv |0\rangle_j$, and $|g\rangle_j \equiv |1\rangle_j$. The transition $|e\rangle \leftrightarrow |g\rangle$ is coupled to the local cavity mode with the coupling strength g and a classical laser field with the Rabi frequency Ω. The detunings of the cavity mode and classical field are Δ and $\Delta-\delta$, respectively. The state $|f\rangle$ is auxiliary and decoupled from the atom-field interaction. In the interaction picture, the Hamiltonian describing the atom-cavity-laser and the fiber-cavity interaction is

$$H = \sum_{j=1}^{2} \{ [g a_j e^{i\Delta t} + \Omega e^{i(\Delta-\delta)t}] |e\rangle_j \langle g|_j + \text{H. C.} \} + \nu [b(a_1^{+} + a_2^{+}) + \text{H. C.}]. \tag{20}$$

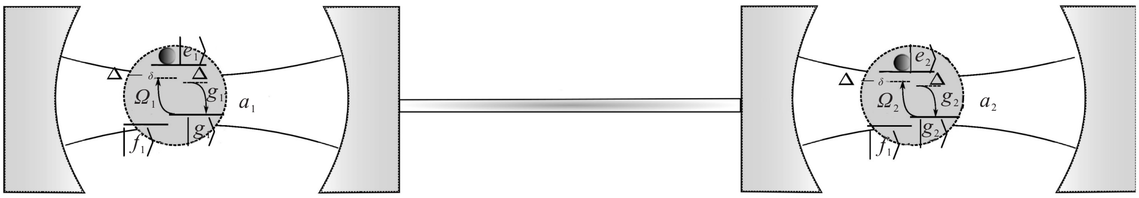

Fig. 3 Atomic levels and laser driving setup for virtual excitation scheme of CPHASE gate

Under the large-detuned condition $\Delta \gg \sqrt{2}\nu$, δ, and Ω, the atoms do not exchange energy with the cavity modes, fiber modes, and laser fields. As the atoms are initially populated in the ground states due to the preset ground state encoding, they cannot exchange excitation with each other through virtual excitation of the cavity modes and they remain in the ground state during the the operation. In order to gain deep physics into such an interaction mechanism, we use the canonical transformations in Eq. (13) and introduce three normal bosonic modes c and $c_{\mp}$. Notice here that by such a transformation, the interaction of the system turns to the case where the three bosonic modes can be coupled to each other as well as to the classical fields through the virtual excitation of the atoms.

Define $\lambda_0 = \frac{\sqrt{2} g\Omega}{4}\left(\frac{1}{\Delta} + \frac{1}{\Delta-\delta}\right)$, $\lambda_{1,2} = \frac{g\Omega}{4}\left(\frac{1}{\Delta \pm \sqrt{2}\nu} + \frac{1}{\Delta-\delta}\right)$, $\xi_0 = \frac{g^2}{4}\left(\frac{1}{\Delta+\sqrt{2}\nu} + \frac{1}{\Delta-\sqrt{2}\nu}\right)$, $\xi_{1,2} = \frac{\sqrt{2} g^2}{4}\left(\frac{1}{\Delta \pm \sqrt{2}\nu} + \frac{1}{\Delta}\right)$. If the conditions δ, $\sqrt{2}\nu$, $\delta \pm \sqrt{2}\nu \gg \lambda_0$, $\lambda_{1,2}$, ξ_0, $\xi_{1,2}$ are further satisfied, the bosonic modes cannot exchange energy with each other and with the laser fields. The nonresonant couplings between the bosonic modes and the classical fields lead to energy shifts depending on the number of atoms in the state $|g\rangle$. Besides, the nonresonant couplings between the bosonic modes result in energy shifts depending on both the excitation numbers of the modes and the number of atoms in state $|g\rangle$. Assume that the cavities and fiber are initially in the vacuum state (the thermal photons are negligible, as it is reasonable at optical frequencies). As the quantum numbers of the bosonic modes are conserved they remain in the vacuum state during the evolution. In such a case, the system can be finally described by the effective Hamiltonian

$$\begin{aligned} H_e = {} & (\mu_1 + \mu_2)(|g\rangle_1\langle g|_1 + |g\rangle_2\langle g|_2)^2 + \\ & \mu_0 (|g\rangle_1\langle g|_1 - |g\rangle_2\langle g|_2)^2 - \eta(|g\rangle_1\langle g|_1 + |g\rangle_2\langle g|_2), \end{aligned} \tag{21}$$

where $\mu_0 = \frac{\lambda_0^2}{\delta}$, $\mu_{1,2} = \frac{\lambda_{1,2}^2}{\delta \pm \sqrt{2}\mu}$ and $\eta = \frac{\Omega^2}{\Delta-\delta}$. The Hamiltonian (21) describes the nonlinearity in the number of the atoms in the ground states $|g\rangle$ due to the nonresonant couplings between the laser fields and vacuum bosonic modes induced by the virtual excitation of the atoms.

For the input $|\Psi(0)\rangle = (|g\rangle_1 + |f\rangle_1) \otimes (|g\rangle_2 + |f\rangle_2)/2$, the unitary transformation dominated by the Hamiltonian (21) produces the output state $|\Psi_Z\rangle = [e^{-i(4\mu_1+4\mu_2-2\eta)t}|g\rangle_1|g\rangle_2 + e^{-i(\mu_1+\mu_2+\mu_0-\eta)t}(|g\rangle_1|f\rangle_2 + |f\rangle_1|g\rangle_2) + |f\rangle_1|f\rangle_2]/2$. After the single-qubit phase shift: $|g\rangle_j \to e^{i(\mu_1+\mu_2+\mu_0-\eta)t}|g\rangle_j$, it gives the two-qubit controlled-Z gate, which gains a phase factor $-2(\mu_1+\mu_2-\mu_0)$ if and only if the two atomic qubits are in $|g\rangle_1|g\rangle_2$.

The idea[9] can be generalized to the more general case which allows coherent control of arbitrary two distant qubits across a quantum network based on such type of linking cavity arrays[18,36].

5 Decoherence and experimental challenge

In the previous sections, we only considered the ideal case without decoherence. In fact, the decoherence is crucial for the quality of the quantum gates. The decoherence sources include the atomic spontaneous emission and photon leakage out of the cavities and fibre. In order to solve the problem, we use the master equation in the Markovian limit[37]. The master equation of motion for the density matrix of the entire system may be written as

$$\dot{\rho} = -i[H,\rho] + \gamma\sum_{j=1}^{2} L[a_j]\rho + \beta L[b]\rho + \kappa\sum_{i=1}^{2}[2\sigma_i^-\rho\sigma_i^+ - \sigma_i^+\sigma_i^-\rho - \rho\sigma_i^-\sigma_i^+], \tag{22}$$

where $L[o]\rho = 2o\rho o^\dagger - o^\dagger o\rho - \rho o^\dagger o$, and κ, γ and β are rates, respectively, for spontaneous emission of the atoms, photon leakage out of the cavity and fibre. The master equation can be solved by the numerical method[8].

It is found that for resonant schemes, the high fidelity quantum gates can only be possible if $g/(\kappa,\gamma,\beta)$ is larger than 10[8], which is practical at the present experimental conditions[38-39].

We can use Ramman transition to adiabatically eliminate the exited state of atoms[23], or use multi-atomic scheme to increase the effective coupling between atoms and the cavity mode[8,40]. By adiabatically eliminating excited states, the atomic spontaneous emission can be suppressed. The virtual excitation schemes can also be used to decrease the effects of photon loss[9]. In real experiments, the parameters are different for different experimental setups. Therefore, we should choose different schemes for different experiments. In order to realize the distributed quantum information processes with fiber coupled cavity QED systems, high efficiency coupling between fiber and cavity modes is also needed. In the passed several years, there are some experimental progresses on this direction[39,41-42]. Trapping atoms in cavity is also necessary, but challenging[43-44]. Therefore, we may choose solid spin qubit instead, such as quantum dots[45-46], Nitrogen-vacancy centers[47-48], and et al.

6 Conclusion

In summary, we have reviewed how to realize distributed quantum information processing in fiber-coupled cavity QED systems. We discussed the basic model of fiber-coupled cavity QED systems at first. Then we summarized the schemes based on kinds of dynamics for realizing quantum information processing, such as quantum state transfer, entanglement generation, and quantum controlled-phase gate. We also discussed the effects of dissipation on the quantum gates. It was found that the schemes are practical for the present experimental conditions. And the different types of the schemes should be used for different experimental conditions.

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