Universality and robustness of revivals in the transverse field XY model

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(Received 3 November 2010; revised manuscript received 2 December 2011; published 12 March 2012)

We study the structure of the revivals in an integrable quantum many-body system, the transverse field *XY* spin chain, after a quantum quench. The time evolutions of the Loschmidt echo, the magnetization, and the single-spin entanglement entropy are calculated. We find that the revival times for all of these observables are given by integer multiples of $T_{\rm rev} \simeq L/v_{\max}$, where L is the linear size of the system and $v_{\max}$ is the maximal group velocity of quasiparticles. This revival structure is universal in the sense that it does not depend on the initial state and the size of the quench. Applying nonintegrable perturbations to the *XY* model, we observe that the revivals are robust against such perturbations: they are still visible at time scales much larger than the quasiparticle lifetime. We therefore propose a generic connection between the revival structure and the locality of the dynamics, where the quasiparticle speed $v_{\max}$ generalizes into the Lieb-Robinson speed $v_{\rm LR}$.

DOI: 10.1103/PhysRevA.85.032114 PACS number(s): 03.65.Yz, 05.30.−d

I. INTRODUCTION

The behavior of quantum many-body systems away from equilibrium has recently become the object of intense experimental study. Ultra-cold atoms in optical lattices feature both large phase coherence times and a high degree of controllability, making it possible to observe quantum coherent dynamics [1]. The potential technological implications are profound because the manipulation of coherent quantum dynamics is at the root of the possibility of building a quantum computer.

In terms of theoretical description, systems away from equilibrium are considerably more complicated than their equilibrium counterparts. Whereas equilibrium systems can be understood by means of standard methods like mean-field theory and renormalization group, we lack analogous methods for understanding nonequilibrium physics [2]. There are no obvious generalizations of such standard methods at equilibrium, and in particular, it is not clear to what extent the nonequilibrium dynamics of quantum systems features universality. An important example is the universality of the Kibble-Zurek mechanism to compute the density of defects as the external temperature is tuned in time [3].

Experiments on cold atomic gases give us the opportunity to observe a genuine quantum evolution in a system that is very close to being isolated. We can then address questions like the possibility of equilibration in a system with unitary evolution and the process of thermalization in a closed quantum system. These topics have recently found a renewed interest along with other fundamental problems in quantum statistical mechanics [4,5]. In the case of such systems, the nonequilibrium dynamics is obtained by making some parameters of the system Hamiltonian time dependent and hence casting the system off equilibrium. The time dependence of the Hamiltonian can be adiabatically slow [6] or it can change abruptly [7,8], in which case the process is called a *quantum quench*. Among other applications, the paradigm of the quantum quench has been recently used to study the behavior of topological phases away from equilibrium [9].

The understanding of nonequilibrium dynamics of isolated interacting quantum systems is of fundamental importance to understanding quantum equilibration, a topic that has lately experienced a new renaissance [4,5,10]. Due to unitary time evolution, a *finite* quantum system with a nontrivial initial state cannot converge to a steady state. However, it has been shown in some remarkable papers [11,12] that thermalization of a finite subsystem is possible in the infinite size limit. The system can be coarse grained by choosing a partial set of local or macroscopic observables, and the expectation values of these observables can in principle converge to the ones in thermal equilibrium. In particular, an integrable system does not thermalize: even local or macroscopic observables undergo oscillations, or at best relax to equilibrium values that are generally not the same as those predicted by the microcanonical ensemble [13,14]. Even for a nonintegrable system, thermalization does not always occur: in some cases there is only relaxation to a nonthermal state depending on the initial conditions [15].

The equilibration process has several characteristic time scales. The largest one is the recurrence time $T_{\rm rec}$ at which the system gets infinitesimally close to its initial state. It only exists for finite systems, and diverges at least exponentially as a function of the system size. The smallest time scale is the relaxation time $T_{\rm rel}$ at which a given observable relaxes to its long-term average value. Furthermore, there is a third important time scale $T_{\rm rev}$ in between at which *revivals* occur: these are brief detachments from the average value of an observable. Revivals typically last for a very short time only (in comparison to the spacing between them), and their magnitude decays in time as the equilibration process nears completion.

In this paper we investigate the structure of the revivals for an integrable model, the transverse field *XY* spin chain. Our main result is that this structure is universal in the sense that it does not depend on the details of the quench and on the initial state. By applying nonintegrable perturbations to the system and finding that the revival structure is surprisingly robust against such perturbations, we argue that the revival structure is a universal nonequilibrium property which follows from the locality of the Hamiltonian.

II. FORMALISM OF THE QUANTUM QUENCH

We consider a closed quantum system whose Hamiltonian $H(\lambda_1,\ldots,\lambda_R)$ depends on parameters λ_i representing the coupling strengths of interactions and external fields. A quantum quench is a sudden change in the Hamiltonian of the system. The quantum system is originally prepared in the ground state ρ_0 of $H(\lambda^{(1)})$, and at time $t = 0$ we switch the parameters to different values $\lambda^{(2)}$. The system then evolves unitarily with the quench Hamiltonian $H(\lambda^{(2)})$ according to $\rho(t) = \mathcal{U}_t(\rho_0)$, where we define the superoperator $\mathcal{U}_t(X) \equiv \exp(-iHt)X\exp(iHt)$. Unitary evolution implies that a finite system cannot converge to a steady state $\overline{\rho}$, even weakly. The limit of $\rho(t)$ for $t \to \infty$ does not exist unless the initial state ρ_0 is trivial, for example, an eigenstate. On the other hand, the time average $\overline{\rho} = \lim_{t\to\infty} t^{-1}\int_0^t \rho(s)ds$ always exists and is given by the ρ_0 totally dephased in the eigenbasis $\Pi_n = |E_n\rangle\langle E_n|$ of the Hamiltonian: $\overline{\rho} = \sum_n \Pi_n\rho_0\Pi_n$. For a finite system, equilibration means that the expectation values of macroscopic observables spend most of their time very close to their average values.

An important quantity describing the time evolution is the Loschmidt echo (LE) defined as $\mathcal{L}(t) = |\mathrm{tr}[\exp(-itH)\rho_0]|^2$ which gives a measure of the distance between the time evolved state $\rho(t)$ and the initial state ρ_0. When the system undergoes a recurrence at $t = T_{\rm rec}$ we have $\mathcal{L}(T_{\rm rec}) \simeq 1$. The general expression for the LE can be written as

$$\mathcal{L}(t) = \sum_{n,m} p_n p_m e^{-i(E_n - E_m)t}, \tag{1}$$

where p_n are the populations of the Hamiltonian eigenstates for the initial state. It follows that the time average of the LE is $\overline{\mathcal{L}} = \mathrm{tr}(\overline{\rho}^2) = \sum_n p_n^2$. The LE typically decays in a short time $T_{\rm rel}$ from 1 to its average value $\overline{\mathcal{L}}$ around which it oscillates. The relaxation time $T_{\rm rel}$ is $O(1)$ for an off-critical quench, while it scales like $O(L^\zeta)$ with the system size L for a critical quench, that is, if $\lambda^{(2)}$ is a critical point, showing a critical *slow down* of the system [16,17].

Revivals are also visible in the LE as deviations from the average value $\overline{\mathcal{L}}$. We define revivals as time instances at which the signal $\mathcal{L}(t)$ differs from $\overline{\mathcal{L}}$ by more than three standard deviations. According to Eq. (1), this happens when an exceptionally large number of weights p_n get partially back in phase. It is not straightforward to understand directly from Eq. (1) when such a situation can occur in a generic quantum system, therefore we consider the particular case of a simple one-dimensional spin chain.

III. REVIVALS IN THE *XY* MODEL

A. Exact solution by free fermions

In this section we consider an integrable (exactly solvable) model, the one-dimensional *XY* model of N spins one half in a transverse magnetic field. Since the model is exactly solvable, we can obtain the whole spectrum and the eigenstate decomposition of the initial state. This leads to an exact expression for the LE, and the revival times can be extracted by inspecting its time dependence.

The Hamiltonian of this spin chain is given by

$$H = -\frac{1}{2}\sum_{l=1}^{N}\left(\frac{1+\eta}{2}\sigma_l^x\sigma_{l+1}^x + \frac{1-\eta}{2}\sigma_l^y\sigma_{l+1}^y + h\sigma_l^z\right), \tag{2}$$

where η is the anisotropy parameter, and h is the external transverse magnetic field. We assume cyclic boundary conditions $\sigma_{N+1} = (-1)^q\sigma_1$. The periodic ($q = 0$) and antiperiodic ($q = 1$) boundary conditions differ in $O(1/N)$ terms and this difference usually does not affect the phase diagram or other quantities in the thermodynamic limit. However, it can be important in the LE that is typically exponentially small in N. In the following we shall see that the boundary conditions can have a dramatic effect in the case of the critical quench. Note that the *XY* model reduces to the quantum Ising model for $\eta = 1$, and to the isotropic *XX* model for $\eta = 0$. The Hamiltonian exhibits two regions of criticality: the *XX* model at $\eta = 0$ has a critical region for $h \in (-1,1)$, while the *XY* regions of criticality are the lines $h = \pm 1$.

The *XY* model can be diagonalized by a standard procedure [18]. In the first step we map σ_l to spinless fermions by using a Jordan-Wigner transformation:

$$\sigma_l^z = 1 - 2c_l^\dagger c_l, \quad \sigma_l^- = (\sigma_l^+)^\dagger = c_l^\dagger e^{i\pi\sum_{j=1}^{l-1}c_j^\dagger c_j}. \tag{3}$$

The translational symmetry is then exploited by the Fourier transform $c_l = \frac{1}{\sqrt{N}}\sum_{l=1}^{N} e^{ikl}c_k$, where the momenta are quantized according to $k_n = \pi(2n+1-q)/N$. Finally, after the Bogoliubov transformation $c_k = \cos\theta_k\gamma_k + i\sin\theta_k\gamma_{-k}^\dagger$, we obtain a Hamiltonian describing noninteracting fermionic degrees of freedom γ_k:

$$H = \sum_{k>0}\Lambda_k(\gamma_k^\dagger\gamma_k + \gamma_{-k}^\dagger\gamma_{-k} - 1). \tag{4}$$

The dispersion relation of these fermionic quasiparticles is given by $\Lambda_k = \sqrt{\epsilon_k^2 + \eta^2\sin^2 k}$ with $\epsilon_k \equiv h - \cos k$, and the angle θ_k appearing in the Bogoliubov transformation is $\theta_k = \tan^{-1}[(\eta\sin k)/(\epsilon_k + \Lambda_k)]$.

B. Loschmidt echo and revival times

In a quantum quench, the system is prepared in the ground state of $H(\lambda^{(1)})$, and then evolved with $H(\lambda^{(2)})$ at $t > 0$. It is useful to write the ground state $|\psi(0)\rangle$ of the initial Hamiltonian in terms of the eigenstates of the quench Hamiltonian:

$$|\psi(0)\rangle = \prod_{k>0}(\cos\chi_k - i\sin\chi_k\gamma_k^\dagger\gamma_{-k}^\dagger)|0_k\rangle, \tag{5}$$

where $\chi_k \equiv \theta_k^{(2)} - \theta_k^{(1)}$, and $|0_k\rangle$ is the vacuum state defined by $\gamma_k|0_k\rangle = \gamma_{-k}|0_k\rangle = 0$. The time evolution then reads $|\psi(t)\rangle = \exp(-iHt)|\psi(0)\rangle$, and the LE takes the form

$$\mathcal{L}(t) = \prod_{k>0}[1 - A_k\sin^2(\Lambda_k t)], \tag{6}$$

where the coefficient $A_k \equiv \sin^2(2\chi_k)$ is a slowly varying positive function of the momentum k.

Now we show how the revivals in the LE can be derived from the dispersion relation Λ_k of the quasiparticles. We first take the logarithm of Eq. (6) to transform the product into a sum over momentum k:

$$\ln \mathcal{L}(t) = \sum_{k>0} \ln[1 - A_k \sin^2(\Lambda_k t)]. \tag{7}$$

Since each $k > 0$ term has a periodicity π/Λ_k in time, and A_k varies sufficiently slowly with k, this expression shows that nearby k modes separated by $\Delta k = 2\pi/N$ add up constructively whenever $\Delta\Lambda_k t = p\pi$ with $p \in \mathbb{Z}$. This rearranges to $t_k = \frac{1}{2} pN|\partial\Lambda_k/\partial k|^{-1}$, and in principle we could expect a revival time t_k corresponding to each k. However, more modes can add up constructively if the dispersion relation Λ_k is closer to a straight line, therefore the most pronounced revivals (in fact, the only revivals that stand out from the background noise) are given by the stationary values of the group velocity $v_g(k) \equiv |\partial\Lambda_k/\partial k|$. The first of these revivals is the one corresponding to the maximal group velocity $v_{\max} = \max_k v_g(k)$ and we can thus give the following estimate for the revival time scale:

$$T_{\text{rev}} \simeq \frac{N}{2\, v_{\max}} = \frac{N}{2} \left| \frac{\partial\Lambda_k}{\partial k} \right|_{\max}^{-1}. \tag{8}$$

The maximum group velocity $v_{\max}$ can be computed exactly from the dispersion relation Λ_k. It takes a particularly simple form in the case of the Ising model ($\eta = 1$): $v_{\max} = h$ when $h < 1$ and $v_{\max} = 1$ when $h \geqslant 1$.

In Fig. 1(a) the LE for a critical quench ($h_2 = 1$) with antiperiodic boundary conditions ($q = 1$) is plotted. The T_{rev} predicted by Eq. (8) is in perfect agreement with the data. The same critical quench with periodic boundary conditions ($q = 0$) is interesting. At the odd revivals there is no signal in the LE due to a destructive (vanishing) contribution from one of the k modes in the product of Eq. (6). Only the even revivals are spotted.

Figures 1(b) and 1(d) verify that the revival times scale like $O(L)$, where $L \sim N$ is the linear size of the system. The quench in Fig. 1(b) is noncritical, and the parameters of the quench Hamiltonian are far away from any phase boundaries. This means that the system is gapped, and a simple spectral analysis would imply that T_{rev} is of $O(1)$. However, even if there is a small reconstruction of the wave function at a time scale $1/\Delta$ (where Δ is a difference between any two energy levels), the weight involved is not sufficiently large to make the corresponding revival strong enough. *Visible revivals* are governed by Eq. (8).

In Fig. 1(c) the system is quenched from different ground states corresponding to different parameter values $\lambda^{(1)}$. As one can see, the details of the evolution and the average values $\overline{\mathcal{L}}$ are different, but the structure of the revivals is the same for all the quenches, and the revival times are consistent with those predicted by Eq. (8). This is the promised *universality* of the revival structure: the initial state and the size of the quench are unimportant. The parameters $\lambda^{(2)}$ determine $v_{\max}$ and therefore T_{rev} but not the fact that revivals happen at time instances spaced evenly at intervals that are linear in system size: $t_p = pL/v_{\max}$.

C. Magnetization and entanglement entropy

Although the behavior of the LE is illuminating for theoretical arguments, it is hardly of experimental relevance because the amplitude of the signal is zero in the thermodynamic limit. On the other hand, it is expected that a revival in the LE corresponds to revivals in macroscopic observables as well. To demonstrate this we study the time evolution of the order parameter (magnetization) $\mu(t)$ [19] and of the single-spin entanglement entropy $S(t)$. If we consider a subsystem A consisting of a single spin, the von Neumann entropy between subsystem A and the rest of the system is $S = -\text{Tr}(\rho_A \log_2 \rho_A)$, where ρ_A is the reduced density matrix of subsystem A.

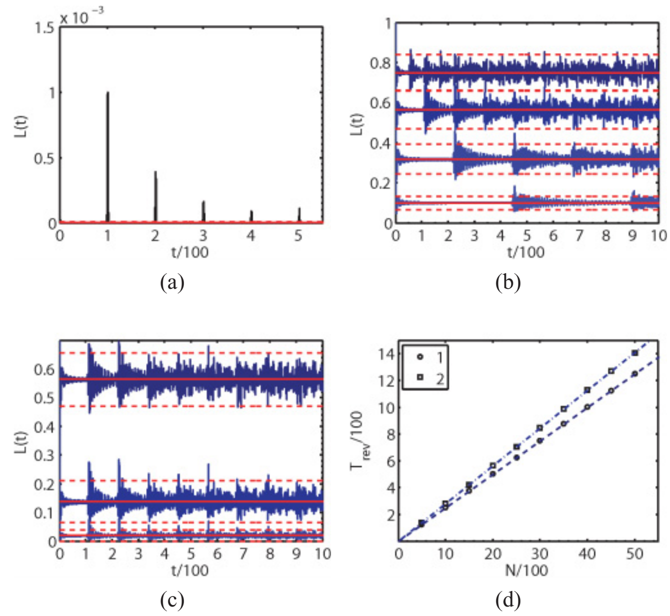


FIG. 1. (Color online) Loschmidt echo for the quenched *XY* model with antiperiodic boundary conditions ($q = 1$). (a) Critical quench with $\eta_1 = \eta_2 = 2.0$, $h_1 = 0.5$, $h_2 = 1.0$, $N = 400$, and $v_{\max} = 2.0$. Equation (8) gives $T_{\text{rev}} = 100.0$, and from the plotted data we get $T_{\text{rev}} \approx 99.7$. (b) Noncritical quenches for different system sizes: $\eta_1 = \eta_2 = 2.0$, $h_1 = 0.7$, $h_2 = 0.8$, $N \in \{200,400,800,1600\}$, and $v_{\max} \approx 1.77$. The equilibrium value of the LE decreases with system size. Equation (8) gives $T_{\text{rev}} \approx 112.7$ for $N = 400$, and from the data we get $T_{\text{rev}} \approx 113.0$. (c) Quenches of different size: $\eta_1 = \eta_2 = 2.0$, $h_1 \in \{0.5,0.6,0.7\}$, $h_2 = 0.8$, and $N = 400$. Group velocities and revival times match those in (b). (d) Linear scaling of the revival times with system size. The lines represent the values given by Eq. (8). Parameters used: 1: those in (a), 2: those in (b). In each plot the red horizontal lines represent average values and three standard deviations thereof.

When the initial state of the quenching process in Eq. (5) is expanded in position basis rather than momentum basis, one finds that the excitations are pairwise correlated between different lattice sites due to the fermionic anticommutation relations. This implies that ρ_A is diagonal, and then we can use translational invariance to establish

$$S(t) = -\mu(t) \log_2 \mu(t) - [1 - \mu(t) \log_2 (1 - \mu t)], \tag{9}$$

where the average magnetization is given by

$$\mu(t) = \frac{1}{N} \sum_l \langle c_l^\dagger c_l \rangle = \frac{1}{N} \sum_k \langle c_k^\dagger c_k \rangle,$$

$$\begin{aligned} \langle c_k^\dagger c_k \rangle = {} & \sin^2(\theta_k)\cos^2(\chi_k) + \cos^2(\theta_k)\sin^2(\chi_k) \\ & - 2\sin(\theta_k)\cos(\theta_k)\sin(\chi_k)\cos(\chi_k)\cos(2\Lambda_k t). \end{aligned} \tag{10}$$

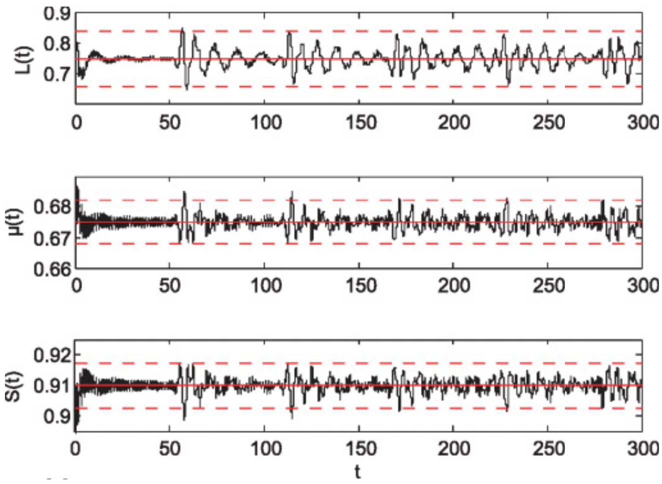


FIG. 2. (Color online) A small quench with $q = 1$, $\eta_1 = \eta_2 = 2.0$, $h_1 = 0.7$, $h_2 = 0.8$, $N = 200$, and $v_{\max} \approx 1.77$. From top to bottom: Loschmidt echo $\mathcal{L}(t)$, magnetization $\mu(t)$, and single-spin entanglement entropy $S(t)$. Equation (8) gives $T_{\rm rev} \approx 56.37$, and from the plotted data we get $T_{\rm rev} \approx 56.58$. The horizontal lines represent average values and three standard deviations thereof.

The entanglement entropy S and the average magnetization μ are thus governed by the interference of the same modes as in Eq. (6) with their frequencies given by the same dispersion relation Λ_k. This means that they share the same time scales as the LE. The results are illustrated in Fig. 2. In fact, since the magnetization is given as a sum rather than a product of different oscillating modes it can be used instead of the LE to determine $T_{\rm rev}$ in the large N limit when the LE has a very small average value. In practice, the magnetization is analogous to the logarithmic Loschmidt echo used in [20].

D. Time evolution of a local disturbance

Now we investigate the time evolution after a local disturbance in the spin chain. In particular, we consider a single-spin flip at position l, which is represented by the operator $F_l = c_l + c_l^\dagger$. The time evolution of the resulting local disturbance is best studied in the Heisenberg picture, where the operator F_l becomes time dependent and takes the form

$$\begin{aligned} F_l(t) &= \sum_{l'} [\Omega_{l-l'}(t) c_{l'} + \Omega^*_{l-l'}(t) c^\dagger_{l'}], \\ \Omega_{l-l'}(t) &= \frac{1}{N} \sum_k e^{ik(l-l')} [e^{i\Lambda_k t} \sin^2\theta_k + e^{-i\Lambda_k t} \cos^2\theta_k \\ &\quad + i(e^{-i\Lambda_k t} - e^{i\Lambda_k t}) \sin\theta_k \cos\theta_k]. \end{aligned} \tag{11}$$

This expression shows that nearby modes at momentum k add up constructively whenever $\Delta\Lambda_k t = 2p\pi \pm \Delta k |l - l'|$ with $p \in \mathbb{Z}$. Due to $\Delta k = 2\pi/N$, this condition can be written as $t = (pN \pm |l - l'|)|\partial\Lambda_k/\partial k|^{-1}$, and one verifies that the disturbance travels with the group velocity $v_g(k) = |\partial\Lambda_k/\partial k|$. Once again, more such modes can add up constructively if the second derivative of Λ_k vanishes, therefore we expect visible wave packets to travel with the stationary values of $v_g(k)$. Indeed, Fig. 3(a) shows that the different wave packets corresponding to the local extrema of $v_g(k)$ propagate through the lattice while maintaining their respective wave forms.

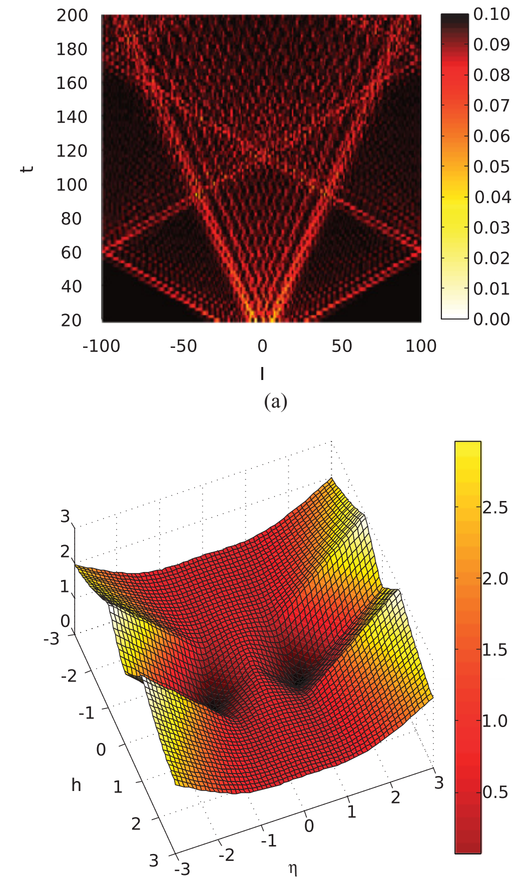
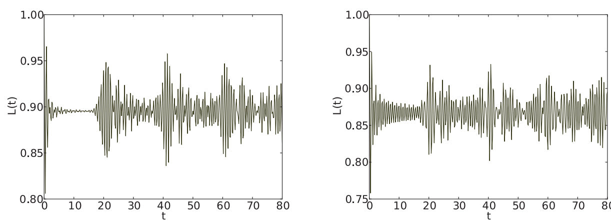


FIG. 3. (Color online) (a) Time evolution of a local disturbance (local quench) $|\Omega_l(t)|^2$ with $q = 1$, $\eta_1 = \eta_2 = 2.0$, $h_1 = 0.7$, $h_2 = 0.8$, and $N = 200$. The maximum speed is $v_{\max} \approx 1.77$. The corresponding global quench is illustrated in Fig. 1(b) for comparison. (b) Maximal group velocity $v_{\max}$ as a function of η and h.

The fastest wave packets travel with the maximal group velocity $v_{\max}$, and the first revivals can be interpreted as constructive interferences between them. Since we assume cyclic boundary conditions, one can think of the spin chain as a closed ring. In this picture the fastest wave packets first meet halfway in the ring at time $t = N/2v_{\max}$, which indeed coincides with the first revival time in Eq. (8).

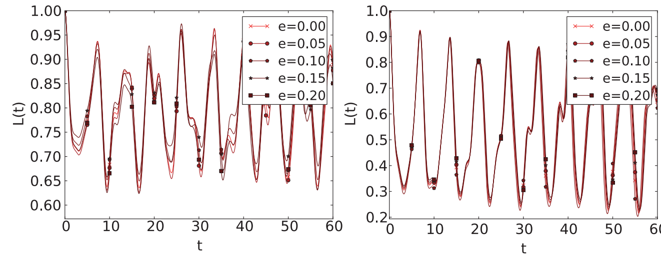
IV. NONINTEGRABLE PERTURBATIONS

In the previous section we showed that the structure of the revivals is governed by the maximal speed of quasiparticles in the system. We found that as long as the quasiparticles exist, the details of the quench are not relevant. There is universality within the integrable behavior of the system. At this point we wonder whether there is universality beyond the quasi-free system. The local physics induced by the local Hamiltonian might imply that as long as information is not completely lost—as in the case of an infinite system—revivals can be observed due to the recombination of the fastest signals even when these are not point-like and do not correspond to quasiparticles. To investigate this possibility, we now study



(a) (b)

FIG. 4. (Color online) Loschmidt echo for the quenched *XZ* spin chain with periodic boundary conditions in the integrable case $g_2 = 0$ (a) and the nonintegrable case $g_2 = 0.3$ (b). The other quench parameters are $\eta_1 = \eta_2 = 1.0$, $h_1 = 10.0$, $h_2 = 4.0$, and $g_1 = 0.0$ in both parts. Since $N = 40$ and $v_{\max} = 1$, Eq. (8) gives $T_{\rm rev} = 20$.



(a) (b)

FIG. 5. (Color online) Loschmidt echo for the transverse field Ising model when the external field has a small site-dependent component of varying magnitude: $\eta_1 = \eta_2 = 1.0$, $N = 13$, periodic boundary conditions. (a) Noncritical case: $h_1 = 0.5$, $h_2 = 0.8$, $\epsilon_1 = 0.0$, $\epsilon_2 = e$. Equation (8) gives $T_{\rm rev} = 8.12$ for $\epsilon = 0.0$. (b) Critical case: $h_1 = 0.5$, $h_2 = 1.0$, $\epsilon_1 = 0.0$, $\epsilon_2 = e$. Equation (8) gives $T_{\rm rev} = 6.5$ for $\epsilon = 0.0$.

the robustness of the revival structure against nonintegrable perturbations.

A. *XZ* spin chain

We start by considering the *XZ* spin chain, which contains an additional $\sigma_l^z \sigma_{l+1}^z$ coupling with respect to the quantum Ising model ($\eta = 1$):

$$H = -\frac{1}{2}\sum_{l=1}^{N}\left(\sigma_l^x\sigma_{l+1}^x + g\sigma_l^z\sigma_{l+1}^z + h\sigma_l^z\right). \qquad (12)$$

This Hamiltonian is nonintegrable, and we simulate the quantum quench by exact numerical diagonalization. To achieve a relatively large system size ($N \simeq 50$), we take the limit of large field ($h \gg 1$), restricting the effective Hilbert space to states where almost all spins are aligned with the field.

As shown in Fig. 4, the structure of the revivals is clearly visible for $|g| \lesssim 0.5$, and hence this structure is universal for a *range* of the nonintegrability parameter g. The revival times scale linearly with N, and the range of visibility is largely independent of both the system size N and the magnetic field h. We find that pronounced revivals gradually disappear in the range $0.3 < |g| < 0.7$ for all $20 \leqslant N \leqslant 50$ when $h = 4$ is fixed, and for all $2 \leqslant h \leqslant 100$ when $N = 20$ is fixed. Since $v_{\max} \sim 1$ in the Ising model for $h \geqslant 1$, we see that the nonequilibrium dynamics is dominated by the $\sigma_l^x\sigma_{l+1}^x$ term in Eq. (12). The *XZ* spin chain is therefore significantly nonintegrable for $|g| \sim 0.5$, even in the $h \to \infty$ limit. This claim is further supported by the fact that the visibility range in g does not depend on h. We finally note that the equilibrium (long-term average) value of the magnetization is strongly dependent on the initial state and so this equilibration is not thermalization, even though the system is nonintegrable [15].

B. Random disorder in the field

Now we consider another integrability breaking perturbation to the *XY* spin chain. We introduce a site-dependent external field component e_l to the Hamiltonian that explicitly breaks the translational invariance and the integrability of the model. The total site-dependent field is $h_l = h + e_l$, where e_l is randomly picked from a uniform distribution with a maximum amplitude ϵ in the sense $-\epsilon < e_l < \epsilon$.

Using exact diagonalization and exploiting the fact that the Hamiltonian decomposes to subspaces of odd and even number of spins, we can assess the effect of the site-dependent field disturbance for modest sized systems $N \leqslant 13$. The simulations run on such systems give supporting evidence that the revival structure is essentially unchanged for a range of the amplitude ϵ. In Fig. 5 the Loschmidt echo for a critical and a noncritical system is plotted with various values of ϵ to illustrate this observation.

V. DISCUSSION

We found in Sec. III that the transverse field *XY* model exhibits a universal structure of revivals that is independent of the initial state and the size of the quench. Since the *XY* model is integrable and its exact solution is in terms of free fermionic quasiparticles, there is a straightforward interpretation for the phenomenon of revivals. The information propagates around the system via wave packets of quasiparticles, and the first revival occurs when the wave packets traveling with the maximal group velocity $v_{\max}$ meet. This interpretation explains the universality of the revival structure since $v_{\max}$ only depends on the dispersion relation of the quasiparticles associated with the quench Hamiltonian (and nothing which would be related to the initial state).

On the other hand, the robustness against nonintegrable perturbations found in Sec. IV suggests something more generic than the quasiparticle interpretation. The quasiparticles of the perturbed system are no longer free, but there is a finite interaction between them. For the *XZ* spin chain, this interaction is on the order of g as can be verified by looking at the exact eigenvalues for finite systems. This implies that the quasiparticles decay on the time scale of $1/g$, and hence one would not expect to see revivals at time scales much larger than $1/g$. However, the evidence is on the contrary: at $N = 50$ and $g \sim 0.5$, revivals are still clearly visible at $t \sim 50$, which is an order of magnitude larger than $1/g \sim 2$.

It appears that the revival structure is a nonequilibrium property that is beyond integrability and the existence of stable quasiparticles. Here we provide a more generic interpretation in terms of *locality*. Quantum many-body physics is generally described by Hamiltonians that can be written as sums of local operators. The locality of the Hamiltonian has profound consequences on the dynamics of the system [21]: there exists an emergent light cone for the propagation of information such that signals outside the light cone are exponentially suppressed. The characteristic speed of the light cone gives the maximal speed of information in the system, and it is called the Lieb-Robinson speed $v_{\rm LR}$. In general, it depends on both the graph of the system and the strengths of the interactions in the Hamiltonian.

We speculate that the revival structure described in the previous sections is much more generic and valid whenever a many-body quantum system has local dynamics, as long as the integrability breaking is not too strong and hence information is not completely lost in the system. Since the propagation of information is governed by $v_{\rm LR}$, the revival time scale becomes $T_{\rm rev} \simeq L/v_{\rm LR}$ in general. In the integrable case, $v_{\rm LR}$ reduces to the quasiparticle speed $v_{\max}$, and hence we recover the revival time scale in Eq. (8). On the other hand, the locality of the dynamics is intact in the nonintegrable case as well, providing a natural explanation for the robustness of the revival structure.

VI. SUMMARY

In this paper we studied the phenomenon of revivals after a quantum quench in the transverse field *XY* model, and found a nontrivial revival structure that cannot be obtained from a simple spectral analysis. It was shown that this structure is universal in the sense that it does not depend on the initial state and the size of the quench. Revivals were shown to be related to quasiparticles propagating around the system with a finite maximum speed $v_{\max}$, and a corresponding estimate for the revival time scale was established.

We also investigated the effect of nonintegrable perturbations on the structure of the revivals. In particular, we considered the *XZ* spin chain and a random disorder in the magnetic field. It was found that the revival structure is clearly visible at time scales far beyond the lifetime of quasiparticles, implying that something more generic than integrability is behind the phenomenon of revivals. In perspective of this, we proposed a generic connection between revivals and locality, where the quasiparticle speed $v_{\max}$ generalizes into the Lieb-Robinson speed $v_{\rm LR}$. We believe that a thorough understanding of this important connection requires further study of nonintegrable systems, for example, studying how the entanglement production in the subsystem is related to the loss of the revival structure.

ACKNOWLEDGMENTS

We thank Lorenzo Campos Venuti and Paolo Zanardi for important discussions. Research at Perimeter Institute for Theoretical Physics is supported in part by the Government of Canada through NSERC and by the Province of Ontario through MRI.

[1] M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, Nature (London) **415**, 39 (2002); M. Greiner, O. Mandel, T. W. Hänsch, and I. Bloch, *ibid.* **419**, 51 (2002).

[2] A. Polkovnikov, K. Sengupta, A. Silva, and M. Vengalattore, Rev. Mod. Phys. **83**, 863 (2011).

[3] T. Kibble, Phys. Today **60**, 47 (2007); W. H. Zurek, Phys. Rep. **276**, 177 (1996).

[4] H. Tasaki, Phys. Rev. Lett. **80**, 1373 (1998); S. Goldstein, J. L. Lebowitz, R. Tumulka, and N. Zanghì, *ibid.* **96**, 050403 (2006); P. Reimann, *ibid.* **99**, 160404 (2007); **101**, 190403 (2008).

[5] S. Popescu, A. J. Short, and A. Winter, Nat. Phys. **2**, 754 (2006); N. Linden, S. Popescu, A. J. Short, and A. Winter, Phys. Rev. E **79**, 061103 (2009).

[6] A. Polkovnikov, Phys. Rev. B **72**, 161201(R) (2005); L. Campos Venuti and P. Zanardi, Phys. Rev. Lett. **99**, 095701 (2007).

[7] A. Silva, Phys. Rev. Lett. **101**, 120603 (2008); M. A. Cazalilla, *ibid.* **97**, 156403 (2006).

[8] P. Calabrese and J. Cardy, Phys. Rev. Lett. **96**, 136801 (2006).

[9] D. I. Tsomokos, A. Hamma, W. Zhang, S. Haas, and R. Fazio, Phys. Rev. A **80**, 060302(R) (2009).

[10] M. Rigol, V. Dunjko, and M. Olshanii, Nature (London) **452**, 854 (2008).

[11] M. Cramer, C. M. Dawson, J. Eisert, and T. J. Osborne, Phys. Rev. Lett. **100**, 030602 (2008); M. Cramer and J. Eisert, New J. Phys. **12**, 055020 (2010).

[12] M. Cramer, A. Flesch, I. P. McCulloch, U. Schollwöck, and J. Eisert, Phys. Rev. Lett. **101**, 063001 (2008).

[13] M. Rigol, V. Dunjko, V. Yurovsky, and M. Olshanii, Phys. Rev. Lett. **98**, 050405 (2007); M. A. Cazalilla, *ibid.* **97**, 156403 (2006).

[14] M. Rigol, Phys. Rev. A **80**, 053607 (2009).

[15] C. Gogolin, M. P. Müller, and J. Eisert, Phys. Rev. Lett. **106**, 040401 (2011).

[16] H. T. Quan, Z. Song, X. F. Liu, P. Zanardi, and C. P. Sun, Phys. Rev. Lett. **96**, 140604 (2006).

[17] L. Campos Venuti and P. Zanardi, Phys. Rev. A **81**, 022113 (2010).

[18] E. Barouch, B. M. McCoy, and M. Dresden, Phys. Rev. A **2**, 1075 (1970).

[19] F. Igloi and H. Rieger, Phys. Rev. Lett. **106**, 035701 (2011).

[20] J.-M. Stéphan and J. Dubail, J. Stat. Mech. (2011) P08019.

[21] E. H. Lieb and D. W. Robinson, Comm. Math. Phys. **28**, 251 (1972); B. Nachtergaele, Y. Ogata, and R. Sims, J. Stat. Phys. **124**, 1 (2006).