When Online Dating Meets Nash Social Welfare: Achieving Efficiency and Fairness

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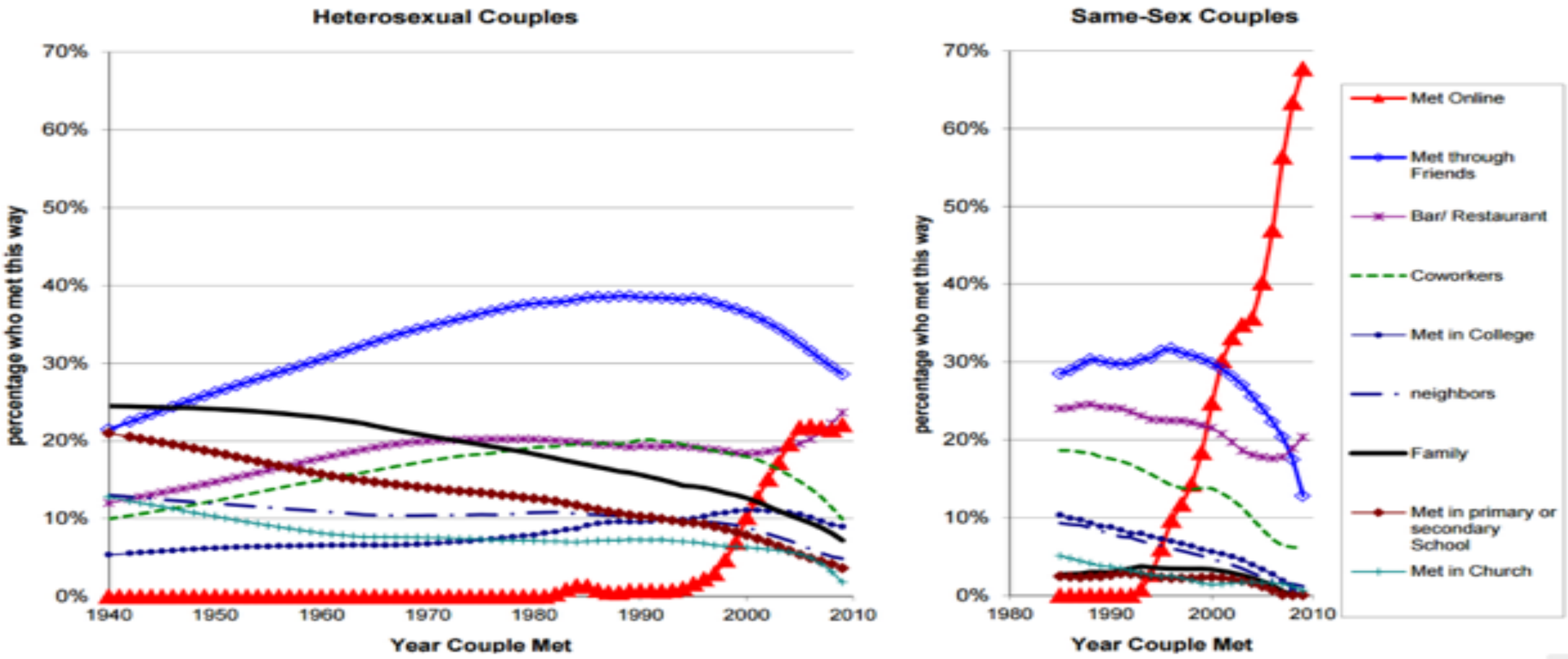
Outlines

- A brief intro to **online dating**.
- Why do we care both **efficiency** and **fairness**?
- How to model a user’s utility?
- How to trade-off efficiency and fairness in online dating?
- Apply our algorithms in real dating apps.

Online Dating Trend: High engagement + High Per-user Value



Per-User Value: 243$/User/Yr (US)

Online Dating: A solid business model based on growing user demands.

Online Dating: Solutions

Online Dating 1.0

- One-sided approach: Filter + Search + Message
- Mostly web-based
- eHarmony, match.com, jiayuan.com, baihe.com
- Advantage: Better for long-term relationships.

Online Dating 2.0

- Two-sided market design
- Mostly mobile-based
- Double Opt-in Mechanism + AI-based recommendations
- Tinder, Badoo, Coffee Meets Bagel, Bumble, TanTan
- Advantage: Simple and Fun

Era of Online Dating 2.0

50M active users
26M daily matches

50,000 couples
997M total matches

17.5M users

6M daily
active users

Double Opt-in Mechanism (two-sided market)

- **Simple** and **fun** user experience through swiping
- Remove the awkwardness of rejection and introducing oneself (only mutual-like users can start to chat)

Online Dating vs. Other Two-sided Markets

Online Ads

Job Markets

Ride-sharing

- Online dating is more **decentralized.**
- Platform can only control **impressions**. (i.e., show who to whom.)
- Hard to predict user behavior: gender differences, individual differences, various motivations, etc.

Online Dating Market Design

- Market design goals

  Efficiency: Maximize total matches (i.e., welfare)

  Fairness: Help each user get a number of matches to keep a high user retention rate.

KPIs: Retention, Engagement, Per-User Value (or LTV)

Fairness is More Important and Difficult

- Fairness is **more** important. (discuss later)
- Online dating markets **cannot be totally fair**.
- Some factors are **uncontrollable** by the platform:

Each user's attractiveness/desirability is the intrinsic unfairness in online dating.

Users tend to like attractive candidates regardless of their own attractiveness

(Hitsch et al. 2010).

Algorithms can help to improve fairness

- Some factors are **controllable** by the platform:
    - Premium features (e.g., boost, superlike, Woo)
    - # of Impressions
    - Recommendation/matching algorithms
- Recommendation algorithms can control the match distribution of the users, and help less attractive users also get a number of matches. Therefore the dating apps can relieve the negative effect of the intrinsic unfairness in the market and satisfy more users.

Challenges to Achieve Efficiency & Fairness

- One systematic framework to trade-off efficiency and fairness.

  Efficiency and fairness do not always align.

- Need to design effective algorithm

  Tremendous user base ==> **Fast** algorithm

  Real-time recs without full information ==> **Online** algorithm

Our Contributions

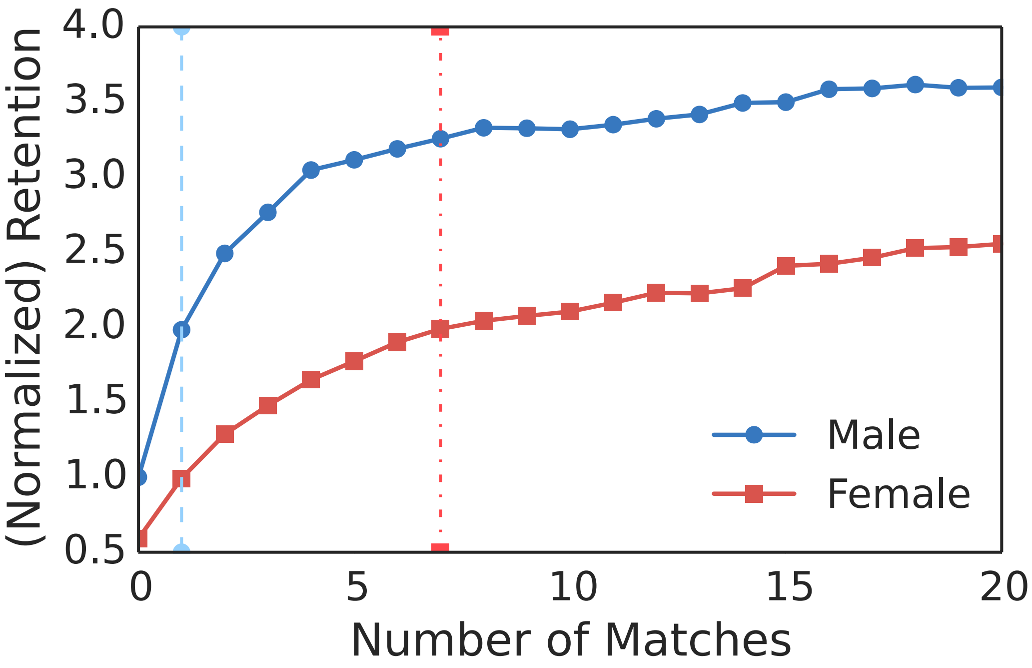
- A systematic framework to capture both efficiency and fairness
  - Use data-driven analysis to model user's utilities
  - The model captures both efficiency and fairness
- Design fast online algorithms to achieve efficiency and fairness
  - Use online submodular maximization to get **online** solutions.
  - Use Nash social welfare to better trade-off efficiency and fairness.

*Our algorithm can improve the efficiency by **26%** and fairness by **99%** in real online dating apps.*

Related Work

- **Online dating** markets and applications: user motivation, gender difference, economics, matching and sorting algorithms, etc.
- Other **two-sided markets**: Airbnb, Uber, Google's Adwords, etc
- **Methodologies**: submodular optimization, fair division, Nash social welfare, Fisher market, etc.

Retentions vs. Matches



- More matches => higher retention
- Males' retention is much more sensitive to matches
- The retention improves fast when a male has<7 weekly matches.

Retention Rate: A widely-used quantitative metric for utility

More Observations

- Improving each male's weekly matches to about 7 (i.e., we call this **the match goal** for males’ matches) will promote the males’ retention rate significantly. If a male gets more matches than the match goal, then the improvement is meaningless.
- The retention curves for both males and females are concave, indicating the **diminishing marginal returns** when a user gets more matches.
- We care more on males’ number of matches as the males’ retention rate is more sensitive to the matches.

Details: Two-sided online dating market settings

- Two-sided users (heterosexual): M males (m), F females (f)
- Total round: T, each round denoted as (t)
- Number of swipes (capacity): $c_m^{(t)}, \bar{c}_f^{(t)}$
- Preference score to another user (swipe-right rate): $p_{m,f}^{(t)}, \bar{p}_{f,m}^{(t)}$
- Match score (probability of a mutual like between each pair):

$$w_{m,f}^{(t)} = p_{m,f}^{(t)} \cdot \bar{p}_{f,m}^{(t)}$$

- Recommendation from m to f: $x_{m,f}^{(t)} \in \{0, 1\}$
- Impression set: $I_m^{(t)} = \{f | x_{m,f}^{(t)} = 1\}$

User's Matches

- Match goal (expected number of matches): $g_m^{(t)}$
- Achieved matches: $$a_m^{(t)} = \sum_{f \in [F]} w_{m,f}^{(t)} \cdot x_{m,f}^{(t)}$$
- Match achievement rate: $$r_m^{(t)} = \frac{a_m^{(t)}}{g_m^{(t)}}$$

From the above observations, 7 weekly matches is a reasonable match goal.

User's Utility Functions

- Symmetric utility function: $u_m^{(t)}$

  Weight parameter for m: $\alpha_m^{(t)}$
- Utility function (degree of satisfaction) for male m:

$$s_m^{(t)} = \alpha_m^{(t)} \cdot u_m^{(t)}(r_m^{(t)}) = \alpha_m^{(t)} \cdot u_m^{(t)}\left(\frac{\sum_{f \in [F]} w_{m,f}^{(t)} \cdot x_{m,f}^{(t)}}{g_m^{(t)}}\right)$$

Paying users / New users may have higher weight parameters.

Maximize users' total utilities

$$max : \sum_{m \in [M]} s_m^{(t)}$$

Objective: maximize total utilities

s.t.,

$$\sum_{m \in [M]} x_{m,f}^{(t)} \le \bar{c}_f^{(t)}, \quad \forall f \in [F];$$

Male's capacity constraint

$$\sum_{f \in [F]} x_{m,f}^{(t)} \le c_m^{(t)}, \quad \forall m \in [M];$$

Female's capacity constraint

$$x_{m,f}^{(t)} \in \{0, 1\}, \quad \forall m \in [M], \forall f \in [F].$$

Define utility functions on impression sets

- Recall a male's impression set: $I_m^{(t)} = \{f | x_{m,f}^{(t)} = 1\}$

  is the set of females whom we show m's profile to.
- The utility function on impression set: $\mu_m^{(t)}(I_m^{(t)})$

$$s_m^{(t)} = \mu_m^{(t)}(I_m^{(t)}) = \alpha \cdot u_m^{(t)}\left(\frac{\sum_{f \in I_m^{(t)}} w_{m,f}^{(t)}}{g_m^{(t)}}\right) \quad \forall m \in [M].$$

Key Property: Monotone Submodular

- **Monotone:** more matches ==> higher utility (implies **efficiency**)

$$\tilde{I}_m^{(t)} \subseteq I_m^{(t)} \Rightarrow \mu_m(I_m^{(t)}) \geq \mu_m(\tilde{I}_m^{(t)})$$

- **Submodular:** Diminishing marginal utility when a user gets more matches (implies **fairness**).

$$\tilde{I}_m^{(t)} \subseteq I_m^{(t)} \Rightarrow \mu_m(I_m^{(t)} \cup \{f\}) - \mu_m(I_m^{(t)}) \leq \mu_m(\tilde{I}_m^{(t)} \cup \{f\}) - \mu_m(\tilde{I}_m^{(t)})$$

Online Submodular Welfare Maximization

Algorithm 1: Greedy Algorithm for Online Submodular Welfare Maximization - *GA*

Initialization: Set each $I_m^{(t)} = \emptyset, \forall m \in [M]$.

When a female $f \in [F]$ logs into the application at round t,

while *f keeps swiping* **do**

    (a) Select the male $m^* \in [M]$, such that

    $m^* = argmax_{m \in [M]} \left(\mu_m(I_m^{(t)} \cup \{f\}) - \mu_m(I_m^{(t)}) \right)$.

    (b) Recommend male m^* to f, $I_{m^*}^{(t)} = I_{m^*}^{(t)} \cup \{f\}$

end

Each time select the recommendation with the **highest marginal utility**.

Theoretical Analysis of the greedy algorithm

- **Offline setting:** Approximation ratio = 1 - 1/e (tight)
- **Online setting:** Competitive ratio = 0.5 (tight)
- **Time Complexity:** Polynomial $O(M\bar{C}_F^{(t)})$

  $\bar{C}_F^{(t)}$ is the total capacities for all females

Nash social welfare: Trade-off Efficiency and Fairness

- Nash social welfare (NSW) definition:

$$\mathtt{NSW}([M]) = \left(\Pi_{m\in[M]} b_m^{(t)}\right)^{\frac{1}{M}} \qquad b_m^{(t)} = {r_m^{(t)}}^{\alpha_m^{(t)}}$$

- NSW is a special case of the generalized mean for $\tau \to 0$

$$A_\tau([M]) = \left(\frac{1}{M} \cdot \sum_{m\in[M]} (b_m^{(t)})^\tau\right)^{\frac{1}{\tau}}$$

$\tau = 1$ average sum (only efficiency)

$\tau \to -\infty$ max-min (only fairness)

$\tau \in [0, 1]$ monotone submodular

Reduce maximizing NSW to submodular maximization

- Maximizing NSW $\qquad \mathtt{NSW}([M]) = \left(\Pi_{m\in[M]} b_m^{(t)}\right)^{\frac{1}{M}}$

Is equivalent to maximizing

$$\sum_{m\in[M]} \alpha_m^{(t)} \cdot log(\epsilon + r_m^{(t)})$$

Thus we reduce it to the submodular maximization problem, and use the greedy algorithm (i.e., Alg. 1) to solve. To guarantee a valid log operation, we set: $epsilon \rightarrow 0_+$

- Utility Cap: define an upper bound of $r_m^{(t)}$ to further improve fairness such that: $\qquad r_m^{(t)} = max(r_m^{(t)}, 1)$

Performance Evaluation

- About 3800 males, 1700 females
- Non paying users with weekly match goal : 7

  Paying users with weekly match goal: 21
- Use $\Psi^{(t)}_{[M]}$ to denote the expectation of each male's match achievement rate:
- In the evaluation, we vary: $\Psi^{(t)}_{[M]} \in [0, 1]$

  In real cases:

$$\Psi^{(t)}_{[M]} \approx 0.5$$

Performance indicators

- Efficiency (Happiness indicator):

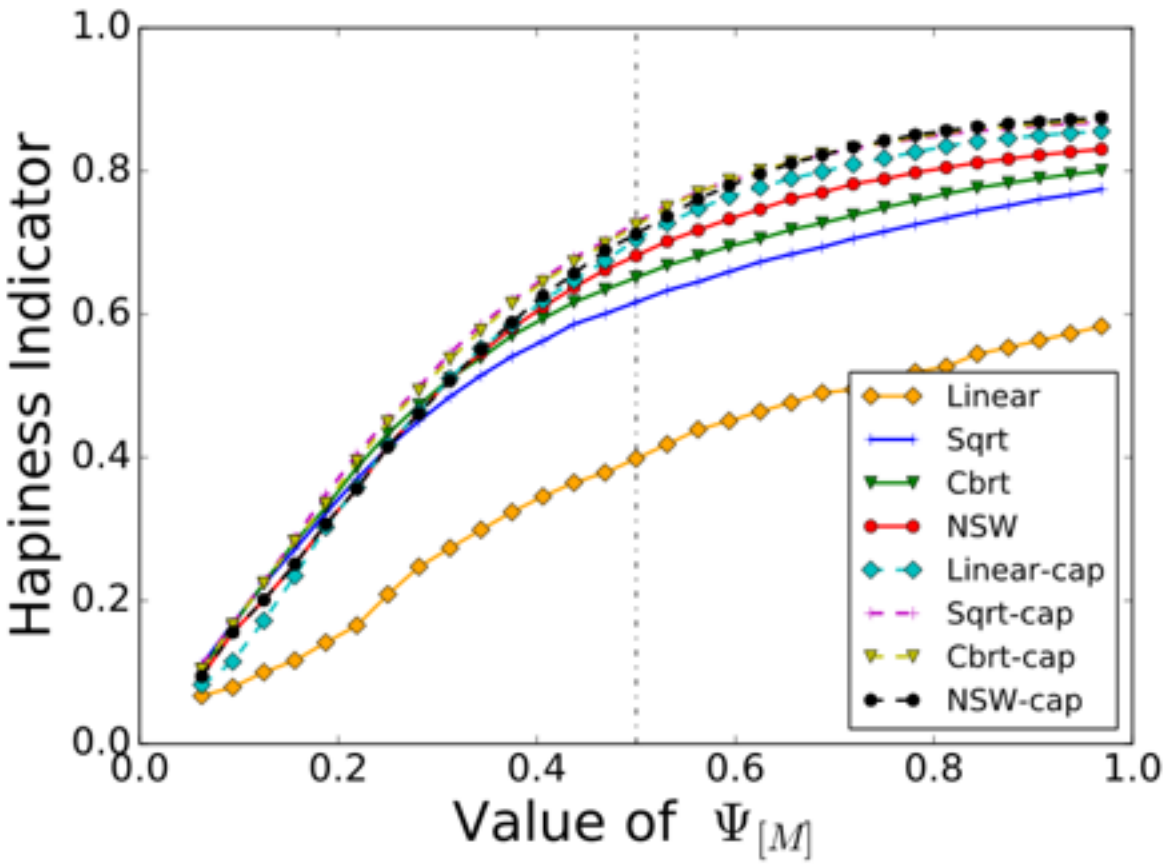
$$H^{(t)}_{[M]} = \frac{1}{M} \cdot \sum_{m \in [M]} max(1, r^{(t)}_m)$$

- Match fairness (Jain's Index):

$$J^{(t)}_{[M]} = \frac{\left(\sum_{m \in [M]} a^{(t)}_m\right)^2}{M \cdot \left(\sum_{m \in [M]} (a^{(t)}_m)^2\right)}$$

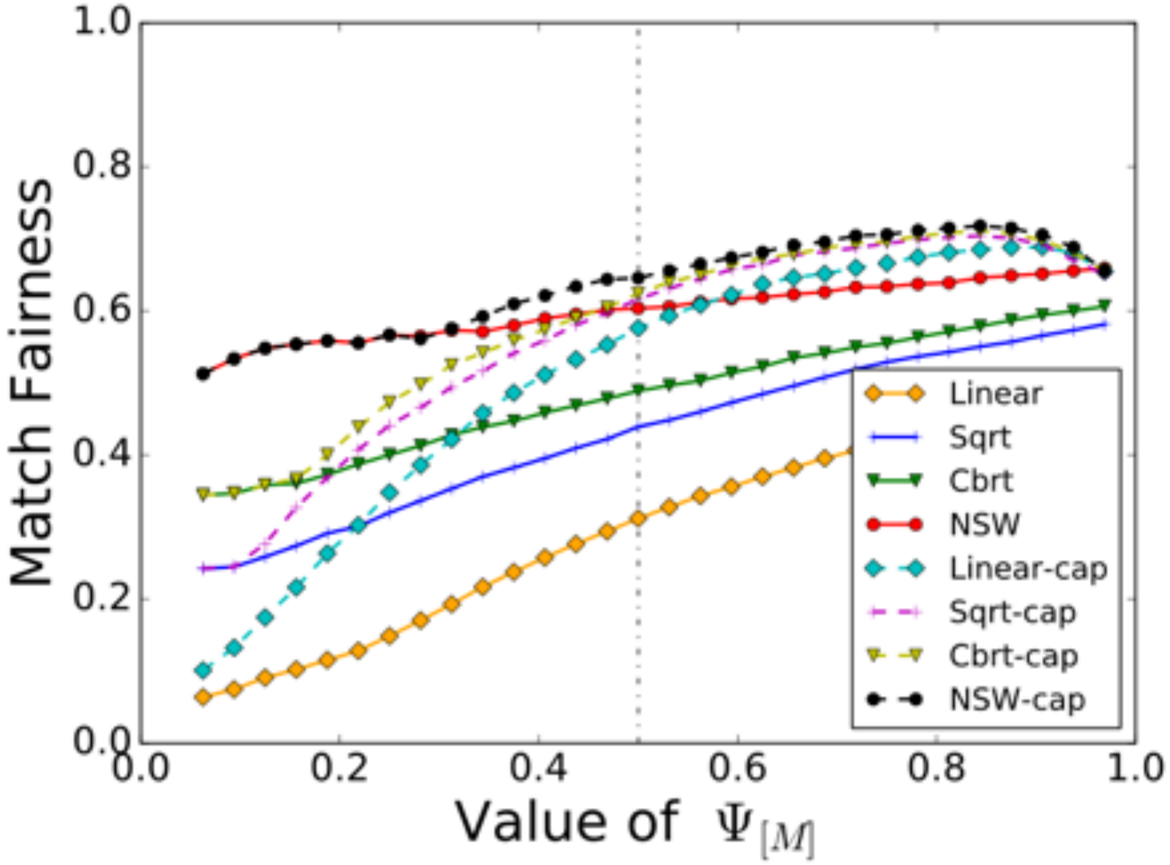
$J^{(t)}_{[M]} \in [0, 1]$ and a higher $J^{(t)}_{[M]}$ indicates a better fairness.

Efficiency



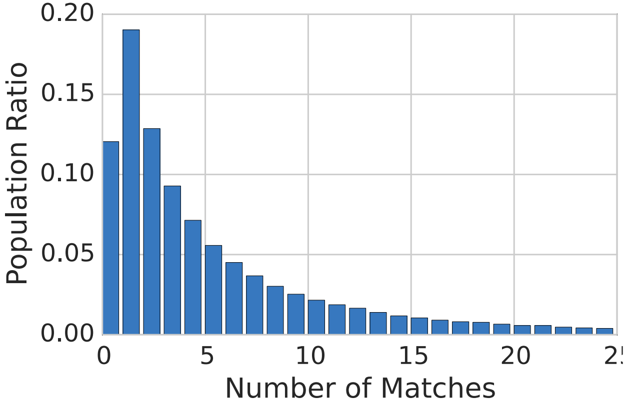
Hapiness Indicator
Value of $\Psi_{[M]}$
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
Linear
Sqrt
Cbrt
NSW
Linear-cap
Sqrt-cap
Cbrt-cap
NSW-cap

Fairness

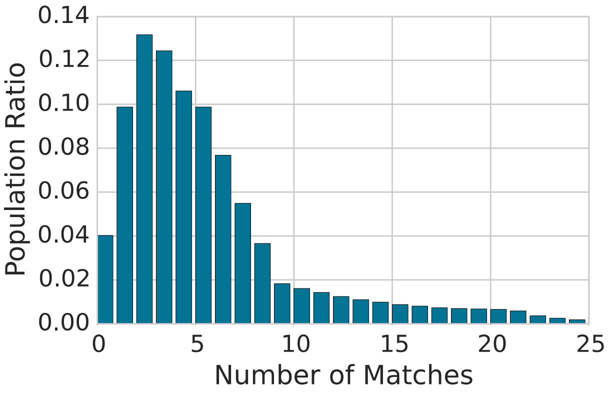


Match Fairness
Value of $\Psi_{[M]}$
Linear
Sqrt
Cbrt
NSW
Linear-cap
Sqrt-cap
Cbrt-cap
NSW-cap

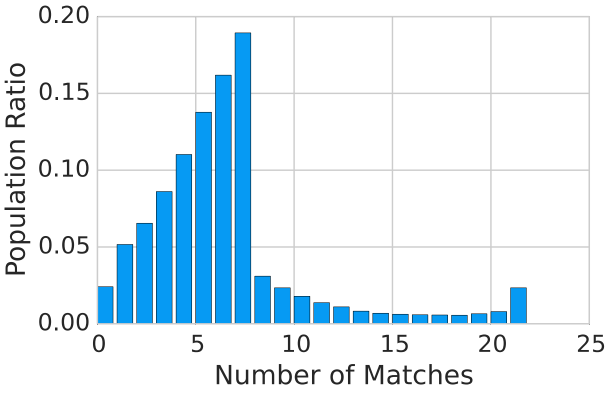
Match Distributions



Dataset



NSW



NSW-cap

Future directions

- Analyze how to improve females' retention rate.
- ML-based algorithm to predict users' swiping behavior.
- Classify the users into different attractiveness levels and design customized recommendation algorithms.
- Build a complete infrastructure to dynamically collect the data and provide efficient parallel computation for the optimization.

Thank You !

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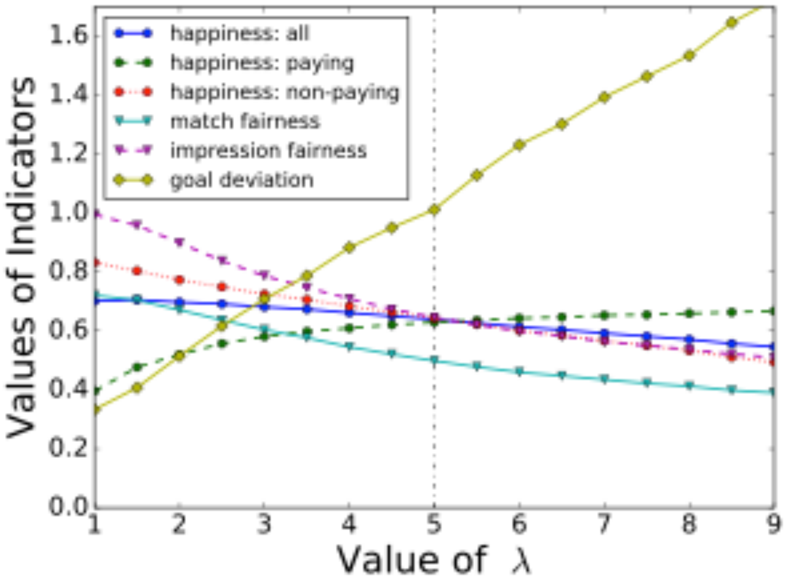
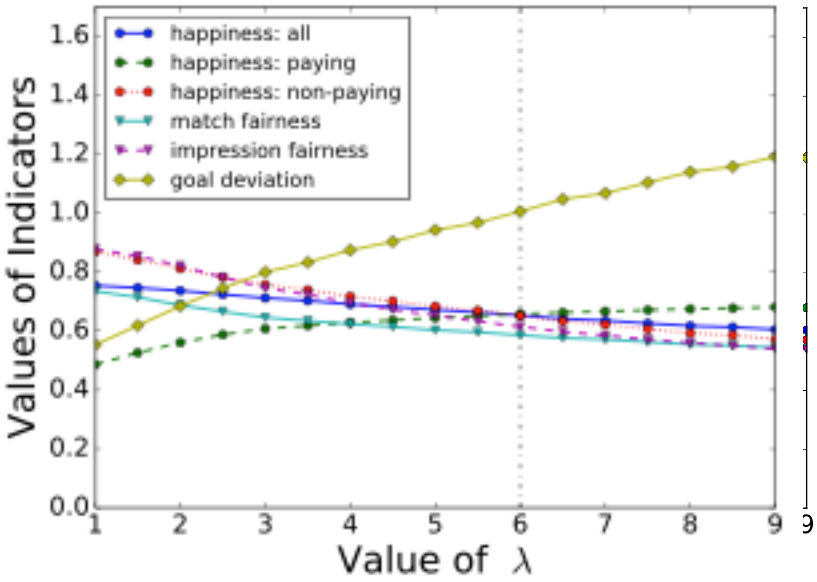
Changing the priority for paying users



NSW



NSW-cap