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Hamiltonian tomography: the quantum (system) measurement problem

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Abstract
To harness the power of controllable quantum systems for information processing or quantum simulation, it is essential to be able to accurately characterise the system's Hamiltonian. Although in principle this requires determining less parameters than full quantum process tomography, a general and extendable method for reconstructing a general Hamiltonian has been elusive. In their recent paper, Wang *et al* (2015 *New J. Phys.* **17** 093017) apply dynamical decoupling to the problem of Hamiltonian tomography and show how to reconstruct a general many-body Hamiltonian comprised of arbitrary interactions between qubits.

Since the development of quantum mechanics, writing down the Hamiltonian of a system has been the first step in understanding its quantum properties. A case in point is the hydrogen atom. While every physics undergraduate is familiar with the Laguerre polynomials and spherical harmonic functions used to solve this problem, there is a more subtle point to be made here. A hydrogen atom is a hydrogen atom is a hydrogen atom. They are all the same. The Hamiltonian is defined by the electrostatic interaction between an electron and a proton. This is also true for many other quantum mechanical problems. The Hamiltonian can and has been written down from fundamental physics principles.

However, over the last few decades, the focus of quantum physics has moved away from solving the classic problems (some might argue because they have all been solved). Now the current challenge is to design, build and control novel quantum systems. For many, a quantum computer and quantum information processing in general is the epitome of this concept. While a quantum computer is still a long term goal, technology based on controllable (typically nanoscale) quantum devices has already resulted in advances in telecommunication, metrology, sensing, and data storage, to name a few. As quantum technology develops, we come up against a problem that is both obvious and subtle. What exactly is the Hamiltonian of the system we have fabricated? While there is a always an ideal design, nanoscale fabrication is still imperfect. Each device must be calibrated. In short, the Hamiltonian of each new quantum device must be measured.

This problem has long been studied in control engineering and is termed system identification. It is the methods and techniques used to *obtain an appropriate mathematical model of a dynamic system on the basis of observed time series and prior knowledge of the system* [1]. However, as always in quantum mechanics, there are complications. Unlike a classical system, the state of a quantum system cannot be measured without that state being altered. More importantly, from a system identification point of view, the size of the quantum mechanical state space grows exponentially with the number of degrees of freedom of the underlying system.

A common method for measuring a quantum mechanical state or process is called quantum state (or process) tomography [2, 3]. This technique involves preparing a complete set of input states and then measuring the system in an equivalently complete set of measurement bases. This technique is particularly well suited to optical implementations [4] but it requires a large number of different combinations of input and measurement settings, and it scales exceptionally badly as the number of qubits increases. Worse, the set of input states and measurements over-constrains the resulting density matrix or process map, which means techniques such as the maximum-likelihood method are required.

Although a full reconstruction of the state of the system (the density matrix) requires determination of many different parameters, the complete density matrix is not always required. If the system is sufficiently coherent that noise processes can be ignored, then reconstructing the Hamiltonian is enough. The problem then boils down to system identification—variously referred to as Hamiltonian identification, Hamiltonian characterisation or Hamiltonian tomography.

The simplest version of this problem is: how does one characterise a single qubit Hamiltonian? Taking systematic measurements from known input states as a function of time allows the Hamiltonian to be reconstructed in its entirety [5, 6]. These early approaches have been extended to include corrections due to decoherence [7–9] additional levels [10] or limited accessibility [11–14] as well as including improved analysis techniques, such a Bayesian analysis [15–17] or compressed sensing [18]. The approach of measuring time traces becomes significantly more complex when considering two-qubits and all possible interactions between them, even when complete control over both qubits is assumed [15, 19–22].

Although the problem of characterising Hamiltonians is of fundamental importance, progress has been slow compared to more brute force process tomography methods. The work of Wang *et al* [23] has completely rewritten the playbook on Hamiltonian tomography by incorporating another common technique from quantum control, *dynamical decoupling*.

Dynamical decoupling (DD) [24–28] involves applying a series of control pulses to the system to decouple it from its environment. This can be thought of as a more sophisticated version of the standard Hahn-echo pulses at the heart of MRI and NMR. The breakthrough that Wang *et al* have made is the realisation that by applying DD to *all but two* of the qubits in the system, the problem of characterisation of the system is reduced to characterisation of each pair of qubits separately. This greatly simplifies the process and means that some of the simplest time-domain Hamiltonian identification techniques developed in the mid-2000s can be applied directly [5, 19, 20]. The DD based approach also allows for the full reconstruction of the relative phases of the pairwise interaction terms. A tailored set of DD sequences is used to reconstruct each of the XX, XY, YZ etc contributions to the Hamiltonian component for a given pair of qubits. This process is then repeated for all relevant pairs within the system. In situations where there are strong symmetry arguments to limit the number of options, either in terms of phases or pairwise interactions, then these contributions do not need to be measured. This results in a very robust, scalable and efficient measurement protocol.

Although the work of Wang *et al* is a major step forward, there are still open questions. How to account for decoherence at short timescales or imperfections in the DD pulses? Can Bayesian analysis or other statistical techniques reduce the total number of measurements required? For an issue that is usually completely side-stepped in quantum mechanics textbooks, measuring the Hamiltonian continues to be an interesting and important problem.

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