LMP-based Real Time Pricing for Optimal Capacity Planning with Maximal Wind Power Integration

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***Abstract*—With the proposed penetration of electric vehicles and advanced metering technology, the demand side is foreseen to play a major role in flexible energy consumption scheduling. On the other hand, over the past several years, there has been a growing interest for the utility companies to integrate more renewable energy resources. Such renewable resources, *e.g.,* wind or solar, due to their intermittent nature, brought great uncertainty to the power grid system. In this paper, we propose a mechanism that attempts to mitigate the resulting grid operational uncertainty by properly exploiting the potentials offered by demand flexibility. To address the challenge, we develop a novel locational marginal price (LMP) based pricing scheme that involves active demand side participation by casting the network objectives as a two-stage Stackelberg game between the local grid operator and several aggregators. We use the solution concept of subgame perfect equilibrium to analyze the resulting game and derive the optimal pricing scheme. Subsequently, we discuss the optimal real time conventional capacity planning for the local grid operator to achieve the minimal mismatch between supply and demand with the wind power integration. Finally, we assess our proposed scheme with field data. The simulation results further confirm the optimality of our scheme and suggest reasonably well long term performance with simplified heuristic approaches.**

***Keywords*—capacity planning, locational marginal price, real time pricing, Stackelberg game, smart grid, subgame perfect equilibrium, wind power integration.**

I. Introduction

To enable a large penetration of renewable energies, the current power system is facing the challenges that the designers haven't thought of before. Due to the intermittent nature of renewable energies, *i.e.,* fast yet uncontrollable ramping rate, their large scale integration need much more reserve capacity than we have today, even with a perfect prediction. One promising solution is to explore the flexibility in the demand side. In particular, with possibly millions of electric vehicles on the road, and the widely implementation of smart meters in the next several decades, the flexibility of residential load would even be 100% with the help of a *potential* distributed electricity storage system provided by the electric vehicles (with level 1 charging). Since the residential load constantly dominates around 30% of the whole electricity consumption, an optimistic estimation is that we can achieve around 20% total demand flexibility in the near future.

In this paper, to design a mechanism using the demand flexibility to help enable the large penetration of renewables, we describe an interaction between a local grid operator and several aggregators. Each aggregator could stand for several neighborhoods of end users, a college, or a hospital, etc. In particular, we notice that the current locational marginal price (LMP) scheme doesn't involve any demand side participation. To encourage such participation, we design a novel pricing scheme based on the LMP of the conventional power, which, by careful selection of parameters, will align the aggregators proper incentives to achieve the desirable overall performance, *i.e.,* minimizing the mismatch between supply and demand with the renewable energy integration. We then show how the parameters can lead to the optimal capacity planning for conventional power supply.

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A. Related Work

We discuss two main bodies of related works. The first one focuses on renewable energy integration with user participation. For example, in [1], Neely *et al.* used Lyapunov theory to obtain a *centralized* optimal queueing system for allocating renewable energy. In [2], He *et al.* proposed a multiple timescale dispatch to integrate wind power. Wu *et al.* investigated how to utilize wind power penetration into the power grid when aggregators use a linear pricing scheme in [3], and proposed a cost sharing game among end users for wind power integration in [4]. Different from the previous work in [1], [2], our focus here is on applying game theory to design a *decentralized* demand side management system, in which aggregators are independent decision makers and are interested in managing their own loads to minimize individual energy expenses. Different from [3] and [4], which introduced an energy consumption game among end users, we propose a two-stage Stackelberg game to fully characterize the interaction between the local grid operator and the aggregators, and obtain the optimal pricing scheme via analyzing the corresponding subgame perfect equilibrium.

Another line of investigation considers demand side response in pricing scheme design. In [5], Strbac *et al.* considered a control framework for the competitive electricity market with load reduction. Weber *et al.* proposed a two-level optimization approach to analyze market bidding strategies in [6]. Rassenti *et al.* showed the impact of demand side bidding in terms of eliminating price spikes in the deregulated electricity market in [7]. Bu *et al.* proposed a game-theoretical decision-making scheme for electricity retailers considering demand response in [8]. Philpott *et al.* analyzed the optimal demand-side bids in the day-ahead electricity market and noticed an interesting conclusion that the purchasers have an incentive to underbid their expected demand so that the day-ahead prices will be below expected real-time prices in [9]. Different from the control perspectives in [5], [6], we use game theory to analyze the interaction between the local grid operators and aggregators. Different from [7], [9], which include the demand side into the bidding process, we align the economic incentives to the aggregators via a pricing scheme and design a two-stage Stackelberg game based on it. Furthermore, by analyzing the game model, we derive the optimal capacity planning for the local grid operator.

B. Our Contributions

In this paper, we assume that the aggregators are non-profit organizations and make decisions to maximize the payoffs of their own group. The local grid operator tries to develop a publicly known real time pricing scheme based on the LMP. Through this pricing scheme, the grid operator aligns incentives to the aggregators and obtains the desirable system performance. The major contributions of the paper may be summarized as follows.

- *Stackelberg Game Formulation:* We formulate the interaction between the local grid operator and the aggregators as a two-stage Stackelberg game, which clearly defines a leader-follower fashion interaction, and also captures each end player's (including both the local grid operator and all the aggregators) selfish nature.
- *LMP-based Pricing Scheme:* We design a novel pricing scheme based on the LMP. Specifically, we incorporate a linear disturbance term into the LMP, which aligns the economic incentives to the aggregators, and makes the interaction a Stackelberg game.
- *Equilibrium Analysis:* We first use backward induction to analyze the two-stage Stackelberg game. Then, we develop the optimal pricing scheme by characterizing the closed form expression of the linear disturbance term. Finally, we propose an algorithm to help the local grid operator make decisions on optimal capacity planning based on the pricing scheme. We establish that such planning will minimize the mismatch between supply and demand given a good wind power prediction.

The rest of this paper is organized as follows. We introduce the system model, in particular, the local grid operator model and the aggregator model in Section II. We review the conventional centralized control approach for maximal wind power integration in Section III. In contrast to the centralized control approach, we formulate a distributed interactive approach based on the Stackelberg game formulation in Section IV. After that, we use backward induction to analyze the two-stage game and obtain the optimal pricing scheme in Section V. Section VI concerns simulation studies in which we propose three heuristic solution approaches and evaluate the long term performance attained by our design methodology. Finally, concluding remarks are presented in Section VII.

II. System Model

As shown in Fig. 1, we consider a system with a local grid operator, several conventional generators, several renewable energy resources, in particular, wind turbines in this paper,

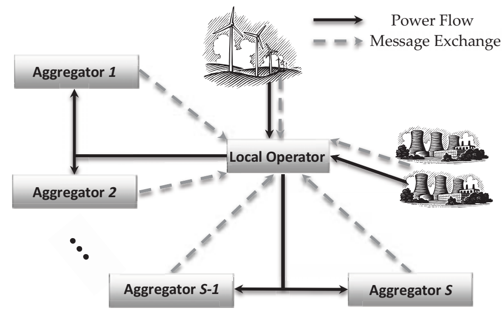


Fig. 1. The system model considered in this paper.

and several aggregators. Based on aggregator load forecasting, there has been extensive work investigating the interaction between the local grid operator and the generators, that conclude LMP to be a desirable network pricing methodology, which is being used in the PJM Interconnection, ERCOT, New York, and New England markets. In this paper, we consider yet another interaction in the system, *i.e.,* the interaction between the local grid operator and all the aggregators. We consider a real time electricity market, and divide each day into H time slots. We denote by $\mathcal{H} = \{1, \ldots, H\}$ all the time slots in a day. Typically, a time slot may correspond to an hour, in which case H is 24. This corresponds to the hour ahead planning for grid operators. Our approach can be readily generalized to a time scale of 15 minutes or half an hour. However, a time scale smaller than 15 minutes would require higher ramping rates for the conventional generators. We further assume that the grid operator handles the fluctuations within each time slot by frequency regulation. In this scenario, we now introduce the local grid operator model and the aggregator model as follows.

A. Local Grid Operator Model

The grid operator uses the LMP information and designs the following pricing scheme at time slot $h \in \mathcal{H}$:

$$p^h = p_m^h + \beta^h \left(\sum_{s \in \mathcal{S}} l_s^h - \hat{w}^h - v_{\max}^h \right), \tag{1}$$

where

- $p_m^h > 0$ is the LMP for the conventional generators at time slot $h \in \mathcal{H}$;
- $\beta^h > 0$ is the incentive control parameter set by the local grid operator. We derive the explicit form of β^h to achieve the optimal system performance in Section IV;
- $\mathcal{S} = \{1, \ldots, S\}$ is the set of S aggregators in the system;
- l_s^h is the energy consumption profile of aggregator $s \in \mathcal{S}$ at time slot $h \in \mathcal{H}$. We will further explain its constraints in Section II-B;
- $\hat{w}^h \geq 0$ is the predicted wind power at time slot h. In this paper, we will only consider a single time slot game. Since the short term wind power prediction is rather accurate (with around 2% prediction error only [10]), for simplicity, we take $\hat{w}^h$ to be the expected value of the upcoming wind power w^h (*i.e.,* $\mathbb{E}\{w^h\} = \hat{w}^h$) and assume the local grid operator will handle the prediction error via frequency regulation;

- $v_{\max}^h > 0$ is the maximal available conventional power at time slot h without starting up any unscheduled cold generator.

Overall, the physical intuition of the pricing scheme is very clear. If the total demand from the aggregators successfully absorbs the renewable energies and in the meanwhile, keeps the request of conventional power in an acceptable range (*i.e.,* not exceeding the maximal available power $v_{\max}^h$), each aggregator will receive a discount of the electricity price. However, if the request for conventional power exceeds $v_{\max}^h$, they are penalized with a higher electricity price.

B. Aggregator Model

Based on the pricing scheme, each aggregator decides its own decision l_s^h at time slot $h \in \mathcal{H}$, based on its own interest. The aggregator may only have limited control of its energy consumption. For example, if an aggregator corresponds to several neighborhoods of end users, it may not be able to completely dictate end user activities. However, it may be able to control some delay-tolerant loads, such as dishwashers, washing machines, and the charging schedule of electric vehicles. As such, we impose the following constraints on l_s^h:

$$l_s^{h,\min} \leq l_s^h \leq l_s^{h,\max}. \tag{2}$$

Note that $l_s^{h,\min}$ and $l_s^{h,\max}$ could be time-varying. However, since we only consider a single time slot game in our formulation, a detailed discussion on the time coupling issue is beyond the scope of this paper. Thus, in our model, we treat them as constants. However, we perform simulation for a long term run and the simulation results show that our pricing scheme works reasonably well with the time coupling constraints, even with simple heuristic approaches. Finally, we define aggregator s's demand flexibility γ_s^h as

$$\gamma_s^h = \frac{l_s^{h,\max} - l_s^{h,\min}}{2 l_s^h}. \tag{3}$$

III. Centralized Control Design

After introducing the local grid operator model and aggregator model, we are now ready to illustrate the system objective. In this paper, we ignore the associated transmission and distribution (T&D) cost, and the generation cost. We assume the local grid operator only wants to ensure system reliability and maintain the conventional generation at a desirable level of $v_d^h \leq v_{\max}^h$ at time slot $h \in \mathcal{H}$. Thus, the system objective at time slot $h \in \mathcal{H}$ is to

$$\begin{aligned} \underset{l_s^h,\ \forall s \in \mathcal{S}}{\textbf{minimize}} \quad & \left(\sum_{s \in \mathcal{S}} l_s^h - \hat{w}^h - v_d^h \right)^2 \\ \textbf{subject to} \quad & l_s^{h,\min} \leq l_s^h \leq l_s^{h,\max}, \quad \forall s \in \mathcal{S}. \end{aligned} \tag{4}$$

The objective function in problem (4) is convex and quadratic, and the constraint sets are convex. Therefore, it can be solved efficiently using various convex programming techniques such as the *interior point method* [11]. However, in many cases, the local grid operator does not have control over aggregators, thus may not be able to select the energy consumption profiles for the aggregators. This motivates us to consider a game-theoretic formulation, in which we treat the aggregators as independent decision makers.

IV. Stackelberg Game Design

A. Local Grid Operator's Payoff

As we have indicated in Section III, the local grid operator wants to constantly match the demand with supply. We model its payoff function $g(\beta^h; l_s^h, \forall s \in \mathcal{S})$ as the expected mismatch at time slot $h \in \mathcal{H}$, *i.e.,*

$$g(\beta^h, v_d^h; l_s^h, \forall s \in \mathcal{S}) = -\left(\sum_{s \in \mathcal{S}} l_s^h(\beta^h) - \hat{w}^h - v_d^h \right)^2. \tag{5}$$

Note that, here we denote l_s^h by $l_s^h(\beta^h)$ to explicitly indicate that any aggregator $s \in \mathcal{S}$ will make its own decision based on β^h, since the latter influences the electricity price. In fact, in this model, we note that there are two parameters that the local grid operator can control. One is β^h to affect aggregators' behaviors. The other one is v_d^h, which is by far the most important task for current grid operators. In fact, these two parameters are often coupled in the design space, as we will see in Section V. Thus, we first assume that the local grid operator fixes a v_d^h and obtains the optimal closed form solution for β^h, and subsequently, for the given closed form β^h, we propose an algorithm for optimal capacity planning to select v_d^h.

B. Aggregator's Payoff

To best capture the aggregators' selfish nature, we model their payoff functions as their electricity costs. That is, for aggregator $s \in \mathcal{S}$, its payoff function $f_s(l_s^h; l_{-s}^h, \beta^h)$ at time slot $h \in \mathcal{H}$ is

$$\begin{aligned} f_s(l_s^h; l_{-s}^h, \beta^h) &= -l_s^h p^h \\ &= -\, l_s^h \left(p_m^h + \beta^h \left(\sum_{s \in \mathcal{S}} l_s^h - \hat{w}^h - v_{\max}^h \right) \right), \end{aligned} \tag{6}$$

where l_{-s}^h denotes the set of all aggregators' energy consumption profile excluding aggregator s at time slot h, *i.e.,* $l_{-s}^h = \{ l_n^h | n \in \mathcal{S} \setminus \{s\} \}$. Such payoff formulation implies that each aggregator's payoff function depends not only on its own decision, but also on all the other aggregators' decisions, which leads to our following two-stage Stackelberg game.

C. Game Formulation

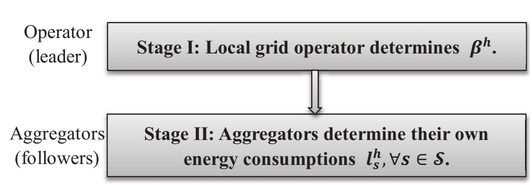


Fig. 2. A Stackelberg game formulation.

As shown in Fig. 2, in this two-stage Stackelberg Game, the local grid operator is the Stackelberg leader: it first decides the pricing parameter $\beta^h > 0$ at time slot $h \in \mathcal{H}$ in

Stage I. Subsequently, the aggregators choose their energy consumption profiles at time slot $h \in \mathcal{H}$ to maximize their individual payoffs in Stage II.

V. Backward Induction for The Two-stage Game

The Stackelberg game falls within the class of dynamic games, and the common solution concept is the subgame perfect equilibrium (SPE). Note that the traditional Nash equilibrium investigates players' simultaneous actions in a static game, and hence, is not applicable to our dynamic model [12]. A general technique for determining the SPE is the backward induction (see, *e.g.,* [13]). We will start with Stage II and analyze the aggregators' behaviors given the local grid operator's pricing scheme. Then we will consider Stage I and analyze how the local grid operator sets the optimal pricing scheme. The backward induction captures the sequential dependence of the decisions in the two stages.

A. Energy Consumption Subgame in Stage II

The subgame in stage II is the energy consumption game among aggregators. We can formally define it as follows:

Energy Consumption Subgame (EC Game):

- *Players:* The set $\mathcal{S}$ of all the aggregators;
- *Strategies:* For each aggregator $s \in \mathcal{S}$, based on the given pricing scheme set by the local grid operator, chooses its own energy consumption profile $l_s^h \in [l_s^{h,\min}, l_s^{h,\max}]$ at time slot $h \in \mathcal{H}$.
- *Payoffs:* For each aggregator $s \in \mathcal{S}$, its payoff function is defined as its electricity cost as in (6) at each time slot.

We first consider the concept of *best response*, which is an aggregator's best choice to maximize its own payoff assuming that all other aggregators' strategies are fixed.

Definition 1: For an aggregator $s \in \mathcal{S}$, its *best response* is:

$$l_s^{h,best}(l_{-s}^h) = \underset{l_s^h \in [l_s^{h,\min}, l_s^{h,\max}]}{\arg\max} \; f_s(l_s^h; l_{-s}^h, \beta^h). \tag{7}$$

Hereafter, for simplicity, we do not explicitly write out the constraints for l_s^h, for all $s \in \mathcal{S}$. Note that the payoff function for each aggregator is quadratic and we can further represent the aggregator s's best response as follows:

$$\begin{aligned} & l_s^{h,best}(l_{-s}^h) \\ =& \arg\min_{l_s^h} \left| l_s^h - \frac{\beta^h(\hat{w^h} + v_{\max}^h - \sum_{n \in \mathcal{S}\setminus\{s\}} l_n^h) - p_m^h}{2\beta^h} \right|. \end{aligned} \tag{8}$$

Next, we consider the solution concept of SPE for the two-stage Stackelberg game.

Definition 2: A strategy profile $\{l_s^{h,*}, \forall s \in \mathcal{S}\}$ is a subgame perfect equilibrium of the Stackelberg game if given any $\beta^h > 0$, the restricted strategy profile $\{l_s^h | \beta^h, \forall s \in \mathcal{S}\}$ is a Nash equilibrium for the *EC subgame* at time slot $h \in \mathcal{H}$. That is, at each time slot $h \in \mathcal{H}$, for any pricing parameter $\beta^h > 0$, *no* aggregator $s \in \mathcal{S}$ can increase its payoff $f_s(\cdot)$ by *unilaterally* changing its own energy consumption profile $l_s^{h,*}$.

By definition of SPE, we can obtain each aggregator's strategy via computing the Nash equilibrium of the *EC subgame*. Note that the Nash equilibrium is a fixed point of all aggregators' best responses, *i.e.,* $l_s^{h,best}(l_{-s}^{h,*}) = l_s^{h,*}$ for all $s \in \mathcal{S}$. It represents a stable solution of the game. We first prove the existence and uniqueness of Nash equilibrium for the *EC subgame*.

Theorem 1: Given each pricing parameter $\beta^h > 0$, there exists a unique Nash equilibrium of the *EC subgame*.

Proof: Note that, for any given $\beta^h > 0$, each player's (*i.e.,* aggregator's) payoff function is concave, and the strategy space involves a set of linear (hence convex) constraints (*i.e.,* $l_s^{h,\min} \le l_s^h \le l_s^{h,\max}, \forall s \in \mathcal{S}$). Theorem 1 then readily follows from [14]. ■

Based on the aggregators' best responses given in (8), we now consider Stage I of the Stackelberg game, where we discuss how to design the optimal pricing parameter β^h, and how to plan the desirable capacity v_d^h for conventional generators at time slot $h \in \mathcal{H}$.

B. Optimal Pricing Strategy in Stage I

To maximize the grid operator's payoff function at the Nash equilibrium $(l_s^{h,*}, s \in \mathcal{S})$ of the *EC subgame*, the local grid operator sets a β^h such that

$$\sum_{s \in \mathcal{S}} l_s^{h,*} = \hat{w^h} + v_d^h. \tag{9}$$

To achieve this goal, we first consider an essential condition, that is the game without demand flexibility constraints. In this case, each aggregator s's best response is simply

$$l_s^{h,best}(l_{-s}^h) = \frac{\beta^h(\hat{w^h} + v_{\max}^h - \sum_{n \in \mathcal{S}\setminus\{s\}} l_n^h) - p_m^h}{2\beta^h}. \tag{10}$$

By solving a system of S equations given by (10), we can obtain the closed form Nash equilibrium as

$$l_s^{h,*} = \frac{\hat{w^h} + v_{\max}^h - p_m^h/\beta^h}{S+1}, \; \forall s \in \mathcal{S}. \tag{11}$$

Together with the local grid operator's objective in (9), we may obtain the optimal parameter β^h as

$$\beta^h = \frac{S p_m^h}{S v_{\max}^h - (S+1) v_d^h - \hat{w^h}}. \tag{12}$$

Note that, we require $\beta^h > 0$ for the pricing scheme to make physical sense. Hence, we need to ensure that

$$S v_{\max}^h - (S+1) v_d^h - \hat{w^h} > 0. \tag{13}$$

That is,

$$v_{\max}^h > v_d^h + \frac{1}{S} v_d^h - \frac{1}{S} \hat{w^h}. \tag{14}$$

It's reasonable to assume that $S > 10$ and $\hat{w^h} < 50\% v_d^h$ in practice. Thus, we only need 15% backup capacity (*i.e.,* $v_{\max}^h > 1.15 v_d^h$) to ensure $\beta^h > 0$. In fact, such backup capacity is also very important and desirable in practice due to the N-1 reliability criteria [15], [16].

Based on the closed form expression of β^h, we now reconsider the general Stackelberg game with demand flexibility constraints and determine the desirable v_d^h based on all the aggregators' behaviors at equilibrium.

Suppose at equilibrium $(l_s^{h,*}, \forall s \in \mathcal{S})$, (9) holds. Then, each aggregator's energy consumption profile $l_s^{h,*}$ satisfies

$$l_s^{h,*} = \arg\min_{l_s^h} \left| l_s^h - \frac{\hat{w^h} + v_d^h}{S} \right|. \tag{15}$$

We now develop an algorithm to show that there exists a v_d^h that leads to a desirable equilibrium satisfying (9) given β^h in its closed form as in (12).

C. Determining the Optimal Conventional Power Supply

We propose an iterative algorithm to determine v_d^h. By denoting $m = (\hat{w^h} + v_d^h)/S$, in our algorithm, we compute a sequence of $m^{(1)}, m^{(2)} \ldots$ to approximate m. Our algorithm could start with any possible $m^{(1)}$, where

$$\sum_{s\in\mathcal{S}} l_s^{h,\min}/S \le m^{(1)} \le \sum_{s\in\mathcal{S}} l_s^{h,\max}/S. \tag{16}$$

Then, given any ϵ,

- At round r, for each aggregator $s \in \mathcal{S}$, it sets
$$l_s^{h(r)} = \arg\min_{l_s^h} |l_s^h - m^{(r)}|. \tag{17}$$
- Set $m^{(r+1)} = \sum_{s\in\mathcal{S}} l_s^{h(r)}/S$.
- If $|m^{(r+1)} - m^{(r)}| \le \epsilon$, return $v_d^h = m^{(r+1)}S - \hat{w^h}$. Otherwise, start round $r+1$.

Theorem 2: The iterative algorithm converges.

Due to the space limitation, the proof of Theorem 2 can be found in [17]. In our proof, we further show that our algorithm will converge exponentially fast after a bounded constant number of rounds. Thus, the optimal capacity planning for the local grid operator is to set v_d^h as obtained from the iterative algorithm and set a corresponding $v_{\max}^h$ to maintain $\beta^h > 0$ as discussed in Section V-B. Note that there could be multiple (or even infinitely many) v_d^h given different $m^{(1)}$ in our algorithm. We propose three approaches on how to select $m^{(1)}$ in Section VI.

VI. Simulation

Since we have already proved the optimality of our approach at each single time slot, we want to test its long term performance in this section. We first introduce the setup for each part of our simulation, then explain the simulation results.

A. Wind Power Setup

We use hourly wind speed data in West Texas as shown in Fig. 3(a). Given the wind speed, the generated wind power is obtained based on the wind power versus wind speed curve in Fig. 3(b). As we mentioned in Section II-A, the real time wind power prediction error is around 2%. Therefore, in our simulation, to focus on the interaction between local grid operator and aggregators, we simply test our approach with perfect wind power prediction.

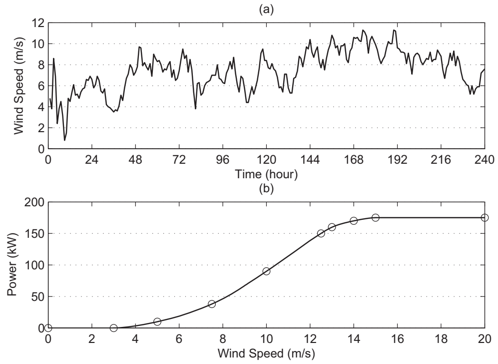


Fig. 3. Wind power prediction based on wind speed: (a) 10 days wind speed measured data [18]. (b) Power versus speed curve [19].

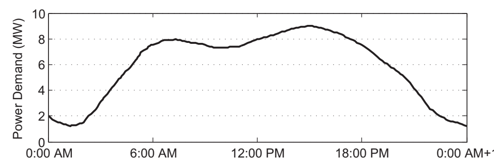


Fig. 4. Typical two peak aggregator's load.

B. Aggregators Setup

In our simulation, we have 12 aggregators. We set each aggregator's peak power randomly around 10 MW. And each aggregator's demand over $\mathcal{H}$ follows the two peak pattern as shown in Fig. 4. To characterize the time coupling between time slots, for each aggregator $s \in \mathcal{S}$, we denote its load prediction at time slot $h \in \mathcal{H}$ as $\hat{l_s^h}$ and the actual load as l_s^h. Thus, we have the following time coupling constraints:

$$(1-\gamma_s^h)\hat{l_s^h} \le l_s^h \le (1-\gamma_s^h)\hat{l_s^h}, \tag{18}$$

$$\hat{l_s^{h+1}} = \hat{l_s^{h+1}} + (\hat{l_s^h} - l_s^h), \tag{19}$$

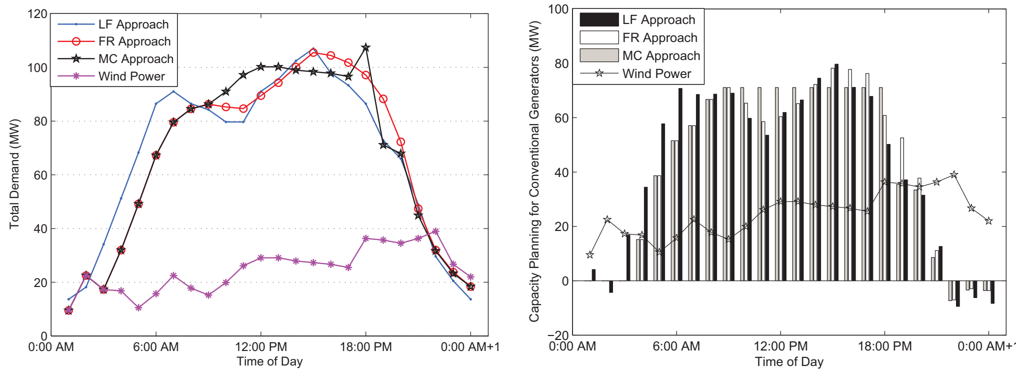
where γ_s^h is aggregator s's demand flexibility as defined in (3), which is assumed to be constant over $\mathcal{H}$ in our simulation.

C. Local Grid Operator's Approaches

We compare three heuristic approaches in the simulation:

- *Load Following (LF) Approach:* In this approach, we do not consider any incentive or interaction issue. That is the grid operator simply performs load following as it currently does.
- *Following Renewables (FR) Approach:* In this approach, the grid operator first sets the initial value of $m^{(1)}$ as the predicted wind power at each time slot and then obtains the optimal demand based on the aggregators' energy consumption profiles at the *closest* equilibrium.
- *Minimal Change (MC) Approach:* In this approach, at each time slot, the grid operator wants a minimal capacity planing change with respect to the capacity planing at the previous time slot. That is, at each time slot $h \in \mathcal{H}$, it sets $m^{(1)} = (\hat{w^h} + v_d^{h-1})/S$.

Fig. 5(a) shows the total demand based on these three approaches. Taking the time coupling constraint (19) into account, we can keep the total demand of different approaches



(a) Total demand with different approaches.

(b) Capacity planning with different approaches.

Fig. 5. Simulation results comparing three approaches, *i.e.,* the load following (LF), the following renewables (FR), and the minimal change (MC) approaches.

over a day almost the same. The LF approach reflects the predicted demand. Since, in our case, we have around 30% of wind power penetration, the FR approach describes the performance of the *minimal* v_d^h at each time slot $h \in \mathcal{H}$, while MC approach further explores the demand flexibility and tries to keep the change of capacity planning between time slots as small as possible. This is better illustrated in Fig. 5(b), where the capacity planning of the MC approach keeps unchanged during 9:00 AM to 18:00 PM. Interestingly, the last two approaches both perform peak shaving in the meanwhile. The FR approach reduces the peak conventional power by 2% while the MC approach reduces it by 10.8%. Note, the negative bars in Fig. 5(b) stand for the case where total demand is less than the available wind power. In practice, we can always disconnect some wind turbines. However, we keep them in the figure to show that our game interaction design can also contribute to minimize such mismatches.

In fact, a carefully designed algorithm considering the time coupling constraints will better utilize our game interaction approach. However, a detailed discussion is beyond the scope of this paper and we leave it as potential future work.

VII. Conclusions

In this paper, we propose an LMP based pricing scheme to align proper incentives to aggregators, based on which, we further formulate a two-stage Stackelberg game model to capture the interaction between the local grid operator and the aggregators. By analyzing the SPE of the game, we obtain the optimal pricing scheme to achieve desirable demand. Simulation results further confirm that, although our design is targeted towards single time slot formulations, it works reasonably well even with simple heuristic approaches in a long term run.

This work may be generalized and extended in several interesting directions. For example, in the future, we would like to consider the case in which the aggregators may not want to share information mutually. This may be handled by considering a game with incomplete information, in which each player in the game knows only a distribution of the other players' states. Also, as mentioned in Section VI, it is of interest to consider the Stackelberg game over multiple time slots, in which case we need to reformulate the game as a repeated sequential one.

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