Experimental measurement-dependent local Bell test with human free will

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A Bell test can rule out local realistic models and has potential applications in communication and information tasks. For example, a Bell's inequality violation can certify the presence of intrinsic randomness in measurement outcomes, which can be used to generate unpredictable random numbers. Nevertheless, a Bell test requires measurements that are chosen independently of environment in the test, as would be the case if the measurement setting choices were themselves intrinsically random. Such situation seems to create a "bootstrapping problem" recently addressed in the BIG Bell Test, a collection of various Bell tests using human choices. Here, we report in detail our experimental methods and results within the BIG Bell Test, specifically for a special type of Bell inequality, known as the measurement-dependent local inequality. With this inequality, even a small amount of measurement independence makes it possible to disprove local realistic models. The experiment utilizes human-generated random numbers in selecting the measurement settings and is implemented with space-like separation between two distant measurement sites. The experimental result violates a Bell's inequality, which cannot be explained by local hidden variable models with independence parameter (as defined in [G. Pütz *et al.*, Phys. Rev. Lett. **113**, 190402 (2014)]) $l > 0.10 \pm 0.05$. This result further quantifies the degree to which a local hidden variable model would need to constrain human choices, if it is to reproduce the experimental results.

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I. INTRODUCTION

Bell tests [1] are designed to rule out local hidden variable models (LHVMs) [2]. By violating Bell's inequalities in experiments that faithfully reproduce the assumptions of Bell's theorem, we can demonstrate that the underlying physical process cannot be explained with LHVMs. In quantum information processing, Bell tests provide device-independent advantages in a variety of tasks, such as quantum key distribution [3–6], randomness amplification [7–11] and expansion [12–15], entanglement quantification [16], and dimension witness [17].

Focusing on the bipartite scenario, a Bell test involves two space-like separated participants Alice and Bob, who implement measurements with settings or "inputs" x and y and generate outputs a and b, respectively. The Bell inequality is defined by a linear combination of the probability distribution $P(ab|xy)$ according to

$$J = \sum_{abxy} c_{abxy} P(ab|xy) \leqslant J_C. \tag{1}$$

Here J_C is the classical upper bound with LHVMs. In quantum mechanics, a Bell value larger than J_C may be achieved. In such a case, we call it a violation of the Bell inequality. In device-independent tasks, the quantum advantage can be certified solely by the Bell value J and the certification can be independent of the implementation of the devices used for state preparation and measurement. For instance, when the Bell inequality is violated, the output cannot be completely predicted, hence the entropy of the output is positive [18]. Complete random bits can be achieved using randomness extraction with the output from several rounds.

Practically, a faithful implementation of a Bell test is conditioned on closing three major loopholes. (a) The locality loophole: the generation of Alice's input a should be space-like separated from Bob's measurement y, and likewise for b and x. If this condition is not satisfied, then a Bell inequality can be violated even with LHVMs by signaling the inputs. (b) The efficiency loophole: the efficiency must be higher than a threshold to ensure the violation. If the realized efficiency is lower than the threshold, the violation cannot be observed without postselecting the outputs. (c) The so-called freedom-of-choice loophole: the inputs should not be influenced by the hidden variables in the LHVM. Clearly, if the LHVM can determine the inputs, then any probability distribution $P(ab|xy)$ can be realized with a LHVM. This motivates efforts to choose the inputs randomly or at least in a manner that cannot be controlled by a LHVM.

The history of testing Bell's inequality is the history of trying to close all the possible loopholes, especially the locality and efficiency loopholes. Only recently, these loopholes were simultaneously closed in Bell tests using physical random number generators [19–22]. Nevertheless, the freedom-of-choice loophole cannot be perfectly closed, as we can never unconditionally certify the randomness without a faithful Bell test. This seems to create a "bootstrapping problem," in which unconditional randomness is required in order to produce unconditional randomness. In principle, it is not possible to rule out superdeterminism, the philosophical argument that the universe is completely deterministic.

When considering the practical case of certifying randomness in the presence of classical noise or an adversary, we can still assume the possibility that the input is random with respect to the measurement devices. In an experiment, well-calibrated quantum random number generators are used, with the assumption that their output values are independent of any prior events. In other words, the measurements performed in Bell tests are independent of the random inputs, named by measurement independence [23]. Moreover, photons from cosmic sources can be considered as random inputs in both theory and experiment, by pushing measurement-dependence constraints back into cosmic history [24–26]. Nevertheless, this randomness is guaranteed by certain physical models, which can be inaccurate and hence make the random inputs partially predictable.

Without relying on any physical model, it is a common belief that humans have free will, by which we mean humans can make choices that are indeterministic consequences of prior physical conditions. Assuming this capacity, human choices as inputs in a Bell test make it possible to circumvent the bootstrapping problem described earlier. At the same time, human choices are not perfectly unpredictable, i.e., sequences of such choices tend to show patterns even when a person is attempting to make random choices. For example, we measured the uniformity of the human-generated random numbers from November 30 to December 1, 2016, in the BIG Bell Test. Using the well-known statistical test suite NIST SP 800-22 ("the NIST tests"), we found that the human random numbers passed only two of the 14 different tests. In fact, we can easily find "000000" or "010101" patterns in the human random numbers. In contrast, our quantum random number generator passed all the tests.

Such patterns by themselves do not open the freedom-of-choice loophole because it is possible for two variables, here xy and λ, to be independent even if one of them, here xy, is biased and thus somewhat predictable. Nevertheless, it is interesting to consider the possibility of some predictability arising in human choices due to the influence of physical environment, which might be correlated to hidden variables. Indeed, John Bell himself discussed this possibility [1]. If such influence was too strong, then a LHVM could explain a Bell inequality violation through a limited freedom of choice.

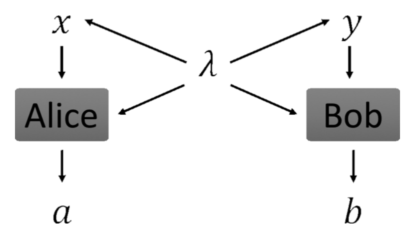

FIG. 1. Bell test with imperfect input randomness.

These qualitative notions regarding partially predictable inputs in Bell tests have been precisely investigated via theoretical analysis [23,27–31].

In this work, we exploit human randomness generated via free will as the input of Bell tests. Human randomness is collected in a worldwide project known as the BIG Bell Test [32], initiated by researchers from the Institute of Photonic Sciences (ICFO). The basic idea of this project is to apply human free will in a state-of-the-art physics experiment, the Bell test. Here, we employ two different Bell inequalities, analyzing the requirements that the human randomness should satisfy to guarantee faithful violations of the Bell tests.

II. MEASUREMENT DEPENDENCE

This section reviews the theory of Bell tests with imperfect inputs, i.e., *measurement dependence*. Given LHVMs, the probability distribution can be represented as

$$P(ab|xy) = \sum_{\lambda} P(\lambda)P(a|x\lambda)P(b|y\lambda). \quad (2)$$

where λ is the predetermined strategy shared by Alice and Bob and $P(\lambda)$ is the probability of choosing λ. In contrast, Eq. (2) cannot generally describe the probability distribution generated by measurement of a quantum state. Bell inequalities can be regarded as witnesses of a quantum probability distribution, which separates it from the probability distributions of Eq. (2).

In the ideal case, the inputs are assumed to be perfectly random; i.e, the adversary does not have extra information of x and y better than blindly guessing. Nevertheless, in practice, input randomness can be imperfect and determined by some local hidden variable λ according to $P(xy|\lambda)$. In this case, as shown in Fig. 1, an adversary that has access to λ can realize the measurement by exploiting such information to have a larger set of LHVMs with probability distributions of the form

$$P(abxy) = \sum_{\lambda} P(\lambda)P(xy|\lambda)P(a|x\lambda)P(b|y\lambda). \quad (3)$$

To understand how $P(xy|\lambda)$ characterizes the input randomness, we consider the lower bounds of $P(xy|\lambda)$ defined as

$$l = \min_{xy\lambda} P(xy|\lambda). \quad (4)$$

Focusing on the case with binary inputs hereafter, we have $l \in [0, 1/4]$. Let us consider two extreme cases. First, when the inputs are perfectly random, we have $P(xy|\lambda) = P(xy)$ and $l = 1/4$. In this case, we recover the probability distribution given in Eq. (2). Second, when the inputs are totally determined by λ, i.e., $(xy) = f(\lambda)$, then the probability becomes

$P(xy|\lambda) = \delta_{(xy),f(\lambda)}$, where δ is the Kronecker delta and we have $l = 0$. In this case, realizing any probability distribution is possible and furthermore, violation of a Bell inequality with a LHVM is possible.

From those scenarios, we can infer that l measures the input randomness. Thus, a smaller l value indicates a less random input or that Alice and Bob have more information of the inputs. In the literature, the problem of Bell tests with imperfect inputs is called measurement dependence. Contrarily, given l, a probability distribution that cannot be described by Eq. (3) is known as measurement-dependent nonlocality. We now see that in a Bell test with human inputs, the assumption that humans have free will can be represented with $l > 0$, implying at least partial measurement independence. In the subsequent texts, we consider two Bell inequalities and show the Bell value dependence on the input randomness parameter l.

The coefficients of the Clauser-Horne-Shimony-Holt (CHSH) [33] are given by $c_{abxy} = (-1)^{a+b+xy}$. Suppose the average probability $P(xy) = \sum_\lambda P(\lambda)P(xy|\lambda)$ equals 1/4 and $P(ab|xy)$ is no-signaling [34], then the classical upper bound $J_C(l)$ with imperfect input randomness characterized by l is [23,31]

$$J_C(l) = 4(1 - 2l). \tag{5}$$

In experiment, the Bell inequality is violated only when the observed Bell value is larger than $J_C(l)$. To do so, a maximally entangled state is prepared

$$|\Psi^+\rangle = (|HV\rangle + |VH\rangle)/\sqrt{2}, \tag{6}$$

and measured in the projecting bases

$$\begin{aligned} A_0(\theta) = Z, \quad B_0(\theta) = \frac{X - Z}{\sqrt{2}}, \\ A_1(\theta) = X, \quad B_1(\theta) = \frac{X + Z}{\sqrt{2}}, \end{aligned} \tag{7}$$

where A_0 and A_1 (B_0 and B_1) are the measurement bases of Alice (Bob) when the inputs are 0 and 1, respectively; $Z = \{|H\rangle, |V\rangle\}$ and $X = \{(|H\rangle + |V\rangle)/\sqrt{2}, (|H\rangle - |V\rangle)/\sqrt{2}\}$.

The other Bell inequality we considered is known as the measurement-dependent local (MDL) inequality proposed by Pütz *et al.* [23], which can be represented by

$$lP(0000) - (1 - 3l)(P(0101) + P(1010) + P(0011)) \leqslant 0. \tag{8}$$

It is proven that when $l \leqslant P(xy|\lambda)$ the probability distribution given in Eq. (3) cannot violate such an inequality. To violate this inequality with quantum settings, we need to prepare a nonmaximally entangled state

$$|\Psi\rangle = \frac{1}{\sqrt{3}}\left(\frac{\sqrt{5}-1}{2}|HV\rangle + \frac{\sqrt{5}+1}{2}|VH\rangle\right) \tag{9}$$

and measure in the bases:

$$\begin{aligned} &A_0(\theta) = \{\cos(\theta)|H\rangle + \sin(\theta)|V\rangle, \sin(\theta)|H\rangle - \cos(\theta)|V\rangle\}, \\ &A_1(\theta) = A_0(\theta - \pi/4), \\ &B_0(\theta) = A_0(\theta + \pi/2), \quad B_1(\theta) = A_1(\theta + \pi/2), \end{aligned} \tag{10}$$

with $\theta = \arccos\sqrt{1/2 + 1/\sqrt{5}} \approx 13.28^\circ$. The state and measurement are optimized by maximizing the violation of the Bell inequality (8). For the detailed discussion of the optimization, please refer to Ref. [23].

III. EXPERIMENTAL SETUP

We experimentally test the Bell inequalities using human-generated random numbers. As shown in Fig. 2(b), we first generate a 780 nm pulsed light as pump via a second-harmonic generation (SHG) process. We set the 1560 nm seed laser to 10 ns pulse width at 100 kHz, which is then amplified and frequency up-converted to 780 nm as the pump. The pump is then focused on a periodically poled potassium titanyl phosphate (PPKTP) crystal to create photon pairs at 1560 nm via spontaneous parametric down conversion. Down-converted photon pairs interfere at the polarizing beam splitter (PBS) in a Sagnac based setup [35] to create entangled pairs. A nonmaximally entangled state can be generated by adjusting the input pump polarization. The entangled pairs are then collected into single mode fibers for detection.

The source laboratory and the measurement laboratory are chosen in a straight line. As shown in Fig. 2(a), the source is in the middle, and Alice's (Bob's) measurement laboratory is 87 ± 2 m (88 ± 2 m) away. Spatial separation ensures the measurements in Alice's laboratory will not affect those in Bob's laboratory, and vice versa. In each pulse period, a random number (if there is one) controls the Pockels cell by applying a zero or half-wave voltage, setting the basis to A_0/A_1 for Alice (or B_0/B_1 for Bob). We compensate polarization drift and align the reference frame using a polarization controller and a half-wave plate. After the measurement using a Pockels cell and a PBS, photons are detected using superconducting nanowire single-photon detectors (SNSPDs). Note that the total response time of the Pockels cell and the SNSPD is around 150 ns, less than the ~290 ns separation between the source and the detection laboratory.

The human-generated random numbers are collected and distributed to our laboratory by ICFO, who initiated the BIG Bell Test project. In this project, people play an online game to generate random numbers, which are then transmitted to the experimental labs. We receive the human-generated random numbers with a Python program and immediately redistribute them to a field-programmable gate array (FPGA) that generates the signal pulses and sync pulses based on the random numbers, which is sent to the measurement station for the experiments. The system is set at 100 kHz frequency to generate signal pulses as soon as a batch of random numbers comes in. We record all the detection results and the random numbers using a time-to-digital convertor (TDC) for off-line data analysis. The peak incoming random number rate is a few thousand bits per second, thus the system waits in most of the time for the incoming random numbers. Note that the human-generated random numbers are directly used as measurement bases without any modification.

IV. EXPERIMENTAL RESULTS

In our experiment, we first adjust pump polarization diagonally to generate a maximally entangled state $|\Psi^+\rangle = (|HV\rangle + |VH\rangle)/\sqrt{2}$. We measure the visibility in the horizontal and vertical basis as $(99.2 \pm 1.1)\%$ and in the

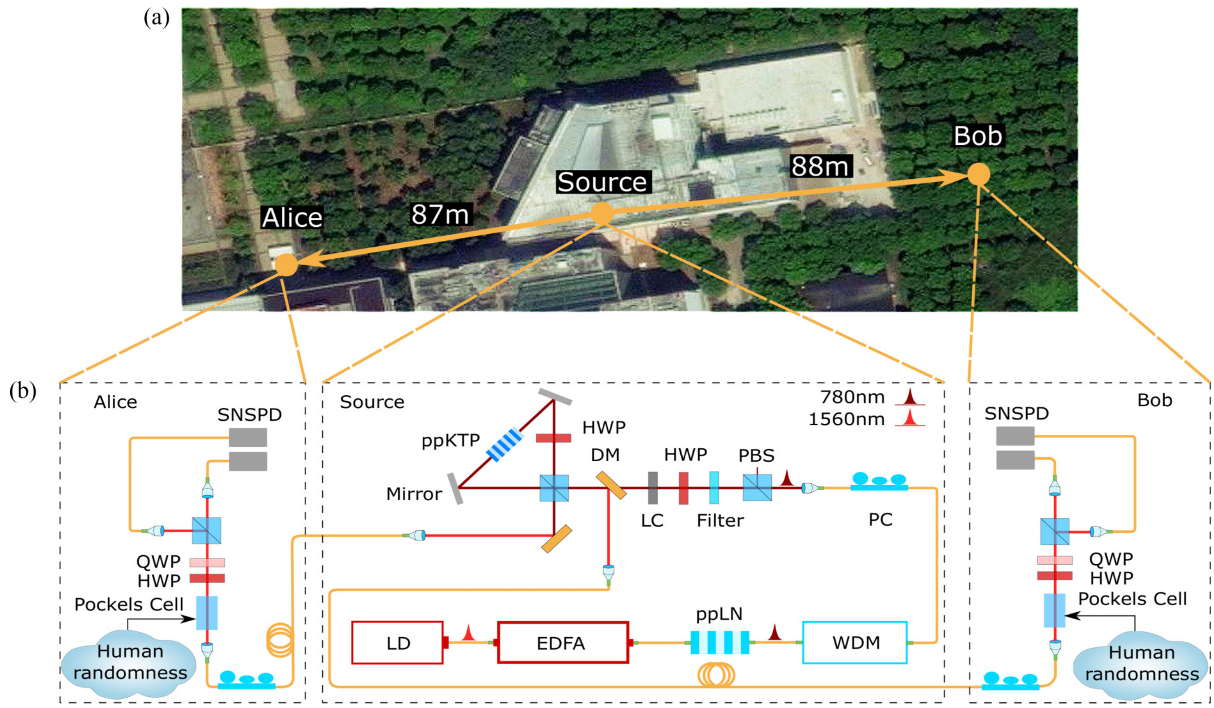

FIG. 2. Bell test using imperfect input randomness. (a) Positions of the entanglement source, Alice's and Bob's detection. Distance between the source and Alice (Bob) is 87 ± 2 m (88 ± 2 m). (b) Schematic setup of the Bell test. A distributed feedback (DFB) laser diode (LD) at $\lambda = 1560$ nm is modulated at 100 kHz with 10 ns width. The pulse is amplified with an erbium-doped fiber amplifier (EDFA) and then up-converted to 780 nm via a second-harmonic generation (SHG) in an in-line periodically poled lithium niobate (PPLN) waveguide. The residual 1560 nm light is filtered with a wavelength-division multiplexer (WDM) and a filter. After polarization adjustment using a half-wave plate (HWP) and a liquid crystal (LC), the 780 nm pump light is focused to a periodically poled potassium titanyl phosphate (PPKTP) crystal in a Sagnac setup to generate entangled pairs. A series of dichroic mirrors (DMs) are used to remove the residual pump light at 780 nm and the fluorescence before the entangled pairs are collected. In Alice's and Bob's detection station, a polarization controller (PC), a quarter-wave plate (QWP), a HWP, and a polarizing beam splitter (PBS) are used to align the reference frame. Random numbers control the Pockels cell to dynamically select the bases. Superconducting nanowire single-photon detectors (SNSPDs) are used to detect the photons after the PBS.

diagonal and antidiagonal basis as $(98.0 \pm 1.0)\%$. We further conduct a state tomography and find the fidelity to the ideal state is approximately 99.1% with maximum likelihood estimation. By setting the measurement bases according to Eq. (7), we measure the S value to be 2.804 ± 0.074, with $E(A_1, B_1) = -0.751$, $E(A_1, B_2) = 0.651$, $E(A_2, B_1) = 0.657$, and $E(A_2, B_2) = 0.745$. Here, the average value is defined by $E(A_x, B_y) = \sum(-1)^{a+b+xy} P(abxy)$. The classical upper bound of the measurement-dependent LHVM is given by Eq. (5). To achieve the experimentally obtained Bell value with measurement-dependent LHVM, we thus have $l < 0.1495 \pm 0.0092$. On the other hand, when the input human randomness has $l > 0.1495$, the experimentally observed data cannot be explained with LHVMs.

Next, we test the MDL inequality using the human random numbers. The MDL inequality had been experimentally tested by Aktas *et al.* [36], but without closing the locality or freedom-of-choice loophole. In our experiment, the freedom-of-choice loophole is closed via the human free will. We separate the measurement station, so the measurement devices cannot communicate with each other. Note that the locality loophole is not fully closed because the random number is not generated locally at the measurement station. In state preparation, we prepare the nonmaximally entangled state given in Eq. (9), by adjusting the pump laser to $\cos(69.1^\circ)\,|H\rangle + \sin(69.1^\circ)\,|V\rangle$. We perform a state tomography to characterize the produced state and calculate the fidelity to be 98.9% with maximum likelihood estimation.

Finally, we analyze the recorded data to count the coincidence events and calculate the probabilities $P(abxy)$.

TABLE I. Experimental values of the MDL inequality measurement using human random numbers. MDL value $l_0 = 0.10 \pm 0.05$ is obtained by calculating all the data. Bases A_0B_0, A_0B_1, A_1B_0, and A_1B_1 are measured to obtain the probabilities of $P(0000)$, $P(0101)$, $P(1010)$, and $P(0011)$ for analysis in Eq. (8). In the probability $P(abxy)$, xy denotes Alice's and Bob's bases while ab denotes the output bits Alice and Bob count for coincidence. The column "Total" refers to the total coincidence count for all Alice's and Bob's outputs, and "Measured" refers to the coincidence count of the specific outputs listed in $P(abxy)$. The result sums all the experimental trials between November 30 and December 1, 2016. Statistical fluctuation is calculated using 1 hour as an integration time.

Basis	Measured	Total	$P(abxy)$
A_0B_0	2833	34408	$P(0000) = 0.02093 \pm 0.01099$
A_0B_1	100	40085	$P(0101) = 0.00074 \pm 0.00048$
A_1B_0	193	41009	$P(1010) = 0.00143 \pm 0.00076$
A_1B_1	86	19853	$P(0011) = 0.00064 \pm 0.00046$

TABLE II. Experimental values of the MDL inequality measurement using quantum random numbers. MDL value $l_0 = 0.106 \pm 0.007$ is obtained by calculating all the data. Column indicators are the same as in Table I. The experiment is performed after the test with human random numbers, using the same setup, but quantum random numbers are substituted instead. Statistical fluctuation is calculated using 5 min as an integration time.

Basis	Measured	Total	$P(abxy)$
A_0B_0	38911	463901	$P(0000) = 0.02101 \pm 0.00062$
A_0B_1	1214	453939	$P(0101) = 0.00066 \pm 0.00009$
A_1B_0	3246	471152	$P(1010) = 0.00175 \pm 0.00023$
A_1B_1	1577	462591	$P(0011) = 0.00085 \pm 0.00007$

The result is summarized in Table I. All experimental trials performed between November 30 and December 1, 2016, were recorded, when the public helped us generate random numbers. Results of the experiment are divided into several sections for statistical analysis, with each section including 1 hour of data. We measure the four probabilities $P(abxy)$ and calculate the MDL Bell inequality in Eq. (8) for a given l. As this inequality cannot be violated with LHVM satisfying $l \leqslant P(xy|\lambda)$, we calculate the smallest possible value of l such that the equal sign is saturated and denote it as l_0. When the input randomness has a larger value of $l > l_0$, the observed result cannot be explained with LVHMs.

We obtain $l_0 = 0.10 \pm 0.05$ for the MDL inequality using human random numbers. For comparison, we use quantum random number generators for the basis selection. With the rest of the setup remaining the same, we obtain $l_0 = 0.106 \pm 0.007$ which gives a similar l_0 value with less fluctuation, as it is easier to accumulate more data using quantum random numbers than using human random numbers. The result is summarized in Table II. For human-generated random numbers, we received several thousand bits per second at peak times, while only tens of bits per second at idle times. For quantum random numbers, we have 200 kHz steady random numbers controlling the basis. In total, we accepted and used around 80 Mbits in the human random number based MDL inequality test in two days. For comparison, we test the inequality with quantum random numbers in around 1.5 hours, consuming more than 1 Gbits in the process.

V. DISCUSSION

In this work, we realized a Bell test with the assistance of human free will to close the freedom-of-choice loophole. We experimentally tested the CHSH and MDL inequalities with the measurement stations space-likely separated. Comparing the results of the two inequalities, we could see that the MDL inequality tolerates more imperfections of input randomness, in the sense of influence by the hidden variables in the LHVM. Future work could extend the results to randomness amplification that amplifies imperfect human randomness to almost uniform randomness.

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