

# Robust gate design for large ion crystals through excitation of local phonon modes

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We propose a scalable design of entangling quantum gates for large ion crystals with the following desirable features: 1) The gate design is universal and applicable for large ion crystals of arbitrary sizes; 2) The gate has no speed limitation and can work outside of the Lamb-Dicke region; 3) The gate operates by driving from either continuous-wave or pulsed laser beams; 4) The gate is insensitive to slow variation of the laser optical phase and works under a thermal state for the ions' motion; 5) The intrinsic gate infidelity can be reduced to a level well below the threshold for fault-tolerant quantum computation under realistic experimental parameters. Different from the previous gate schemes, here we propose a gate design based on driving of the local oscillation mode of the ions instead of the collective normal modes and develop a formalism based on the Heisenberg equations to deal with the many-body quantum dynamics outside of the Lamb-Dicke region.

Trapped ions constitute one of the most promising systems for realization of large-scale quantum computing [1–4]. To scale up the trapped ion quantum computer, several approaches have been considered, including the ion shuttling approach based on the QCCD (quantum charge-coupled device) architecture [5, 6], the quantum network approach based on the photon entanglement links of ions in different traps [7–9], and the direct approach based on entangling gates in large ion crystals of 1D (one-dimensional) [10], 2D [11], or 3D [12] geometry. The latter approach, when it works, would be most convenient and cost-saving for experiments. Even with the first two approaches in mind for the ultimate scaling, it is always cost-effective for large-scale computing to have as many ions as possible in local ion crystals. With use of the cryogenic traps, one can stably control any large ion crystals with negligible influence from the background gas collisions [13]. The major challenge is then the design of robust entangling gates in large ion crystals of arbitrary sizes.

The entangling gates play a central role for implementation of quantum computing. For trapped ion systems, the original Cirac-Zoller gate assumes the ground state cooling of the ions [1]. The Molmer-Sorensen gate [14, 15] and the phase gate [16, 17], which are widely used in experiments, alleviate this requirement and replace it with the Lamb-Dicke condition. These gates typically still assume sideband addressing of individual canonical normal modes during the gates. For large ion crystals, in particular for more scalable gates based on excitation of the transverse phonon modes [18], it is challenging to satisfy the sideband addressing condition, and the gate scheme based on segmentally modulated laser pulses was introduced in Ref. [18, 19], which removes the sideband addressing condition and finds wide applications in recent experiments for gates based on excitation of multiple phonon modes [20–27]. These gates can be extended to 2D and 3D ion crystal architectures by incorporating the effects of micromotion in the gate design [11, 12], and the segmental modulation can be applied on either laser

amplitude [18–23], frequency detuning [24, 25], or phase [26, 27]. One complexity for these segmentally modulated gates is that the optimal gate parameters have to be calculated based on the detailed experimental configuration, including the number of ions in the crystal, the equilibrium positions of every ion, and the normal mode spectrum. An important constraint for all these gates is the requirement of the Lamb-Dicke condition, assuming the conditional position shift of each ion much less than the laser wavelength. Breach of this condition is a major cause of the gate infidelity when one increases the gate speed [23]. Another paradigm for the gate design is based on application of a number of discrete momentum kicks from a train of ultra-short pi-pulses [28, 29]. This approach does not require the Lamb-Dicke condition, however, the complexity and the error accumulation in applying many pi-pulses from an ultrafast laser make the experiment for this approach quite challenging [30], and the achieved fidelity so far is still significantly lower compared with those of the other approaches and the threshold for fault-tolerant quantum computing.

In this paper, we propose a robust and scalable gate design with the following features: 1) the gate applies to any large ion crystals without the requirement of sideband addressing, and at the same time the gate design is universal and does not require detailed knowledge of the ion number and configuration in the crystal and the spectrum of the normal modes; 2) the gate has no speed limitation and can work outside of the Lamb-Dicke region; 3) the gate can operate by driving from conventional continuous-wave laser beams and is insensitive to the optical phase fluctuation from driving laser beams coming in different directions. Different from the previous gate designs, here we achieve the conditional phase in entangling quantum gates based on driving of the local phonon modes instead of the collective normal mode of the ion crystal. We develop a formalism for the gate design based on the Heisenberg equations to deal with nonlinear dynamics outside of the Lamb-Dicke region and use the interaction picture to calculate the gate infi-

delity from the quantum many-body dynamics. We find that the intrinsic gate infidelity can be reduced to a level well below the error threshold for fault-tolerant quantum computing under reasonable experimental parameters for any large ion crystals in both 1D and 2D configurations. The scheme also directly applies to a scalable 2D array architecture of microtraps [31–34] to achieve entangling gates with fast enough gate speed under large ion spacing and moderate laser power.

Now let us consider the gate design for a large ion crystal of arbitrary size. A key concept in this design is the local phonon mode for the ion oscillation, with its frequency defined as the oscillation frequency  $\omega_l$  of the target ion  $l$  when all the other ions in the crystal are fixed at their equilibrium positions. The value of  $\omega_l$  includes contribution from the trapping potential and the Coulomb interaction from all the ions. We achieve the entangling gate by driving two target ions in the many-ion crystal with the laser frequency resonant (or near-resonant) with the local oscillation frequency  $\omega_l$ .

The total Hamiltonian of the system can be written as  $H = H_0 + H_1$  with

$$H_0 = \sum_{\mu} \left( \frac{p_{\mu}^2}{2m} + \frac{1}{2} m \omega_{\mu}^2 x_{\mu}^2 \right) + \sum_{i=1,2} \sigma_i V(x_i) \quad (1)$$

where  $x_{\mu}$  denotes the coordinate operator (displacement from the equilibrium position) and  $p_{\mu}$  is the corresponding momentum operator. The summation  $\mu$  is over all the ions in the crystal, which should be understood as  $\mu = (\mu_1, \mu_2)$  for a 2D crystal. The summation  $i$  is only over the two target ions 1, 2 on which we want to apply the entangling gate through application of a spin-dependent potential. The local oscillation frequency  $\omega_{\mu}$  is defined by  $m\omega_{\mu}^2 \equiv \frac{\partial^2}{\partial x_{\mu}^2} (V_T + V_C)$  for the  $\mu$ th ion in the crystal, where  $V_T$  denotes the trapping potential, and  $V_C$  denotes the Coulomb energy with  $V_C = \frac{1}{2} \sum_{\mu \neq \mu'} \frac{k_c e^2}{|x_{\mu} - x_{\mu'}|}$  ( $|x_{\mu} - x_{\mu'}|$  should be understood as distance between the two vectors  $x_{\mu}$  and  $x_{\mu'}$  for the 2D case). Note that if  $V(x_i)$  is a linear function of  $x_i$ , it reduces to a spin-dependent force, which is the case when we apply the Lamb-Dicke condition (expansion of  $V(x_i)$  to the first order of  $x_i$ ) [14–16]. Here we consider a general spin-dependent potential which could be outside of the Lamb-Dicke region (fast gates require large spin-dependent position shifts of the target ions which drive them outside of the Lamb-Dicke region). If the gate is driven by a pair of traveling-wave laser beams, the potential  $V(x_i)$  has the form

$$V(x_i) = 2\hbar |\Omega| \cos[kx_i + \phi_t + \phi_0] \quad (2)$$

where  $k$  denotes the wavevector difference of the two beams along the  $x_i$  direction,  $\phi_t$  is a time-dependent phase of  $\Omega$  that will be controlled during the gate, and  $\phi_0$  is the slowly varying optical phase difference from laser

beams coming in different directions which is assumed to be fixed and unknown during each gate but fluctuates from gate to gate. The spin operator  $\sigma_i$  reduces to  $\sigma_{iz}$  if we apply a spin-dependent ac-Stark shift [17] and to  $\sigma_{ix}$  if we apply the phase-insensitive Raman driving [35]. The Hamiltonian  $H_1$  has the form

$$H_1 = -\frac{1}{2} \sum_{\mu \neq \mu'} m \omega_{\mu, \mu'}^2 x_{\mu} x_{\mu'} \quad (3)$$

where  $m \omega_{\mu, \mu'}^2 \equiv \frac{-\partial^2 V_C}{\partial x_{\mu} \partial x_{\mu'}}$ .

We solve the dynamics in the interaction picture. With respect to  $H_0$ , the Heisenberg equations for  $x_i$  and  $p_i$  are given by

$$\dot{x}_i = \frac{p_i}{m}, \quad \dot{p}_i = -m \omega_i^2 x_i - \sigma_i \frac{\partial V(x_i)}{\partial x_i} \quad (4)$$

From the driving of the spin-dependent potential  $\sigma_i V(x_i)$ , the  $i$ th ion will follow a spin-dependent trajectory in the phase space. We decompose the operators  $x_i, p_i$  as

$$x_i = x_{ic} + x_{iq}, \quad p_i = p_{ic} + p_{iq}, \quad (5)$$

where  $x_{ic}, p_{ic}$  denote the classical spin-dependent trajectory (proportional to  $\sigma_i$ , but otherwise a classical function with  $x_{ic}(0) = p_{ic} = 0$  at the initial time), and  $x_{iq}, p_{iq}$  with  $[x_{iq}, p_{iq}] = i\hbar$  denote the small quantum fluctuation around the spin-dependent trajectory. With  $V(x_i)$  given by Eq. (3), we can expand

$$\begin{aligned} -\frac{\partial V(x_i)}{\partial x_i} &= 2\hbar |\Omega| k \sin(kx_i + \phi_t + \phi_0) \\ &\simeq 2\hbar |\Omega| k [\sin(kx_{ic} + \phi_t + \phi_0) \\ &\quad + kx_{iq} \cos(kx_{ic} + \phi_t + \phi_0)] \end{aligned} \quad (6)$$

where  $kx_{ic}$  may not be small under large driving and thus we keep the exact function to all the orders of  $kx_{ic}$ . However,  $kx_{iq} = \eta_i(a_i + a_i^{\dagger})$  with the Lamb-Dicke parameter  $\eta_i \equiv k\sqrt{\hbar}/(2m\omega_i)$  representing the small quantum fluctuation around the spin-dependent trajectory, which is determined by the initial thermal fluctuation and is small under a small Lamb-Dicke parameter  $\eta_i$ . In Eq. (7), we expand  $\frac{\partial V(x_i)}{\partial x_i}$  to the linear order of  $kx_{iq}$  (this is equivalent to expansion of  $V(x_i)$  to the quadratic order of  $kx_{iq}$ , one order higher than the conventional spin-dependent force approximation already). Later, we will consider expansion of  $V(x_i)$  to even higher orders of  $kx_{iq}$  and show that they give only small contribution to the gate infidelity under typical experimental parameters. With the above expansion, the Heisenberg equation (4) becomes

$$\begin{aligned} \dot{x}_{ic} &= \frac{p_{ic}}{m}, \\ \dot{p}_{ic} &= -m \omega_i^2 x_{ic} + 2\hbar |\Omega| k \sigma_i \sin(kx_{ic} + \phi_t + \phi_0), \end{aligned} \quad (7)$$

and

$$\begin{aligned} \dot{x}_{iq} &= \frac{p_{iq}}{m}, \\ \dot{p}_{iq} &= -m\omega_i^2 x_{iq} + 4m\omega_i \eta_i^2 |\Omega| \sigma_i x_{iq} \cos(kx_{ic} + \phi_t + \phi_0). \end{aligned} \quad (8)$$

Equation (7) is a set of nonlinear differential equations for the classical variables  $x_{ic}, p_{ic}$  which can be solved exactly (numerically) to give the spin-dependent trajectory. The spin operator  $\sigma_i$  takes the eigenvalue  $\pm 1$  under the component eigenstates  $|0\rangle, |1\rangle$ . Equation (8) is a set of linear differential equations for the operators  $x_{iq}, p_{iq}$ , which can also be solved exactly with knowledge of  $x_{ic}$ . Typically, the amplitude  $4\eta_i^2 |\Omega| \ll \omega_i$ , so the second term in Eq. (8) representing the spin-dependent modulation of the oscillation frequency is only a small perturbation, and its effect can be included into the Hamiltonian  $H_1$  instead of  $H_0$  by the following transformation

$$\begin{aligned} H_0 &\rightarrow H_0^t = H_0 + \sum_{i=1,2} H_i^p, \\ H_1 &\rightarrow H_1^t = H_1 - \sum_{i=1,2} H_i^p, \end{aligned} \quad (9)$$

where  $H_i^p \equiv 2m\omega_i \eta_i^2 |\Omega| \sigma_i x_{iq}^2 \cos \phi_{xt}$  and  $\phi_{xt} \equiv kx_{ic} + \phi_t + \phi_0$ . Under this transformation, the second term in Eq. (8) is gone and therefore the motion of  $x_{iq}, p_{iq}$  is spin independent under the transformed Hamiltonian  $H_0^t$ . We choose a basic time interval  $\tau$  for the gate so that the solution of  $x_{ic}(\tau), p_{ic}(\tau)$  from Eq. (7) is independent of the value of  $\sigma_i$  (a convenient choice is the solution with  $x_{ic}(\tau) = p_{ic}(\tau) = 0$ ). In this case, the motional dynamics from  $H_0$  gets decoupled from the spin state at  $\tau$ . The unitary operator  $U_0(\tau) \equiv T \exp(-i \int_0^\tau H_0^t(t) dt / \hbar)$ , where  $T \dots$  denotes time-ordered integration, therefore is independent of the spin state of the ions. The interaction Hamiltonian is then given by

$$\begin{aligned} H_I(t) &= U_0^\dagger(t) H_1^t(x_\mu) U_0(t) \\ &= H_1^t(U_0^\dagger(t) x_\mu U_0(t)) \\ &= - \sum_{\mu \neq \mu'} (m/2) \omega_{\mu, \mu'}^2 x_\mu(t) x_{\mu'}(t) \\ &\quad - \sum_{i=1,2} 2\eta_i^2 \sigma_i m \omega_i |\Omega| x_{iq}^2(t) \cos \phi_{xt}, \end{aligned} \quad (10)$$

where  $x_\mu(t), x_{iq}(t)$  represent the solution from the corresponding Heisenberg equations under the Hamiltonian  $H_0$ . With  $x_\mu(t) = \sqrt{\hbar/(2m\omega_\mu)}(a_\mu + a_\mu^\dagger)$  for  $\mu \neq 1, 2$  and  $x_\mu(t) = x_{\mu c}(t) + x_{\mu q}(t) = \sqrt{\hbar/(2m\omega_\mu)}(\alpha_{\mu c} + a_\mu + \alpha_{\mu c}^* + a_\mu^\dagger)$  for  $\mu = 1, 2$ , the solution from  $H_0$  has the form

$$a_\mu(t) = a_\mu(0) e^{-i\omega_\mu t}. \quad (11)$$

In term of  $\alpha_{\mu c} = \sqrt{m\omega_\mu/(2\hbar)} x_{\mu c} + i\sqrt{1/(2m\hbar\omega_\mu)} p_{\mu c}$ , ( $\mu = 1, 2$ ), the equation (7) takes the form

$$\begin{aligned} \dot{\alpha}_{\mu c} &= -i\omega_\mu \alpha_{\mu c} \\ &\quad + i(2\eta_\mu |\Omega| \sigma_\mu) \sin[\eta_\mu(\alpha_{\mu c} + \alpha_{\mu c}^*) + \phi_t + \phi_0] \end{aligned} \quad (12)$$

with the initial condition  $\alpha_{\mu c}(0) = 0$ . We take a constant amplitude  $|\Omega|$  for the Raman laser beams and only tune the relative phase  $\phi(t)$  between the beams to satisfy the condition  $\alpha_{\mu c}(\tau) = 0$ .

In the Hamiltonian  $H_I(t)$ , the term  $m\omega_{1,2}^2 x_{1c}(t) x_{2c}(t)/2$  only depends on the spin operators  $\sigma_1, \sigma_2$  and does not couple to the motional modes. The integration of this term over time gives the desired entangling gate on the ion spin qubits. The other terms in  $H_I(t)$  represent remaining spin-motion coupling, which contribute to the gate infidelity. The rate for these spin-motion coupling terms is of the order

$$m\omega_{\mu, \mu'}^2 / (2m\sqrt{\omega_\mu \omega_{\mu'}}) = k_c e^2 / (md^3 \omega_\mu) \equiv \omega_I. \quad (13)$$

where we have taken  $\mu, \mu'$  as the nearest neighbor (the one with the highest interaction rate) in the lattice with lattice distance  $d$  and the local oscillation frequency  $\omega_\mu = \omega_{\mu'}$ . The rate  $\omega_I$  is a basic quantity that characterizes the interaction rate in the ion lattice. We can also define a quantity  $t_p \equiv \sqrt{md^3/(k_c e^2)} = d/v_p$  to characterize the phonon propagation time to the neighboring lattice site, where  $v_p = \sqrt{k_c e^2/(md)}$  characterizes the phonon propagation speed in a large lattice [34]. With this definition, we have  $\omega_I = 1/(\omega_\mu t_p^2)$ .

We take the basic time interval  $\tau$  to satisfy the condition  $\omega_I \tau \ll 1$  (the exact condition will be specified below when we derive the expression for the gate infidelity). The evolution operator in the interaction picture is expressed as

$$U_I(\tau) = T e^{-i \int_0^\tau H_I(t) dt / \hbar}. \quad (14)$$

Without loss of generality, we assume the target ions 1, 2 are at the neighboring sites with  $\eta_1 = \eta_2 = \eta$ ,  $\omega_1 = \omega_2 = \omega$ , and  $\alpha_{1c} = \alpha_{2c} = \alpha_\pm$ , where  $\alpha_\pm$  corresponds to the solution of Eq. (12) with  $\sigma_\mu = \pm 1$ . The conditional phase term in  $U_I(\tau)$  has the form  $e^{i\Phi}$  with

$$\Phi = \varphi_c \sigma_1 \sigma_2 + \varphi_s (\sigma_1 + \sigma_2) \quad (15)$$

where  $\varphi_c = \omega_I \int_0^\tau (\alpha_{R+} - \alpha_{R-})^2 dt$ ,  $\varphi_s = \omega_I \int_0^\tau (\alpha_{R+}^2 - \alpha_{R-}^2) dt$ ,  $\alpha_{R\pm} \equiv \text{Re} \alpha_\pm(t)$ , and we have dropped the spin-independent global phase  $\frac{1}{2} \int_0^\tau \omega_I (\alpha_{R+} + \alpha_{R-})^2 dt$  in  $\Phi$ .

For fast gates with  $\eta\alpha \sim 1$ , the nonlinear equation (12) can be solved numerically. There is a convenient choice of  $\phi_t$  to satisfy the condition  $\alpha(\tau) = 0$ . We take  $\phi_t = \phi(t - \tau/2)$  to be an even function of  $t - \tau/2$ , and the equation (7), which is equivalent to Eq. (12), has a solution with  $x_{ic}(t - \tau/2) = x_{ic}(\tau/2 - t)$  (even) and  $p_{ic}(t - \tau/2) = -p_{ic}(\tau/2 - t)$  (odd). We can divide the duration  $\tau/2$  into several time segments, and for each time segment  $j$ , we take  $\phi_j(t) = \pm \omega t + \phi_{j0}$  with appropriate  $\phi_{j0}$  so that we have  $\text{Im} \alpha(\tau/2) \propto p_{ic}(t = \tau/2) = 0$ . Due to the symmetry, it is then obvious  $\alpha(\tau) = \alpha(0) = 0$ .

When the system is in the Lamb-Dicke region with  $\eta\alpha \ll 1$ , we can derive an analytic expression for the

solution  $\alpha(t)$ . We take  $\phi_t = \omega t$  when  $0 \leq t \leq \tau/2$  and  $\phi_t = \omega t + \pi$  when  $\tau/2 < t \leq \tau$ , the solution  $\alpha(t)$  is given by  $\alpha(t) = \eta |\Omega| \sigma e^{-i\omega t} [e^{i(\omega t + \phi_0)} \sin(\omega t) - \omega t e^{-i\phi_0}] / \omega$  for  $0 \leq t \leq \tau/2$ , and  $\alpha(t) = \eta |\Omega| \sigma e^{-i\omega t} [-e^{i(\omega t + \omega\tau/2 + \phi_0)} \sin \omega(t - \tau/2) + e^{i(\omega\tau/2 + \phi_0)} \sin(\omega\tau/2) + \omega(t - \tau)e^{-i\phi_0} / \omega]$  for  $\tau/2 \leq t \leq \tau$ . We have  $\alpha(\tau) = 0$  when  $\omega\tau = 2K\pi$ , where  $K$  is an integer. The conditional phase shift in this case is given by

$$\Phi = \eta^2 \omega_I \tau |\Omega|^2 / (6\omega^2) [\omega^2 \tau^2 + 36 \cos^2 \phi_0 - 6] \sigma_1 \sigma_2. \quad (16)$$

Apart from the above conditional phase shift term  $\Phi$ , the other terms in the interaction Hamiltonian  $H_I$ , denoted as  $H_{I_r}$ , generate residue spin-motion entanglement at time  $\tau$  which contributes to the gate infidelity. The evolution operator in the interaction picture can be expressed as

$$U_I(\tau) = e^{i\Phi} T e^{-i \int_0^\tau H_{I_r} dt / \hbar} = e^{i(\Phi + A_+ + A_-)}, \quad (17)$$

where  $A_\pm$  denotes the part of the generator that doesn't (does) flip a sign when we flip the sign of the spin operator  $\sigma_i$  ( $i = 1, 2$ ). To suppress the spin-motion entanglement after the gate, similar to the idea of dynamical decoupling, we compose  $2^n$  ( $n = 1, 2, 3, \dots$ ) segments of the basic time step  $\tau$ . For each segment, we control  $\phi(t)$  to be identical except for an additional phase  $\phi_{aj}$  ( $j = 1, 2, \dots, 2^n$ ) of 0 or  $\pi$  (a phase  $\pi$  corresponds to a sign flip of  $\sigma_i$ ). With  $n = 1, 2, 3$ , we call the resultant scheme the  $\phi_2$ ,  $\phi_4$ ,  $\phi_8$  protocol, with the explicit sequence of phase  $\phi_{aj}$  for each segment:  $\phi_2: \phi_{aj} = [0, \pi]$ ,  $\phi_4: \phi_{aj} = [0, \pi, \pi, 0]$ ,  $\phi_8: \phi_{aj} = [0, \pi, \pi, 0, \pi, 0, 0, \pi]$ . Denote the corresponding evolution operator for the  $\phi_2$ ,  $\phi_4$ ,  $\phi_8$  protocol by  $U_I(2\tau) = e^{i(2\Phi + A_2)}$ ,  $U_I(4\tau) = e^{i(4\Phi + A_4)}$ ,  $U_I(8\tau) = e^{i(8\Phi + A_8)}$ , respectively. The explicit expressions for  $A_2$ ,  $A_4$ ,  $A_8$  can be derived using the Baker-Hausdorff formula and are given in the supplementary materials. We calculate the gate infidelity with the resultant  $U_I$ . Using the  $\phi_8$  protocol as an example, as derived in the supplementary materials, the gate infidelity  $\delta F$  is given by

$$\delta F \simeq \omega_I \tau (\eta |\Omega| \tau)^2 (2n_c \omega_I \tau)^7 (2\bar{n} + 1), \quad (18)$$

where  $\bar{n}$  denotes the mean thermal phonon number of the local mode and  $n_c$  is a dimensionless parameter roughly estimated by the lattice coordination number with  $n_c \simeq 2.0$  (5.6) for 1D (2D) ion lattice.

The conditional phase in Eq. (16) acquired by this gate has a dependence on the unknown optical phase  $\phi_0$  through its second term (coming from the oscillating terms usually neglected by the rotating-wave approximation). Such a dependence becomes negligible for conventional slow gates with  $\omega\tau \gg 2\pi$ , however, its magnitude is comparable with the first term when  $\omega\tau = 2\pi$ . To remove the dependence on the phase  $\phi_0$ , we combine two

$\phi_8$  sequence in succession, while adding a phase of  $\pi/2$  to  $\phi(t)$  for the second  $\phi_8$  sequence. The conditional phase acquired by the two combined  $\phi_8$  sequences is then

$$\Phi = (8/3) \omega_I \tau (\eta |\Omega| \tau)^2 [1 + 12/(\omega\tau)^2] \sigma_1 \sigma_2. \quad (19)$$

To realize a controlled phase flip (CPF) gate, we need  $\Phi = \pi \sigma_1 \sigma_2 / 4$ , and this condition sets the time  $\tau$  (or with a fixed  $\tau$  sets the magnitude of the Raman Rabi frequency  $|\Omega|$ ). In this case, the intrinsic gate infidelity is twice the value given in Eq. (18).

Let us estimate the gate performance under some experimental parameters. For ions with the Lamb-Dicke parameter  $\eta \approx 0.05$  and the local oscillation frequency  $\omega = 2\pi \times 3$  MHz (typical for driving transverse modes of 1D or 2D ion crystals), we have  $\omega_I \simeq 2\pi \times 10$  (3.6) KHz under an ion spacing of  $d = 8.8$  (12.4)  $\mu\text{m}$ . Let us take  $2n_c \omega_I \tau \simeq 1/4$  ( $\omega\tau = 6\pi$  with  $n_c \simeq 2.0$  (5.6) for 1D (2D) ion crystals). To realize a CPF gate, the required  $|\Omega| \simeq 2\pi \times 6.8$  (11.5) MHz for 1D (2D) crystals. The intrinsic gate infidelity is  $\delta F = 1.0 \times 10^{-4}$  with the mean thermal phonon number  $\bar{n} \sim 1$ . The gate time in this case is  $T_g = 16\tau = 16 \mu\text{s}$ . For 2D arrays of microtraps with large ion spacing  $d \sim 50 \mu\text{m}$  [31–34], we can get the gate time  $T_g = 192 \mu\text{s}$  and the intrinsic gate infidelity  $\delta F = 0.92 \times 10^{-7}$  by choosing  $\omega\tau = 144\pi$  and  $|\Omega| \simeq 2\pi \times 1.1$  MHz using a single  $\phi_8$  cycle.

Another contribution to the gate infidelity is from the higher-order Lamb-Dicke expansion in  $kx_{iq} = \eta(a_i e^{-i\omega t} + a_i^\dagger e^{i\omega t})$ . In Eq. (6), we have included the expansion to the first two orders in  $kx_{iq}$  and all the orders in  $kx_{ic}$ . In the supplementary materials, we show that the higher-order expansion terms in  $\sin(kx_i + \phi_t + \phi_0)$  of Eq. (6) contribute to the gate infidelity by the form  $\delta F \simeq \frac{\pi^2}{2} \eta^4 (\bar{n} + 1/2)^2 \simeq 0.38 \times 10^{-4}$  under  $\bar{n} \sim 1$ . For 2D ion crystals, the ions also have micromotion around their equilibrium positions. As shown explicitly in Ref. [11], the micromotion has well defined dynamics which does not give additional gate infidelity but leads to a renormalization of the effective Raman Rabi frequency  $|\Omega|$  by an average over the Gaussian beam profile in the 2D plane (assuming the wavevector difference of the Raman beams is perpendicular to the ion lattice to drive the transverse modes). After this correction of the effective magnitude  $|\Omega|$ , the above formalism remains unchanged.

In summary, we have developed an approach for designing robust and scalable entangling gates for ions in arbitrarily large lattices based on excitation of the local phonon modes. The scheme has a number of desirable features, removes some key limitations in the current approach, and may have wide applications in future experiments as one scales up the ion trap quantum computers.

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