Optimal Control for Electricity Storage Against Three-Tier ToU Pricing

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***Abstract*—The emerging sharing economy has deeply changed our daily lives. Recently, there has been an increasing interest in exploiting the new business models in electricity sector. Most existing research focused on the arbitrage against two-tier Time-of-Use (ToU) pricing (namely, the ToU pricing only contains peak period and off-peak period). A simple greedy control policy can achieve the optimal performance. However, this greedy approach cannot be straightforwardly generalized to other types of ToU pricing. In this paper, we consider the optimal control policy for electricity storage to enable arbitrage against various three-tier ToU pricing. We offer both explicit expressions for the control policies and their economic insights. We believe our work is an essential attempt to exploit the possible opportunities for sharing economy in the electricity sector and will sharp our understanding on the impacts of three-tier ToU pricing.**

***Index Terms*—Control policy, electricity storage, time-of-use pricing, Optimal Control**

I. Introduction

Traditionally, without the help of storage devices in the system, power system operators constantly suffer from the mismatch between supply and demand during peak time. Hence, the system operators often utilize Time of Use (ToU) pricing to incentivize large consumers (and in the last decade, to incentive even residential consumers) to change their energy consumption behaviors. However, over the last decade, the cost for installing storage systems has steadily declined. Figure 1 illustrates the pricing trends and forecasts for behind-the-meter energy storage by different technologies [1]. Figure 2 uses lithium-ion storage as an example and further plots its prices in Germany from Q4 2014 to Q1 2017 [2]. It can be observed that the median lithium-ion system price for German customers has fallen by 60% since Q4 2014. This leads to an increasing witness of storage systems in the power grid all over the world.

A. Opportunities and Challenges

These newly installed storage systems can contribute to the power system in a number of ways [3]: to perform peak shaving and hence reduce the critical peak demand, to conduct frequency regulation and improve the system stability, etc. In this paper, we consider a specific scenario where the storage system owners try to arbitrage against the ToU pricing. This arbitrage partially aligns with the peak shaving service since the peak mostly occurs during the peak period defined by the ToU pricing. However, since ToU pricing only specifies different tariffs for specific periods, it may not acurrately reduce

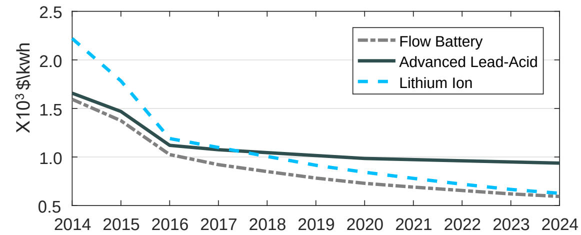

Fig. 1: Utility-Scale Energy Storage System Cost Trends by Technology, Global Averages: 2014 - 2024 [1].

the critical peak. Nonetheless, the analytical understanding for arbitraging is already a very delicate task.

The challenges mainly comes from various structures of ToU pricing. Kalathil *et al.* laid out the analytical framework for arbitraging against the two-tier ToU pricing, and exploited the sharing economy business model based upon the framework [4]. The simplicity of the two-tier scheme allows a greedy control policy (i.e., fully charge the battery during off peak period, and first use the energy in the battery during peak period) to achieve the optimal performance. By contrast, the greedy control policy often does *not* work for multi-tier ToU pricing. In this paper, we propose an (M, C) control policy to handle different kinds of three-tier ToU pricing.

B. Literature Review

Capacity investment and charging operation are the two key questions for electricity-storage control. The current studies on the optimal-investment decision adopt the electricity-planning approach to explore the optimal size of electricity storage [5], [6], [7]. Those studies mainly focus on the storage-size planning for cooperating intermittent renewable energys utilization. Some planning research has further noticed the possibility that the storage owners market strategy can fundamentally influence their investment decisions [8], [9]. The researches on the optimal charging operation for the storage control focus two key questions. A sequence of studies proposes optimal control policies for utilizing storage to improve the transmission-grid operation. In those studies, a great number of literature suggest the controls to facilitating the integration of intermittent renewable-energy into the grid and providing operation services [10], [11]. The other part of relative studies examines the mechanism using storage provide grid-operation services [12], [13], [14]. The other literature

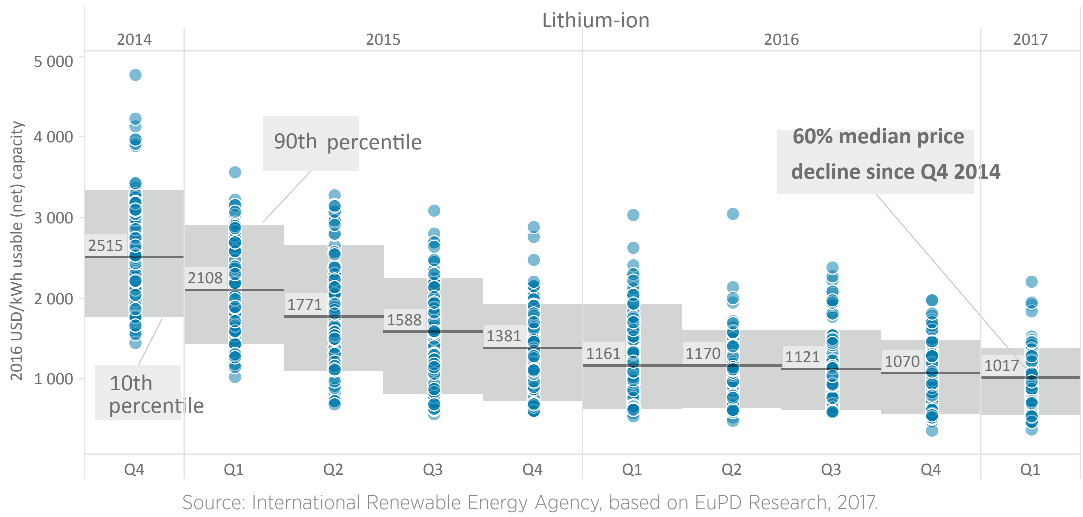

Fig. 2: Home storage lithium-ion system offers in Germany from Q4 2014 to Q1 2017 [2].

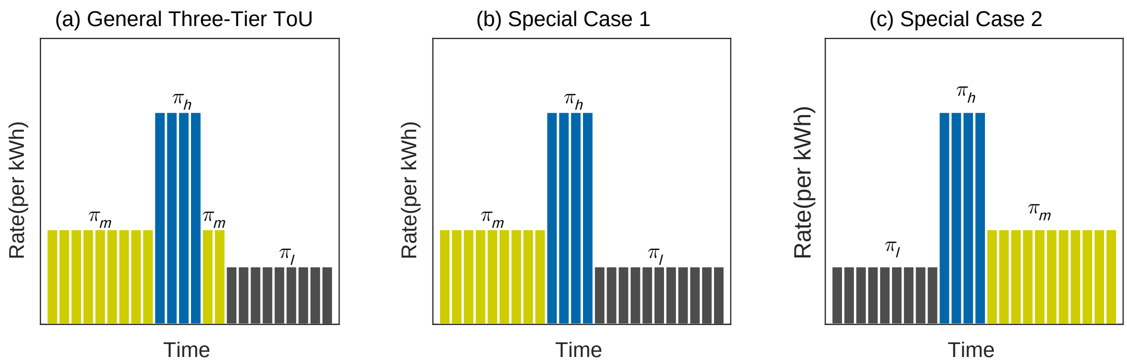

Fig. 3: Sample three tier ToU pricing.

stream on the optimal storage-charging policy addresses the optimal charging-strategy for mitigating the pricing risks in the distribution networks [15], [16], [17]. Those researches consider the scenario when consumers face a time-varying retail rate and utilize the storage to hedge against the price-changing risks. Compared with the previous literature, our work considers a more stylized model and focus on exploiting the possible sharing opportunities in the electricity sector. As for the methodology, we offer the solution from both computational and economic perspectives, which deeps our understanding of the structure of the problem.

The rest of the paper is organized as follows. We introduce the system model in Section II. We analyze the two types of three-tier ToU pricing schemes in Section III and IV, respectively. Throughout the analysis, we offer both explicit optimal control policies and their economic insights. Section V concludes the paper and discusses interesting future directions.

II. System Model

Consider a collection of firms that use electricity. An aggregator interfaces between these firms and the grid. The aggregator itself does not consume electricity. It purchases the net electricity consumed by the firms from the grid, and resells this to the firms.

In practice, each day is usually divided into four contiguous periods, with two partial-peak periods, as shown in Fig. 3(a). To gain more insights on the impacts of three-tier pricing on decision making, we would like to consider two special cases, as shown in Fig. 3(b) and (c), each of which has only one partial peak period.

The aggregator faces time-of-use prices – π_h during peak hours, π_m during partial-peak hours, and π_l during off-peak hours.

Firms can trade electricity with each other, or purchase from the grid through the intermediary aggregator. All firms face the same prices. The distribution system within the aggregators purview allows firms to trade electricity.

Firm k may choose to invest in C_k kWh of storage to arbitrage against the ToU price. Then, the aggregator could set up a mechanism to enable the sharing of unused energy in each firm's storage. In this paper, we focus on the specific standalone problem, where each firm makes its own decision.

This also serves as a performance upper bound for the sharing economy business model with three-tier ToU pricing, since the standalone problem can simulate the combined firm case, where all the firms are merged into a single firm, and all the sharing transactions are internalized.

Recall that with two-tier ToU pricing [4], by ignoring the storage losses and inefficiencies, storage will be charged during off-peak hours, and discharged during the peak hours. This simple greedy control strategy is guaranteed to achieve the optimal performance for each single firm standalone operation. However, the three-tier ToU pricing will dramatically change the structure of the problem, which warrants a carefully designed new control policy for each standalone firm. Specifically, the greedy control policy would *not* work for the three-tier ToU pricing schemes shown in Fig. 3(a) and (b) since there is a partial peak period before the peak. This will force the storage owner to trade-off between using the energy during the partial peak period or reserving it for the upcoming peak. By contrast, the greedy control policy can be easily extended to the ToU pricing scheme in Fig. 3(c), which simply requires the storage owner to fully charge the battery during off-peak, then satisfy the peak demand as much as possible. If there is still leftover energy for the partial peak demand, use the energy in the storage with high priority.

Based on the above observations, in the next two sections, we will first discuss the (M, C) control policy for the first special three tier pricing (Fig. 3(b)), then generalize our results to the more general pricing scheme (Fig. 3(a)).

III. The (M, C) Control Policy

We are interested in a specific type of control policy. Each firm k will first fully charge the battery (of capacity C_k) during off peak hours. And then, it will use the energy in the battery up to $C_k - M_k$ to support the partial peak energy consumption. Finally, the firm will use the remaining energy in the battery to support the energy consumption during peak hours. To highlight the two key parameters in this class of control policy, we term it the (M_k, C_k) policy. Since we will focus the analysis on the standalone problem, we will drop the subscript k for the subsequent analysis.

Denote the energy consumptions for firm k during partial peak, peak, and off peak hours, by X, Y, and Z, respectively. Denote the actual energy purchase from the grid through the aggregator during the three periods by P^m, P^h, and P^l, respectively. The control policy yields the following conditions:

$$P^m = \begin{cases} X - (C - M), & \text{if } X > C - M, \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

$$P^h = \begin{cases} Y - M, & \text{if } X > C - M, Y > M, \\ Y - (C - X), & \text{if } X < C - M, Y > C - X, \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

$$P^l = Z + \begin{cases} X + Y, & \text{if } X < C - M, Y < C - X, \\ Y + (C - M), & \text{if } X > C - M, Y < M, \\ C, & \text{otherwise.} \end{cases} \quad (3)$$

Hence, the daily expected operational cost for firm k can be obtained as follows:

$$J(C, M) = \pi_s C + \pi_m \mathbf{E}[P^m] + \pi_h \mathbf{E}[P^h] + \pi_l \mathbf{E}[P^l], \quad (4)$$

where π_s is the daily amortized cost for the storage system. Thoughout this paper, we assume X and Y are independent. By solving the KKT conditions (details can be found in the Appendix A), we can obtain the optimal control policy (M^*, C^*):

$$\mathbf{P}(Y \le M^*) = \frac{\pi_h - \pi_m}{\pi_h - \pi_l}, \text{ i.e., } M^* = F_Y^{-1}\left(\frac{\pi_h - \pi_m}{\pi_h - \pi_l}\right), \quad (5)$$

$$\mathbf{P}(X \le C^* - M^*, M^* \le Y \le C^* - X) = \frac{\pi_m - \pi_l - \pi_s}{\pi_h - \pi_l}, \quad (6)$$

where $F_Y(\cdot)$ is the cummulative density function of the random variable Y. Actually, we can write the optimal control policy in a denser way:

$$M^* = F_Y^{-1}\left(\frac{\pi_h - \pi_m}{\pi_h - \pi_l}\right), \quad (7)$$

$$C^* = F_{X+Y|Y>M^*}^{-1}\left(\frac{\pi_m - \pi_l - \pi_s}{\pi_m - \pi_l}\right), \quad (8)$$

where $F_{X+Y|Y>M^*}(\cdot)$ is the conditional commulative density function of random variable $X+Y$ given $Y > M^*$. Note that M^* can be solely determined by (5).

Finally, we would like to verify that the obtained (M^*, C^*) is truely the unique optimal control policy. To achieve this goal, we can check the Hessian matrix $H|_{(M^*, C^*)}$ at (M^*, C^*) and prove it is positive definite (Details are deferred to Appendix B). This guarantees that (M^*, C^*) is the unique optimal (M, C) control policy.

Remark: We want to first establish the relationship between the new control policy and the classical news-vendor type solution for two-tier ToU pricing. We understand this relationship with two corner cases when the three-tier ToU degenerates to two-tier ToU pricing.

Case 1: If $\pi_m = \pi_h$, then the partial peak period collapses to peak period.

$$\mathbf{P}(Y \le M^*) = 0 \Rightarrow M^* = 0. \quad (9)$$

That is, firm k does not need to reserve anything for the peak period. And

$$\begin{aligned} &\mathbf{P}(X \le C^*, 0 \le Y \le C^* - X) \\ &= \mathbf{P}(X + Y \le C^*) = \frac{\pi_h - \pi_l - \pi_s}{\pi_h - \pi_l}. \end{aligned} \quad (10)$$

It collapse to the classical result.

Case 2: If $\pi_m = \pi_l$, then the partial peak period collapses to off peak period. In this case, $X \equiv 0$. Note that, the energy consumption during the off peak period does not affect the decision making. Specifically, we can directly check the KKT conditions (22)-(23). Note that, since $X \equiv 0$, we have

$$f_X(x) = \delta(x), \quad (11)$$

where $f_X(\cdot)$ denotes the probability density function of random variable X. Hence,

$$\int_0^{C-M} f_X(x)dx = 1, \quad \int_{C-M}^{\infty} f_X(x)dx = 0. \tag{12}$$

The latter directly implies that $\frac{\partial J}{\partial M} = 0$. Combining with the analysis in Appendix A, we know that the KKT condition for M automatically holds. The other KKT condition for C immediately yields the classical results.

Remark: We want to close this session by exploiting the economic insights behind the explicit control policy. Right now, control policy (5)-(6) is easy to compute and hence ready for implementation. And we have just shown that at corner cases it is equivalent to the classical results, which verifies its correctness. However, the intuition is still missing. By rearranging the control policy, we could obtain the following two equations:

$$\pi_m - \pi_l = (\pi_h - \pi_l)\mathbf{P}\left(Y \le M^*\right) \tag{13}$$

$$\pi_s = (\pi_m - \pi_l)\mathbf{P}\left(X + Y \ge C | Y > M^*\right) \tag{14}$$

The first equation (13) indicates that the marginal cost of reserving 1 unit energy for peak period use (left hand side) should equal to the *expected* marginal profit. And the second equation (14) essentially states that the marginal cost for purchasing 1 unit of capacity (left hand side) should equal to the *expected* marginal profit for having the additional capacity. Note that the right hand side precisely describes the condition that when the additional capacity will make profits if maintaining the reserved capacity for peak period use unchanged.

IV. GENERALIZATION

Now we are ready to generalize the (M, C) control policy to the most general three-tier ToU pricing (Fig. 3(a)). Note that the storage owner does not need to reserve any capacity for the second partial peak period, and this allows us to again apply the (M, C) control policy. However, in this case, capacity M will be reserved for the peak use and the second partial peak use. The analysis precisely parallels with that in Section III, and hence we offer the following theorem without a detailed proof. Instead, we provide the economic insights to illustate the correctness of the theorem.

Theorem 1. *Denote the energy consumptions for firm k during the first partial peak, peak, and second partial peak periods by $X, Y,$ and Z, repesctively*[1]*. The optimal (M, C) control policy is the solution to the following system of equations:*

$$0 = (\pi_m - \pi_l)\boldsymbol{P}(Y + Z \le M) - (\pi_h - \pi_m)\boldsymbol{P}(Y \ge Z) \tag{15}$$

$$\begin{aligned} 0 = \pi_l + \pi_s - \pi_m + (\pi_m - \pi_h)\boldsymbol{P}(X \le C - M, X + Y \ge C) \\ + (\pi_m - \pi_l)\boldsymbol{P}(X \le C - M, X + Y + Z \le C)\,. \end{aligned} \tag{16}$$

[1]It turns out that the energy consumption during off peak period is irrelavent in this problem.

Remark: This is one of the results that is hard to decipher. However, we can again smartly rearrange the terms to gain economic insights and all the intuition will become clear:

$$(\pi_m - \pi_l)\mathbf{P}\left(Y + Z \le M\right) = (\pi_h - \pi_m)\mathbf{P}\left(Y \ge Z\right) \tag{17}$$

$$\begin{aligned} \pi_s = (\pi_h - \pi_m)\mathbf{P}\left(X \le C - M, X + Y \ge C\right) \\ + (\pi_m - \pi_l)\mathbf{P}\left(X \ge C - M, \text{ or } X + Y + Z \ge C\right). \end{aligned} \tag{18}$$

The first equation simply states the expected marginal cost should equal to the expected marginal profit for reserving a single unit energy for peak and the second partial peak use, and the second equation states this principle also holds for purchasing additional storage capacity. Note that, when considering to purchase additional storage capacity, we need to first fix M, which is determined soly by (17). By fixing M, there are only two conditions under which the additional capacity could gain profits: a) the additional capacity being used in any of the non off-peak period will achieve a profit of at least $\pi_m - \pi_l$, b) the additional capacity being used in the peak period will achieve an additional profit of $\pi_h - \pi_m$.

V. CONCLUDING REMARKS

This paper provides the first step towards understanding the sharing economy opportunities in electricity sector with three tier ToU pricing schemes. We offer both the explicit solutions as well as the economic insights for the standalone optimal decision making, which illuminates the upper bound that the sharing economy could possibly achieve.

We plan to understand the spot market for the sharing opportunities under three-tier ToU pricing. It is challenging because sharing can happen in two periods - the partial peak and the peak. However, it is not very straightforward to analyze the competitive market during the partial peak, since the firms' behavior during the partial peak period will affect their decision making during the peak period, and hence the two spot markets are heavily coupled together.

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APPENDIX

A. Optimal (M, C) Control Policy

By assuming X and Y are independent, we have

$$\begin{aligned}\mathbf{E}[P^m] &= \mathbf{E}[(X-(C-M))^+] \\ &= \int_{C-M}^{\infty} (x-(C-M)) f_X(x)dx,\end{aligned} \tag{19}$$

$$\begin{aligned}\mathbf{E}[P^h] = &\int_{M}^{\infty} f_Y(y)(y-M)dy \int_{C-M}^{\infty} f_X(x)dx \\ &+ \int_0^{C-M}\int_{C-x}^{\infty} (x+y-C) f_X(x) f_Y(y) dydx\end{aligned} \tag{20}$$

$$\begin{aligned}\mathbf{E}[P^l] =&\mathbf{E}[Z] + \int_0^{C-M}\int_0^{C-x} (x+y) f_X(x) f_Y(y) dydx \\ &+ \int_{C-M}^{\infty}\int_0^{M} (y+C-M) f_X(x) f_Y(y) dydx \\ &+ C\int_{C-M}^{\infty} f_X(x)dx \int_M^{\infty} f_Y(y)dy \\ &+ C\int_0^{C-M}\int_{C-x}^{\infty} f_X(x) f_Y(y) dydx\end{aligned} \tag{21}$$

Substituting (19)-(21) into (4) leads to the complete integral expression for $J(C, M)$, which allows us to observe the KKT conditions:

$$\begin{aligned}0 = \frac{\partial J}{\partial C} = \pi_s + (\pi_l - \pi_m)&\int_{C-M}^{\infty} f_X(x)dx \\ - (\pi_h - \pi_l)&\int_0^{C-M}\int_{C-x}^{\infty} f_X(x) f_Y(y) dydx,\end{aligned} \tag{22}$$

$$\begin{aligned}0 = \frac{\partial J}{\partial M} = &\left(\pi_m - \pi_h \int_M^{\infty} f_Y(y)dy - \pi_l \int_0^M f_Y(y)dy\right) \\ &\times C\int_{C-M}^{\infty} f_X(x)dx.\end{aligned} \tag{23}$$

Solving this system of equations determines the optimal control policy (M^*, C^*):

$$\mathbf{P}(Y \le M^*) = \frac{\pi_h - \pi_m}{\pi_h - \pi_l}, \text{ i.e., } M^* = F_Y^{-1}\left(\frac{\pi_h - \pi_m}{\pi_h - \pi_l}\right),$$

$$\mathbf{P}(X \le C^* - M^*,\ M^* \le Y \le C^* - X) = \frac{\pi_m - \pi_l - \pi_s}{\pi_h - \pi_l}.$$

B. Positive Definiteness

At (M^*, C^*), the Hessian matrix $H|_{(M^*,C^*)}$ is given by:

$$H|_{(M^*,C^*)} = (\pi_h - \pi_l)\times \begin{bmatrix} \int_0^{C^*-M^*} f_X(x) f_Y(C^*-x)dx & 0 \\ 0 & f_Y(M^*)\int_{C^*-M^*}^{\infty} f_X(x)dx \end{bmatrix},$$

which is positive definite. This guarantees that (M^*, C^*) is the unique optimal (M, C) control policy.