

Experimental Distillation of Quantum Nonlocality

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We report the first experimental demonstration of distillation of quantum nonlocality, confirming the recent theoretical protocol [Phys. Rev. Lett. **102**, 120401 (2009)]. Quantum nonlocality is described by a correlation box with binary inputs and outputs, and the nonlocal boxes are realized through appropriate measurements on polarization entangled photon pairs. We demonstrate that nonlocality is amplified by connecting two nonlocal boxes into a composite one through local operations and four-photon measurements.

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The seminal paper by Bell in 1964 reveals that spatially separated quantum systems can have stronger correlation impossible to be explained by any local hidden variable (LHV) theory based on shared randomness [1,2]. This correlation is known thereafter as quantum nonlocality, which has been tested by a number of remarkable experiments [3–5]. Quantum nonlocality is not only the critical concept for foundational research of quantum mechanics, but also finds important applications in recent development of quantum information theory. Nonlocality emerges as a key resource for realization of various quantum information protocols, such as device-independent quantum key distribution [6–8], nonlocal computation [9], and self-certified random number generators [10,11]. Compared with entanglement, nonlocality represents a stronger form of correlation as nonlocality implies entanglement but the reverse is not true.

Similar to entanglement, nonlocal correlation is more useful if it gets stronger. Entanglement purification protocols have been proposed [12] and demonstrated by several experimental groups [13–15]. An interesting question is whether nonlocality can be distilled. Can we get stronger nonlocality from local operations on multiple weakly nonlocal systems? The answer is far from being obvious as entanglement purification does not automatically yield nonlocality distillation. Entanglement purification protocols in general use both local operation and classical communication [12], while classical communication is not allowed for distillation of nonlocality as it violates the locality requirement. Note that nonlocality is a more stringent resource not directly implied by entanglement [16]. It is well known that there are entangled states where the quantum correlation can be described by the LHV theory with no nonlocality [17]. Because of this difference, it has been proven that for a large class of nonlocality, distillation is actually impossible [18,19]. Only until recently theoretical advance finds examples to show that certain nonlocality described by correlation boxes can be distilled through only local operations [18,20].

In this paper, motivated by this intriguing theoretical advance [18,20], we report the first experimental demonstration of distillation of quantum nonlocality using the photonic system. By controlling the bases of binary measurements on entangled photon pairs, we realize the nonlocal boxes proposed by Forster *et al.* [18] that allow distillation of nonlocality. The nonlocality is quantified by its violation of the Clauser-Horne-Shimony-Holt (CHSH) inequality [2]. We optimize the experimental parameters to maximize the nonlocality difference between the distilled box and the original ones. The difference is typically small (about a few percents) even for the optimized box, so the experiment needs to be precisely controlled. To measure nonlocality of the distilled box formed with local operations on two identical nonlocal boxes, we use linear optical manipulation of two entangled photon pairs and simultaneous detection of four photons through single-photon detectors. The experimental data unambiguously confirm the nonlocally increase of the distilled box compared with the original individual ones.

The nonlocality revealed by violation of Bell's inequality can be described by a correlation box shared between two parties, typically called Alice and Bob [21]. We consider a Bell scenario with binary inputs and outputs. Each party has an input bit, denoted by $x, y \in \{0, 1\}$, that determines his or her measurement basis, and an output bit, denoted by $a, b \in \{0, 1\}$, that corresponds to the measurement outcome. Let $P(ab|xy)$ denote the conditional probability to get the measurement outcomes a and b when Alice and Bob measure in the bases x and y , respectively. $P(ab|xy)$ completely determines the correlation of the box. Under a given basis (x, y) , the correlation of the measurement outcomes (a, b) is described by

$$C_{xy}(P) = P(00|xy) + P(11|xy) - P(01|xy) - P(10|xy). \quad (1)$$

From this correlation function, one can define the CHSH nonlocality of the box [2,18,20], which is characterized by

$$\mathcal{N}(P) = \max_{xy} |\mathbf{C}_{xy}(P) + \mathbf{C}_{\bar{x}\bar{y}}(P) + \mathbf{C}_{\bar{x}y}(P) - \mathbf{C}_{x\bar{y}}(P)|, \quad (2)$$

where \bar{x} (\bar{y}) denotes NOT of the binary number x (y), i.e., $\bar{x} = (1, 0)$ for $x = (0, 1)$. The algebraic maximum of $\mathcal{N}(P)$ is 4; however, $\mathcal{N}(P)$ is bounded by tighter values for physical theories, and the value of $\mathcal{N}(P)$ characterizes the maximum nonlocality achievable in such a theory. The LHV theory based on the assumption of local realism requires that the correlation in $P(ab|xy)$ is from preshared randomness, that is, $P(ab|xy)$ can be written in the form $P(ab|xy) = \int P(a|x, v)P(b|y, v)p(v)dv$, where v is the hidden random variable with the probability distribution $p(v)$. With this restriction on the correlation in $P(ab|xy)$, the well-known CHSH inequality shows that the nonlocality measured by $\mathcal{N}(P)$ is bounded from above by $\mathcal{N}(P) \leq 2$ for any models based on local realism [2]. Quantum mechanics allows stronger correlation, and any violation of the CHSH inequality with $\mathcal{N}(P) > 2$ is a signature of nonlocality. However, nonlocality in quantum mechanics is still bounded by the Tsirelson bound with $\mathcal{N}(P) \leq 2\sqrt{2}$ [22]. It is interesting to note that the nonsignalling condition alone from the relativity theory in principle could allow even stronger nonlocality. In term of the correlation matrix $P(ab|xy)$, the nonsignalling condition requires the marginal distribution $P(a|xy) \equiv \sum_b P(ab|xy) = P(a|x)$, independent of y , and $P(b|xy) \equiv \sum_a P(ab|xy) = P(b|y)$, independent of x . This condition guarantees that two remote parties can not signal (change the marginal distribution of the other side) by choosing different measurement bases. With the nonsignalling condition alone, Popescu and Rohrlich (PR) have constructed a nonlocal box, the so-called PR box, that achieves the maximum algebraic violation of the CHSH inequality with $\mathcal{N}(P) = 4$ [21]. The nonlocality in the range of $2\sqrt{2} < \mathcal{N}(P) \leq 4$, although not attainable by quantum mechanics, can be discussed in the general framework of nonsignalling theory [18,20,21,23].

To distill nonlocality shared between two parties using only local operations, we consider a particular type of nonlocal correlation boxes proposed in Ref. [18], for which the conditional probability matrix $P(ab|xy)$ is parametrized in the following way

$$P \equiv \begin{pmatrix} P(00|00) & P(01|00) & P(10|00) & P(11|00) \\ P(00|01) & P(01|01) & P(10|01) & P(11|01) \\ P(00|10) & P(01|10) & P(10|10) & P(11|10) \\ P(00|11) & P(01|11) & P(10|11) & P(11|11) \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 - \eta & \eta & \eta & 1 - \eta \\ 1 - \eta & \eta & \eta & 1 - \eta \\ 1 - \eta & \eta & \eta & 1 - \eta \\ 1 - \gamma & \gamma & \gamma & 1 - \gamma \end{pmatrix}, \quad (3)$$

with $0 < \eta, \gamma < 1$. It is easy to check that the CHSH nonlocality for this matrix is given by $\mathcal{N}(P) = 2 + 2\gamma - 6\eta$.

This correlation box is nonlocal when $\gamma > 3\eta$. However, not every box is realizable with a physical system, even when $\mathcal{N}(P) \leq 2\sqrt{2}$ which satisfies the Tsirelson bound. A necessary and sufficient condition for a set of correlation functions $\mathbf{C}_{xy}(P)$ to be attainable by quantum mechanics has been derived in Refs. [24,25], which implicitly determine the physical region of η, γ [18]. For two nonlocal boxes characterized by the same conditional probability matrices $P(a_1b_1|xy)$ and $P(a_2b_2|xy)$ in the form of Eq. (3) with the same input x, y for the bases of detection but different measurement outcomes a_1, b_1 and a_2, b_2 , the local distillation operation is done through a mod 2 addition of the measurement outcomes on each side as illustrated in Fig. 1, that is, the distilled box is characterized by the condition probability $P_d(ab|xy)$, with $a = a_1 \oplus a_2$ and $b = b_1 \oplus b_2$ [18]. It is easy to check that the matrix for $P_d(ab|xy)$ still has the form of Eq. (3), but with η, γ replaced by η', γ' , where

$$\eta' = 2(\eta - \eta^2), \quad \gamma' = 2(\gamma - \gamma^2). \quad (4)$$

The distilled box has stronger nonlocality compared with the original box if $\mathcal{N}(P_d) > \mathcal{N}(P)$. A necessary condition for this is that the parameters η, γ are in the region $0 < \eta < \gamma/3 < 1/6$. For experimental implementation of the nonlocality distillation, it is better to have $\mathcal{N}(P_d) - \mathcal{N}(P)$ as large as possible. Under the constraint that the box characterized by the conditional probability in the form of Eq. (3) is physically attainable, we numerically maximize the nonlocality increase $\mathcal{N}(P_d) - \mathcal{N}(P)$ under different parameters η, γ and find that the optimal values are $\eta_o \approx 0.019$ and $\gamma_o \approx 0.164$. Under this optimal choice of η, γ , the nonlocality increase $\mathcal{N}(P_d) - \mathcal{N}(P) \approx 2.324 - 2.214 = 0.110$, representing about a 5% improvement. As the relative increase in nonlocality is small, the experiment needs to be done with a good precision for an unambiguous demonstration of nonlocality distillation.

To experimentally realize distillation of two nonlocal boxes, we first need to implement a correlation box where the conditional probability $P(ab|xy)$ has the form of Eq. (3) with tunable η, γ . We assume Alice and Bob share

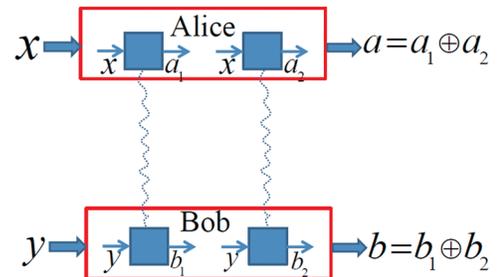


FIG. 1 (color online). Illustration of nonlocality distillation of two correlation boxes. The two boxes share the same input x, y , and the output a, b of the distilled box is given by the mod-2 addition $a = a_1 \oplus a_2, b = b_1 \oplus b_2$, where a_1, b_1, a_2, b_2 denote the outputs of the two individual boxes.

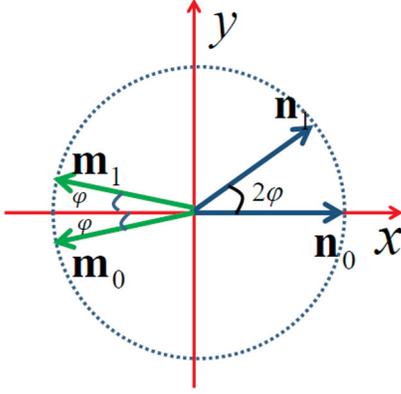


FIG. 2 (color online). The directions of the measurements of the Pauli spins for Alice ($\mathbf{n}_0, \mathbf{n}_1$) and Bob ($\mathbf{m}_0, \mathbf{m}_1$), which realize the correlation box characterized by the conditional probabilities in the form of Eq. (3) that is optimal for demonstration of nonlocality distillation from two copies.

singlet entangled states given by $|\psi^-\rangle = (|01\rangle - |10\rangle)/\sqrt{2}$, and Alice (Bob) measures a Pauli spin σ_A (σ_B) along the $\mathbf{n}_0, \mathbf{n}_1$ ($\mathbf{m}_0, \mathbf{m}_1$) direction when the input bit $x = 0, 1$ ($y = 0, 1$). The output bit is taken as $a = 0, 1$ ($b = 0, 1$) if the measurement outcome of the Pauli spin $A_x \equiv \mathbf{n}_x \cdot \sigma_A$ ($B_y \equiv \mathbf{m}_y \cdot \sigma_B$, $x, y = 0, 1$) is $+1, -1$, respectively. In this implementation, one can check that the conditional probability $P(ab|xy)$ has the form of Eq. (3) if $\mathbf{n}_0 \cdot \mathbf{m}_0 = \mathbf{n}_0 \cdot \mathbf{m}_1 = \mathbf{n}_1 \cdot \mathbf{m}_0$, with $\eta = (1 + \mathbf{n}_0 \cdot \mathbf{m}_0)/2$ and $\gamma = (1 + \mathbf{n}_1 \cdot \mathbf{m}_1)/2$. To satisfy this constraint, we take the directions of $\mathbf{n}_0, \mathbf{n}_1, \mathbf{m}_0, \mathbf{m}_1$ as specified by the angle φ in Fig. 2. In this case, $\mathbf{n}_0 \cdot \mathbf{m}_0 = \mathbf{n}_0 \cdot \mathbf{m}_1 = \mathbf{n}_1 \cdot \mathbf{m}_0 = -\cos(\varphi)$ and $\mathbf{n}_1 \cdot \mathbf{m}_1 = -\cos(3\varphi)$. We find that with $\varphi = 15.95^\circ$, this implementation realizes the optimal choice of η, γ with $\eta = (1 - \cos(\varphi))/2 \approx 0.019$ and $\gamma = (1 - \cos(3\varphi))/2 \approx 0.164$ that maximize the nonlocality increase for distillation of two nonlocal boxes.

To realize the two nonlocal correlation boxes specified with the above conditions, we experimentally generate two pairs of entangled photons through the spontaneous parametric down-conversion (SPDC) setup shown in Fig. 3. Two pieces of the type-II BBO crystals are pumped by femtosecond laser pulses, generating two pairs of entangled photons in the singlet state $|\psi^-\rangle = (|HV\rangle - |VH\rangle)/\sqrt{2}$ [26], where $|H\rangle$ and $|V\rangle$ denote horizontal and vertical polarization of a single photon. After the entangled photons are generated, Alice (Bob) applies the measurement A_x (B_y) by using half-wave plates (HWP) to rotate the polarization of her (his) photons. The angles for the wave plates HWP5, HWP6, HWP7, HWP8 are specified in the Supplemental Material corresponding to four different inputs $(x, y) = (0, 0), (0, 1), (1, 0), (1, 1)$ of the correlation box [27]. For the nonlocal boxes 1 and 2, the measurement outcomes for the conditional probabilities $P_1(a_1 b_1 | xy)$ and $P_2(a_2 b_2 | xy)$ are recorded in Table I. From the measurements, we find the CHSH nonlocality

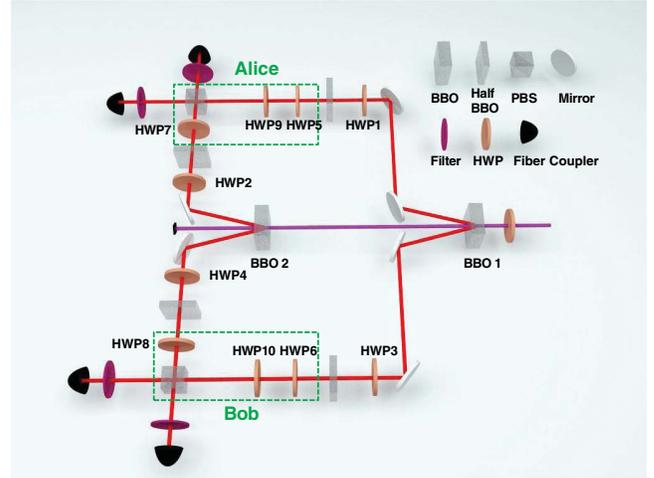


FIG. 3 (color online). The schematic experimental setup to implement the nonlocality distillation and to measure the properties of the distilled box. Femtosecond pulses (with the wavelength at 390 nm and a repetition rate of 76 MHz) from a frequency-doubled Ti:Sa laser pump two BBO crystals (with type-II cutting of 2 mm depth) to generate two pairs of photons with perpendicular polarization. Four additional BBO crystals of 1 mm depth are used to compensate the spatial and temporal walk-off between the photons, which, together with the four half-wave plates (HWP1, HWP2, HWP3, HWP4) set at 45° , prepare the two pairs of photons each in the maximally entangled singlet state $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|HV\rangle - |VH\rangle)$. Alice and Bob then use rotation of HWP5, HWP6, HWP7, and HWP8 to choose the measurement bases. By measuring the spins along the directions of $\mathbf{n}_0, \mathbf{n}_1, \mathbf{m}_0, \mathbf{m}_1$ specified in Fig. 2, Alice and Bob realize two correlation boxes which allow maximum distillation of nonlocality using the protocol illustrated in Fig. 1. The two polarization beam splitters (PBSs) at Alice's and Bob's sides, together with HWP9 and HWP10, realize effectively the required mod-2 addition. The output modes of the PBSs are coupled into single-mode fibers and then detected by four single-photon detectors. The results are registered through a four-port coincidence circuit with a 3 ns coincidence window to reduce the accidental coincidence counts. There is no need of background subtraction of accidental coincidences for this experiment. The typical two-photon coincidence rate from each BBO crystal is about 15 kHz and the four-photon coincidence rate is about 2.2 Hz for this experiment. To reduce the statistical error, we accumulate the photon counts for 9.3 h for each data point.

$\mathcal{N}(P_1) = 2.1440 \pm 0.0001$ and $\mathcal{N}(P_2) = 2.1356 \pm 0.0001$, where the error bar accounts for the statistical error associated with the photon counts under the assumption of a Poissonian distribution. The realized two nonlocal boxes are close to the optimal box specified above for distillation. The small difference is due to the infidelity of the entangled singlet states as well as the imprecision in controlling the angles of the wave plates.

To measure the conditional probabilities $P_d(ab|xy)$ for the distilled box realized in Fig. 3, we note that the mod 2 addition $a = a_1 \oplus a_2$ and $b = b_1 \oplus b_2$ required in the

TABLE I. The experimental results for the conditional probabilities $P(a_1 b_1 | xy)$ and $P(a_2 b_2 | xy)$ measured for each individual nonlocal box, and $P(ab | xy)$ measured for the distilled box. The CHSH nonlocality is calculated for each box from the measured conditional probabilities. The distilled box has a nonlocality larger than that of each individual box, confirming distillation of the nonlocality.

Initial Box 1	Initial Box 2				Final Box							
ab	00	01	10	11	00	01	10	11	00	01	10	11
00	0.49858(3)	0.01553(1)	0.01408(1)	0.47181(3)	0.48672(2)	0.01779(1)	0.01999(1)	0.47550(2)	0.4772(41)	0.0298(36)	0.0291(36)	0.4639(40)
01	0.48714(3)	0.01242(1)	0.01074(1)	0.48970(3)	0.48278(3)	0.01789(1)	0.01947(1)	0.47986(2)	0.4773(43)	0.0313(38)	0.0343(38)	0.4571(42)
10	0.48850(3)	0.01567(1)	0.01527(1)	0.48056(3)	0.47063(2)	0.01561(1)	0.02156(1)	0.49221(3)	0.4670(39)	0.0323(35)	0.0324(35)	0.4683(40)
11	0.41605(3)	0.07914(2)	0.07662(2)	0.42819(3)	0.41206(2)	0.08596(1)	0.09420(1)	0.40778(2)	0.3510(33)	0.1564(30)	0.1359(30)	0.3567(33)
Nonlocality: 2.1440 ± 0.0001	Nonlocality: 2.1356 ± 0.0001				Nonlocality: 2.206 ± 0.021							

distillation protocol can be simply implemented with a polarization beam splitter (PBS). A PBS transmits (reflects) the photon when it is in H (V) polarization. The outputs of the PBS are coupled into single mode fibers and detected by single photon detectors. To verify the protocol, the measurement outcomes from Alice and Bob are compared through a coincidence circuit to construct the correlations. When we register a four-photon coincidence between the two output modes of the PBS on both Alice's and Bob's sides, the photons at the two input modes of each PBS must have identical polarization, which means $a = a_1 \oplus a_2 = 0$ and $b = b_1 \oplus b_2 = 0$. The count rate of this coincidence is therefore proportional to the conditional probability $P_d(a = 0, b = 0 | xy)$. To measure other components of $P_d(a, b | xy)$, we rotate the HWP9 (HWP10) at Alice's (Bob's) side by 45° , which exchanges H and V and thus flips a_1 (b_1). With a bit flip on a_1 , b_1 , or both, the coincidence measures the relative conditional probabilities $P_d(a = 1, b = 0 | xy)$, $P_d(a = 0, b = 1 | xy)$, and $P_d(a = 1, b = 1 | xy)$, respectively. A technical problem for this measurement is that the four-photon coincidence could also be caused by the events with two entangled photon pairs from the same BBO crystal and no photon from the other crystal [28]. To deduct the coincidence due to these unrelated events, we measure their contribution to the four-photon coincidence rate simply by blocking the down converted photons from one of the BBO crystals. After this correction, the four-photon coincidence rate is directly proportional to the conditional properties $P_d(a, b | xy)$ for the distilled box.

The measured conditional probabilities $P_d(a, b | xy)$ for the distilled box are shown in Table I. From these data, we find the CHSH nonlocality $\mathcal{N}(P_d) = 2.206 \pm 0.021$. The error bar gets larger since to measure the properties of the distilled box we need to record four-photon coincidence, which has a significantly smaller count rate and thus a larger statistical error. Apparently, $\mathcal{N}(P_d) > \mathcal{N}(P_1)$ and $\mathcal{N}(P_d) > \mathcal{N}(P_2)$, where the nonlocality increases by more than three times the standard deviation (error bar), so the experiment unambiguously demonstrate distillation of quantum nonlocality.

In summary, we have reported the first experimental demonstration of distillation of quantum nonlocality through only local operations on two correlation boxes. The experiment unambiguously confirms that quantum nonlocality is enhanced without any communication between the remote parties. Quantum nonlocality has emerged as a key resource for implementation of self-certified device-independent quantum information protocols, and an experimental demonstration of nonlocality amplification provides a useful step for future application along this direction.

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