



# Fault-tolerant routing: $k$ -inconnected many-to-one routing in wireless networks

Deying Li<sup>a,b,\*</sup>, Yuexuan Wang<sup>c</sup>, Qinghua Zhu<sup>a,b</sup>, Huiqiang Yang<sup>a,b</sup>

<sup>a</sup> Key Laboratory of Data Engineering and Knowledge Engineering, Renmin University of China, MOE, China

<sup>b</sup> School of Information, Renmin University of China, Beijing, China

<sup>c</sup> Institute for Theoretical Computer Science, Tsinghua University, Beijing, China

## ARTICLE INFO

### Keywords:

Wireless networks  
Fault-tolerant  
 $k$ -inconnected many-to-one routing  
Approximation algorithm

## ABSTRACT

This paper addresses the problem of fault-tolerant many-to-one routing in static wireless networks with asymmetric links, which is important in both theoretical and practical aspects. The problem is to find a minimum energy subgraph for a given subset and a destination node such that there are  $k$  node-disjoint paths from each node in the subset to the destination node in the subgraph. Firstly, we prove that the problem is NP-hard, and then propose two approximation algorithms: the minimum weight  $k$  node-disjoint paths based (MWkNDPB) algorithm and the minimum energy  $k$  node-disjoint paths based (MEkNDPB) algorithm. Extensive simulations have been conducted to show that proposed algorithms are efficient.

© 2009 Elsevier B.V. All rights reserved.

## 1. Introduction

All-to-one or many-to-one routing is one of the important and primary communication methods in wireless networks for data collection. In most cases, wireless networks are deployed under a harsh environment, thus it is easy for wireless nodes and links to experience frequent failures. However, because node or link failures have a significant impact on the performance and reliability of wireless networks, how to ensure fault tolerance all-to-one or many-to-one routing becomes a very important issue in wireless networks. On the other hand, energy efficiency is also an important issue in wireless networks since nodes are powered by batteries that may not be possible to be recharged or replaced during a mission. How to provide energy efficient fault tolerance all-to-one or many-to-one routing in wireless networks is very challenging.

There are four classes of the communication models in wireless networks: anycast; unicast; all-to-one or one-to-all communication; and many-to-one or one-to-many communication. Previous work related to fault tolerant topology control in static wireless networks focus on the following cases: (1)  $k$ -fault tolerant anycast, (2)  $k$ -fault tolerant unicast, (3)  $k$ -fault tolerant all-to-one problem and  $k$ -fault tolerant one-to-all problem.

For minimum total power all-to-all  $k$ -fault tolerance problem, authors in [2,13,14] used the algorithm BICONNECTED-KR proposed in [5] to construct a minimum weight 2-connected subgraph as an approximation for minimum power all-to-all 2-connected problem, and analyzed its approximation ratio. The authors in [2] gave the best approximation ratio 4. The authors in [1] proposed a localized algorithm FTCBTC, which generalized the well-known Cone-Based Topology Control [20,11]. Ramanathan and Hain [16] proposed greedy algorithms for minimum maximal power consumption for  $k$ -node connectivity. Some approximation algorithms for constructing minimum power  $k$  node-disjoint paths between any two nodes are proposed in [3,6,15]. And Li and Hou [15] proposed a localized implementation of the centralized algorithm.

\* Corresponding author at: Key Laboratory of Data Engineering and Knowledge Engineering, Renmin University of China, MOE, China.  
E-mail address: [deyingl@hotmail.com](mailto:deyingl@hotmail.com) (D. Li).

Srinivas and Modiano [17] proposed a novel polynomial time algorithm that optimally solves the minimum energy 2-link-disjoint paths problem for unicast, as well as a polynomial time algorithm called Source Transmit Power Selection (STPS) for minimum energy  $k$  node-disjoint paths problem for unicast, based on the minimum weight  $k$  node-disjoint S–D path algorithm [18]. In addition, Srinivas and Modiano [17] presented efficient heuristic algorithms for link/node-disjoint paths problems.

The minimum energy broadcast/multicast problem [7–9,12] is a special case of the one-to-all(one-to-many)  $k$ -fault-tolerant problem. The work in [7] proved that the minimum energy broadcast problem is NP-hard. And gave three algorithms, and one of them is  $1 + 2\ln(n - 1)$ -approximation algorithm. For minimum total power all-to-one and/or one-to-all  $k$ -fault tolerance, Wang et al. [19] proposed two approaches: minimum weight based approach and nearest neighbor augmentation approach. And they gave theoretical analysis for the proposed algorithms. The minimum weighted based approach is a  $k$ -approximation algorithm for all-to-one  $k$ -fault tolerant topology control.

There are few works on the fault tolerant many-to-one routing. However, it is very meaningful and practical in wireless networks. For example, the important and main tasks of wireless sensor networks are monitoring a geographical region and collecting the relevant data. We need to collect data from sensors in a relevant region to sink. In this paper, we address the  $k$ -inconnected many-to-one routing problem: to construct a routing with minimum total energy in which there are  $k$  node disjoint paths from any node in a given subset of nodes to one specific node. We first prove that the  $k$ -inconnected many-to-one problem is NP-hard, and then propose two approximations for the problem. There is an extended abstract for the problem in [10].

The rest of the paper is organized as follows: In Section 2 we describe the network model and problem definitions. In Section 3, we first prove the problem is NP-hard. Then two approximation algorithms for the problem are proposed. Simulation results about proposed algorithms are exhibited in Section 4. Finally, we give a conclusion in Section 5.

## 2. Problem definitions

In this paper, we use the following network model. A wireless network consists of  $N$  nodes, each of which is equipped with an omni-directional antenna with a maximal transmission range  $r_{\max}$ . The power required for a node to attain a transmission range  $r$  is at least  $Cr^\alpha$ , where  $C$  is a constant,  $\alpha$  is the power attenuation exponent and usually chosen between 2 and 4. For any two nodes  $u$  and  $v$ , there exists a directed edge from  $u$  to  $v$  if the Euclidean distance  $d(u, v) \leq r_u$ , where  $r_u$  is the transmission range of node  $u$  determined by its power level. In this paper, we consider asymmetric networks in which the existence of a directed edge from  $u$  to  $v$  does not guarantee the existence of a directed edge from  $v$  to  $u$ .

The network can be modeled by an edge-weight directed graph  $G = (V, E, c)$ .  $V$  is a set of  $N$  nodes. For any pair of nodes  $u$  and  $v$ , there is a directed edge from  $u$  to  $v$  if and only if  $d(u, v) \leq r_{\max}$ . We assign a weight to a directed edge  $(u, v)$  by  $c(u, v) = Cd(u, v)^\alpha$ . In fact, since each node has the same maximal transmission range  $r_{\max}$ , the graph  $G$  is a symmetric directed graph, i.e., if there is a directed edge from  $u$  to  $v$ , then there is a directed edge from  $v$  to  $u$ . But, because each node may not have the same transmission assignment, the graph induced by the assignment will be a directed graph.

Suppose  $H$  is a subgraph of  $G$ . Let  $p(u)$  be the power assignment of node  $u$ ,  $c(u, v)$  be the cost of a directed edge  $(u, v)$ , and  $p(H)$  be the total energy of  $H$ ,  $c(H)$  be the total cost of  $H$ , then we have:

$$p(u) = \max_{(u,v) \in H} C(u, v) \quad (1)$$

$$c(H) = \sum_{(u,v) \in H} c(u, v) \quad (2)$$

$$p(H) = \sum_{u \in H} p(u). \quad (3)$$

**Minimum Energy  $k$ -inconnected Many-to-One Routing Problem:** Given a directed graph  $G = (V, E, c)$ , a root  $r$  and a subset  $S$  of nodes along with a positive integer  $k$ , find the power assignment of each node such that the sub-graph  $H$  induced by the power assignment has minimum energy, and for any  $s \in S$ ,  $H$  contains  $k$  node-disjoint paths from  $s$  to  $r$ . We call this problem as MEkinMRP in short.

## 3. Algorithms

In the section, we first prove the minimum energy  $k$ -inconnected many-to-one routing problem is NP-hard. Then propose two algorithms to solve the problem.

**Theorem 1.** For any  $k \geq 2$ , the minimum energy  $k$ -inconnected many-to-one routing problem (MEkinMRP) is NP-hard.

**Proof.** We will prove it by two cases:

**Case 1:**  $k = 2$ , we transfer the vertex-cover problem to the MEkinMRP problem.

**The vertex-cover problem** is formally represented as follows: A given graph  $G = (V, E)$ , to find minimum cardinality set of nodes  $C$  such that each edge  $e \in E$  has at least one of its endpoints in  $C$ .

Suppose  $G = (V, E)$  is instance of vertex-cover problem. We construct a new directed graph  $G_1 = (V_1, E_1)$  as followings:

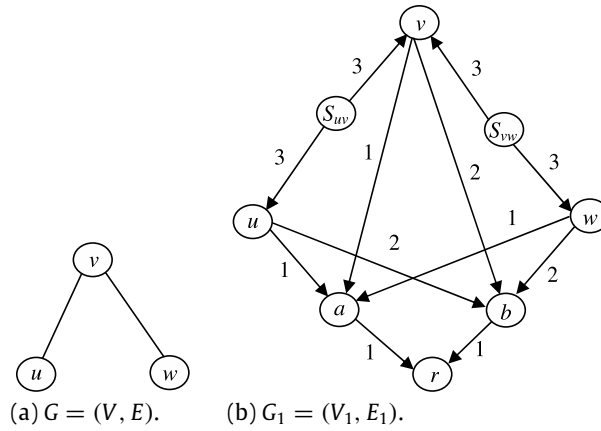


Fig. 1. An example of the transformation for  $k = 2$ .

- (1) For each edge  $(u, v)$  of  $G$ , add a node  $s_{uv}$  at the middle. i.e.,  $\{u, v, s_{uv}\} \subseteq V_1$  and  $(s_{uv}, u) \in E_1, (s_{uv}, v) \in E_1$ . Assign distance 6 between  $u$  and  $v$  such that  $d(s_{uv}, u) = 3$  and  $d(s_{uv}, v) = 3$ .
- (2) Let  $r, a, b$  be three additional new nodes in  $G_1$ . Connect each node  $u$  of  $G$  to  $a$  and  $b$  with distance 1 and 2 respectively, i.e. for any  $u \in V, (u, a) \in E_1, (u, b) \in E_1$ , and  $d(u, a) = 1, d(u, b) = 2$ .
- (3) Connect  $a$  and  $b$  to  $r$  with distance  $d(a, r) = 1$  and  $d(b, r) = 1$ .

Therefore we get an edge-weight directed graph  $G_1 = (V_1, E_1)$ , where,

$$V_1 = V \cup \{s_{uv} | \forall e = (u, v) \in E\} \cup \{a, b, r\}$$

$$E_1 = \{(s_{uv}, u), (s_{uv}, v) | \forall (u, v) \in E\} \cup \{(u, a), (u, b) | \forall u \in V\} \cup \{(a, r), (b, r)\}.$$

It is easy to know that transferring graph  $G$  to  $G_1$  takes time  $O(n^2)$ . An example is shown in Fig. 1.

Set  $S = \{s_{uv} | \forall (u, v) \in E(G)\}$ . In the following, we prove that  $G$  has a vertex cover of size  $h$  if and only if there is 2-inconnected routing from  $S$  to  $r$  in  $G_1$  using power at most  $|E(G)| \times 3^\alpha + (|V| - h) \times 1^\alpha + h \times 2^\alpha + 2 \times 1^\alpha$ .

Firstly, suppose that  $G$  has a vertex cover  $C$  of size  $h$ . According construction of graph  $G_1$ ,  $C$  is also a subset of  $V_1$ . We set a subgraph  $H = (V_h, E_h)$  of  $G_1$ , where  $V_h = V_1$ , and  $E_h = \{(s_{uv}, u), (s_{uv}, v) | \forall (u, v) \in E\} \cup \{(c, b), (c, a) | c \in C\} \cup \{(x, a) | x \in V - C\} \cup \{(a, r), (b, r)\}$ . In the following, we prove that for each node  $s_{uv}$ , there are two vertex-disjoint paths from  $s_{uv}$  to  $r$  in  $H$ . Since  $C$  is a vertex cover, for each edge  $(u, v) \in E(G)$ , there must be  $u \in C$  or  $v \in C$ . Without loss generality, we assume that  $u \in C$ . For each node  $s_{uv} \in V_1$ , there must be two node-disjoint paths from  $s_{uv}$  to  $r$ :  $s_{uv}ubr$  and  $s_{uv}var$ . Therefore, subgraph  $H$  is a 2-inconnected routing from  $S$  to  $r$ . And  $H$  consumes total energy is at most  $|E(G)| \times 3^\alpha + (|V| - h) \times 1^\alpha + h \times 2^\alpha + 2 \times 1^\alpha$ .

Secondly, suppose  $H$  is a subgraph of  $G_1$  such that for each node  $s_{uv}$  in  $S$ , there are two node-disjoint paths from  $s_{uv}$  to  $r$  and  $H$  consumes at most  $|E(G)| \times 3^\alpha + (|V| - h) \times 1^\alpha + h \times 2^\alpha + 2 \times 1^\alpha$ . We let  $C$  be a set of  $b$ 's in-neighbors in  $H$ , which is a subset of  $V$  with size  $h$ . It is easy to know that  $C$  is a vertex cover of  $G$ .

Because the vertex-cover problem is NP-hard [4], the minimum energy 2-inconnected many-to-one routing problem (MEkinMRP) is NP-hard.

**Case 2:** general  $k$ , we transfer the hitting set of hypergraph problem to the MEkinMRP problem.

**The hitting set of hypergraph problem** is formally represented as follows: A given hypergraph  $G = (V, E)$ , where  $V$  is a set of nodes, and for each  $e \in E, e$  is a subset of  $V$ . Find minimum cardinality set of nodes  $S$  that covers all edges of  $G$ , i.e. every edge contains at least one node of  $S$ .

Suppose  $G$  is a hypergraph with uniform  $k$ -edge (i.e. each edge contains  $k$  nodes), we construct a new directed graph  $G_1 = (V_1, E_1)$ . For each edge  $e$  in hypergraph, we add a new node  $v(e)$ , let  $v(e)$  connect each node of edge  $e$ , and assign distance 3 for these edges. Let  $r, a_1, a_2, \dots, a_{k-1}, b$  be  $k + 1$  additional nodes, connect each node  $u$  of  $G$  to  $a_1, a_2, \dots, a_{k-1}$  and  $b$  with distance 1, 1,  $\dots$ , 1 and 2, respectively. And connect  $a_1, a_2, \dots, a_{k-1}$  and  $b$  to  $r$  with distance 1.

We get an edge-weight directed graph  $G_1 = (V_1, E_1)$ , where

$$V_1 = V \cup \{v(e) | \forall e \in E\} \cup \{r, a_1, a_2, \dots, a_{k-1}, b\}$$

$$E_1 = \{(v(e), u) | \forall e \in E, \forall u \in e\} \cup \{(u, a_1), (u, a_2), \dots, (u, a_{k-1}), (u, b) | \forall u \in V\} \cup \{(a_1, r), \dots, (a_{k-1}, r), (b, r)\}.$$

Transferring from hypergraph  $G$  to graph  $G_1$  takes time  $O(n^2)$ . An example for this construction is shown in Fig. 2.

Set  $S = \{v(e) | \forall e \in E\}$ . In the following, we prove that  $G$  has a hitting set of size  $h$  if and only if there is  $k$ -inconnected routing from  $S$  to  $r$  in  $G_1$  with using power at most  $|E(G)| \times 3^\alpha + (|V| - h) \times 1^\alpha + h \times 2^\alpha + k \times 1^\alpha$ .

Firstly, suppose  $C$  is a hitting set with size  $h$  in hypergraph  $G$ . According construction of graph  $G_1$ ,  $C$  is also a subset of  $V_1$ . We set a subgraph  $H = (V_h, E_h)$  of  $G_1$ , where  $V_h = V_1$ , and  $E_h = \{(v(e), u) | \forall e \in E, \forall u \in e\} \cup \{(c, a_1), (c, a_2), \dots, (c, a_{k-1}), (c, b) | c \in C\} \cup \{(x, a_1), (x, a_2), \dots, (x, a_{k-1}) | x \in V - C\}$ . Then  $H$  is a  $k$ -inconnected routing from  $S$  to  $r$ . It is because:

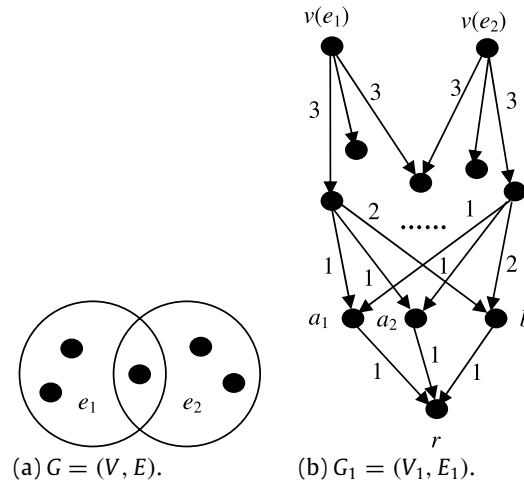


Fig. 2. An example of the transformation for general  $k$ .

for each node  $v(e)$ , since  $C$  is a hitting set of  $G$ , there must be a node  $u_e \in e \cap V$  such that  $u_e \in C$ , therefore there are  $k$  disjoint paths  $v(e)u_ebr$  and  $v(e)u_iar$ , where there are  $(k - 1)$  nodes  $u_i$  except  $u_e$  in  $e$ . Subgraph  $H$  consumes total energy at most  $|E(G)| \times 3^\alpha + (|V| - h) \times 1^\alpha + h \times 2^\alpha + k \times 1^\alpha$ .

Secondly, suppose  $G_1$  has  $k$ -inconnected routing from  $S$  to  $r$  with using power at most  $|E(G)| \times 3^\alpha + (|V| - h) \times 1^\alpha + h \times 2^\alpha + k \times 1^\alpha$ . That is there is a subgraph  $H$  of  $G_1$ , for each node  $v(e) \in S$ , there are  $k$  disjoint paths from  $v(e)$  to  $r$ . Because there are only  $k$  nodes to  $r$ , so there must be nodes which connect to  $b$ . Set  $C$  is a in-neighbors of  $b$  in  $H$ . Because  $H$  consumes at most  $|E(G)| \times 3^\alpha + (|V| - h) \times 1^\alpha + h \times 2^\alpha + k \times 1^\alpha$ ,  $C$  has size at most  $h$ . For any edge  $e \in E$ , there must be  $e \cap C \neq \emptyset$ . Otherwise,  $e \cap C = \emptyset$ , then according to construction of  $G_1$ ,  $v(e)$  can not connect to  $b$ , that is,  $v(e)$  can not have path through  $b$  to  $r$ , and since  $r$  has only  $k$  in-neighbors including  $b$ , therefore there are not  $k$  node-disjoint paths from  $v(e)$  to  $r$ , contradicts to  $H$  is a  $k$ -inconnected routing from  $S$  to  $r$ . Therefore  $C$  is a hitting set of  $G$ .

Because the hitting set of uniform hypergraph problem is NP-hard [4], the minimum energy  $k$ -inconnected many-to-one routing problem (MEkinMRP) is NP-hard, for any  $k > 2$ .

The theorem is proved.  $\square$

Since the MEkinMRP is NP-hard, we need to find approximation algorithms or heuristic algorithms for this problem. In the following, we will propose two heuristic algorithms: minimum weight  $k$  node-disjoint paths based (MWkNDPB) algorithm and minimum energy  $k$  node-disjoint paths based (MEkNDPB) algorithm.

### 3.1. MWkNDPB algorithm

Before introducing MWkNDPB algorithm, we first address the minimum weight  $k$  node-disjoint paths of node pair problem.

**Minimum Weight  $k$  Node-disjoint Paths Problem (MWkNDP):** Given an edge-weighted digraph  $G = (V, E, c)$  and a pair nodes  $(s, r)$  in  $V$ , find a subgraph  $H$  of  $G$  with minimum  $c(H) = \sum_{(u,v) \in H} c(u, v)$  such that  $H$  contains at least  $k$  node-disjoint paths from  $s$  to  $r$ .

In the following, we propose minimum weight  $k$  node-disjoint paths flow based (MWkNDPFB) algorithm for MWkNDP problem.

It is well known that all the classical flow algorithms are applied to solve edge-disjoint paths problem, but the MWkNDP problem is to find node-disjoint paths. Therefore we first construct a new graph  $G_a = (V_a, E_a, c_a)$ , and transform MWkNDP problem in  $G$  to the edge-disjoint paths problem in  $G_a$ .

Given a graph  $G = (V, E, c)$ , the graph  $G_a = (V_a, E_a, c_a)$  is constructed as following:

1. For each node  $i \in V$ , there are two nodes  $i_t$  and  $i_h$  in  $V_a$  corresponding to  $i$ . And  $(i_t, i_h) \in E_a$  with  $c_a(i_t, i_h) = 0$ .
2. For each edge  $(i, j)$  in  $G$ , there is an edge  $(i_h, j_t) \in E_a$  with  $c_a(i_h, j_t) = c(i, j)$ .

Therefore, we got  $G_a = (V_a, E_a, c_a)$ , where

$$V_a = \cup_{i \in V} \{i_t, i_h\};$$

$$E_a = \cup_{i \in V} \{(i_t, i_h)\} \cup \{(i_h, j_t) | \forall (i, j) \in E\}.$$

For a pair nodes  $(s, r)$  in  $V$ , there is a corresponding pair nodes  $(s_h, r_t)$  in  $G_a$ . The minimum weight  $k$  node-disjoint paths problem for  $(s, r)$  in  $G$  is transformed to the minimum weight  $k$  edge-disjoint paths problem for  $(s_h, r_t)$  in  $G_a$ , which can be modeled to the following minimal cost flow problem:

$$\begin{aligned} & \text{minimize } \sum c_a(i, j) \cdot x_{ij} \\ & \sum x_{ij} - \sum x_{ji} = \begin{cases} k & \text{if } i = s_h \\ 0 & \text{if } i \neq s_h, r_t \text{ (for any } i \in V_a) \\ -k & \text{if } i = r_t \end{cases} \\ & x_{ij} = 0, 1, \text{ (for any } (i, j) \in E_a). \end{aligned} \tag{4}$$

Note that: we split each node  $i \in V$  into two nodes  $i_t$  and  $i_h$  along with an edge  $(i_t, i_h)$  in  $V_a$ , which guarantees that the subgraph obtained from the solution of ILP (4) is node-disjoint after contracting all the  $(i_t, i_h)$  edges.

Because the relaxation of ILP has an integer solution since the ILP (4) is an integer flow problem, ILP can be solved in polynomial time. Therefore, there is an optimal algorithm for the MEkinMRP.

**MWkNDPFB Algorithm for the MEkinMRP:**

**Input:**  $G(V, E, c)$ , a pair of nodes  $(s, r)$  and constant  $k$ .

**Output:** A subgraph  $H$ , in which there are  $k$  node-disjoint paths from  $s$  to  $r$  with minimum  $c(H)$ .

1. Construct a new graph  $G_a = (V_a, E_a, c_a)$  according  $G$ .
2. Solve the relaxation linear programming of (4) to get the optimal solution of (4).
3. Construct a subgraph of  $G_a$  by the solution of (4).
4. Contract all the  $(i_t, i_h)$  edges in the subgraph into a node, and we get the subgraph  $H$  of  $G$ .

Based on the MWkNDPFB algorithm, we propose MWkNDPB algorithm for the minimum energy  $k$ -inconnected many-to-one routing problem. The main idea of MWkNDPB algorithm is that: given the  $(S, r)$ , for each  $s \in S$ , we invoke the MWkNDPFB to get a subgraph  $H_{sr}$ . Then we can get a subgraph  $H = \bigcup H_{sr}$ .  $H$  guarantees that there are  $k$  node-disjoint paths from any  $s \in S$  to  $r$ .

**MWkNDPB Algorithm:**

**Input:**  $(G, k, r, S)$ , where  $G = (V, E, c)$  is a directed weight graph of network topology and  $r$  is the root,  $S \subseteq V - r$ ,  $k$  is a constant integer.

**Output:** A subgraph  $H$  that is  $k$ -inconnected from  $S$  to  $r$ .

1. For each  $s \in S$ , invoke the MWkNDPFB routine to get a subgraph  $H_{sr}$ .
2. Union all the subgraphs  $H_{sr}$  got in step 1, that is  $H = \bigcup_{s \in S} H_{sr}$ .

**Theorem 2.** *The time complexity of MWkNDPB algorithm is  $O(n^8)$ , where  $n$  is the number of nodes in the network.*

**Proof.** Since graph  $G$  is a directed graph, there are at most  $n(n - 1)$  edges in  $G$ . Considering that we split each node in  $G$  into two nodes  $i_t$  and  $i_h$  along with an edge  $(i_t, i_h)$  in  $G_a$ , there are at most  $n(n - 1) + n = n^2$  edges in  $G_a$ . Since each edge in  $G_a$  corresponds to one variable in ILP (4), there are  $n^2$  variables in ILP (4). The time complexity of solving a LP problem with  $n$  variables is  $O(n^{3.5})$ , so, the time complexity of MWkNDPFB algorithm is  $O(n^{2 \cdot 3.5}) = O(n^7)$ . There are at most  $n - 1$  nodes in subset  $S$ , so, the time complexity of MWkNDPB algorithm is  $O((n - 1)n^7) = O(n^8)$ .  $\square$

**Theorem 3.** *The MWkNDPB algorithm has  $k|S|$ -approximation ratio, where  $S$  is a given set of sources nodes in the network.*

**Proof.** It is easy to know that subgraph  $H = \bigcup_{s \in S} H_{sr}$  produced by the MWkNDPB algorithm is a  $k$ -inconnected routing from  $S$  to  $r$ , for given  $(S, r)$ .

Suppose  $H_{opt}$  is optimal solution for the minimum energy  $k$ -inconnected many-to-one routing problem. And  $D_{sr}$  is an optimal solution for minimum energy  $k$  node-disjoint paths problem form  $s$  to  $r$ . Because  $H_{sr}$  is an optimal solution for minimum weight  $k$  node-disjoint paths problem for  $(s, r)$ , then,

$$\begin{aligned} p(D_{sr}) & \leq p(H_{opt}) \\ c(H_{sr}) & \leq c(D_{rs}) \leq kp(D_{sr}). \end{aligned}$$

From above inequalities, we have:

$$p(H) \leq \sum_{s \in S} p(H_{sr}) \leq \sum_{s \in S} c(H_{sr}) \leq k \sum_{s \in S} p(D_{sr}) \leq k|S|p(H_{opt}).$$

Therefore, the MWkNDPB algorithm is an  $k|S|$ -approximation algorithm.  $\square$

### 3.2. MEkNDPB algorithm

Because the MWkNDPB algorithm has high time complexity, in this section, we propose another algorithm: MEkNDPB algorithm, which has lower time complexity.

Before introducing the algorithm, we first introduce the minimum energy  $k$  node-disjoint paths of node pair problem.

**Minimum Energy  $k$  Node-disjoint Paths Problem:** Given an edge-weighted digraph  $G = (V, E, c)$  and a node pair  $(s, r)$  in  $V$ , find a subgraph  $H$  of  $G$  such that  $H$  contains at least  $k$  node-disjoint paths from  $s$  to  $r$  and  $p(H) = \sum_{u \in H} p(u)$  is minimized where  $p(u) = \max_{(u,v) \in E} c(u, v)$ .

Suppose  $H$  is a subgraph which contains  $k$  node-disjoint paths from  $s$  to  $r$ . Each node in  $H$  except  $s$  and  $r$  has exactly one ingoing edge and one outgoing edge.  $s$  has at least  $k$  outgoing edges and no ingoing edge.  $r$  has at least  $k$  ingoing edges and no outgoing edge. Therefore,  $p(s) = \max_{(s,v) \in H} c(s, v)$ ,  $p(r) = 0$ , and  $p(i) = \max_{(i,v) \in H} c(i, v)$ ,  $i \in H$  and  $i \neq s, r$ . Then  $p(H) = p(s) + \sum_{u \neq s, (u,v) \in H} c(u, v)$ . The authors in [17] proposed the STPS algorithm to the minimum energy  $k$  node-disjoint paths of node pair problem by using that the value  $p(s)$  must be  $s$ 's some outgoing edge's weight  $c(s, v)$ .

#### STPS Algorithm:

**Input:**  $G(V, E, c)$ , a pair of nodes  $(s, r)$  and a constant integer  $k$ .

**Output:** A subgraph  $H$ , in which there are  $k$  node-disjoint paths from  $s$  to  $r$  with minimum  $p(H)$ .

**Initialize:** order  $s$ 's  $M$  outgoing edges as  $m_1, m_2, \dots, m_M$ , where  $c(m_i) > c(m_j) \Leftrightarrow i > j$ , where  $c(m_i)$  is the weight of the edge  $m_i$ . Let  $p_i(s)$  represent the current iteration transmission power of  $s$ , corresponding to  $i$ -th closest nodes "reached" by  $s$ . Initialize  $i = k$  and thus  $p_i(s) = c(m_k)$ . Note that starting with  $i < k$  would be fruitless, as the existence of  $k$  node-disjoint paths requires at least  $k$  outgoing edges from  $s$ . Finally, let  $E_{\min}$  represent the overall energy cost of the  $k$  minimum energy node-disjoint paths. Initialize  $E_{\min}$  to  $\infty$ .

1. Construct a new graph  $G_i$ , where  $G_i$  is a modified version of  $G$ . Accordingly, let  $G_i$  be equal to  $G$ , except removing the outgoing edges  $m_{i+1}, m_{i+2}, \dots, m_M$  for  $s$  whose cost are bigger than  $p_i(s)$ , and setting the costs of all the other outgoing edges  $m_1, m_2, \dots, m_i$  of  $s$  to zero.
2. Run a minimum weight  $k$  node-disjoint paths algorithm on  $G_i$ . Let  $c(H_i) = \sum_{(u,v) \in H_i} c(u, v)$  represent weight of subgraph  $H_i$ . If  $k$ -disjoint paths cannot be found by the minimum weight  $k$  node-disjoint paths algorithm, then set  $c(H_i) = \infty$  and continue.
3. Evaluate the following condition: If  $c(H_i) + p_i(s) < E_{\min}$ , then set  $E_{\min} = c(H_i) + p_i(s)$ .
4. Increment  $i = i + 1$  and correspondingly increase  $s$ 's transmission power, i.e.  $p_{i+1}(s) = c(m_{i+1})$ . Repeat steps 1–4 until  $i > M$ , at which point all relevant  $p(s)$  will have been considered, and the overall minimum energy  $k$  node-disjoint subgraph  $H$  for node pair  $(s, r)$  is determined.

Based on the STPS algorithm, we propose MEkNDPB algorithm for the minimum energy  $k$ -inconnected many-to-one routing problem. The main idea of MEkNDPB algorithm is that: given the  $(S, r)$ , for each  $s \in S$ , we invoke the STPS algorithm in [17] to get a subgraph  $H_{sr}$ , and STPS guarantees that subgraph  $H_{sr}$  is minimum energy cost for a pair nodes  $(s, r)$ . Then we can get a subgraph  $H = \bigcup_{s \in S} H_{sr}$ .  $H$  guarantees that there are at least  $k$  node-disjoint paths from any  $s \in S$  to  $r$ .

#### MEkNDPB Algorithm:

**Input:**  $(G, k, r, S)$ , where  $G = (V, E, c)$  is a directed weight graph of network topology and  $r$  is the root,  $S \subseteq V - r$ ,  $k$  is a constant integer.

**Output:** A subgraph  $H$  that is  $k$ -inconnected from  $S$  to  $r$ .

1. For each  $s \in S$ , invoke the STPS routine to get a subgraph  $H_{sr}$ .
2. Union all the subgraphs  $H_{sr}$  got in step 1, that is  $H = \bigcup_{s \in S} H_{sr}$ .

**Theorem 4.** The time complexity of MEkNDPB algorithm is  $O(kn^4)$ , where  $n$  is the number of nodes in the network and  $k$  is the number of node-disjoint paths required.

**Proof.** Since the time complexity of STPS algorithm [17] is  $O(kn^3)$  and there are at most  $n - 1$  nodes in subset  $S$ , the time complexity of MEkNDPB algorithm is  $O((n - 1)kn^3) = O(kn^4)$ .  $\square$

**Theorem 5.** The MEkNDPB algorithm has  $|S|$ -approximation ratio, where  $S$  is a given set of sources nodes in the network.

**Proof.** It is easy to know that subgraph  $H = \bigcup_{s \in S} H_{sr}$  produced by the MEkNDPB algorithm is a  $k$ -inconnected routing from  $S$  to  $r$ , for given  $(S, r)$ .

Suppose  $H_{\text{opt}}$  is optimal solution for the minimum energy  $k$ -inconnected many-to-one routing problem. Since  $H_{sr}$ , for any  $s \in S$ , produced by the STPS algorithm is an optimal subgraph for minimum energy  $k$  node-disjoint paths problem for  $(s, r)$ , then,  $\max_{s \in S} p(H_{sr}) \leq p(H_{\text{opt}})$ .

We have:

$$p(H) = p(\bigcup_{s \in S} H_{sr}) \leq \sum_{s \in S} p(H_{sr}) \leq |S| \max_{s \in S} p(H_{sr}) \leq |S| p(H_{\text{opt}}).$$

Therefore, the MEkNDPB algorithm is an  $|S|$ -approximation algorithm.  $\square$



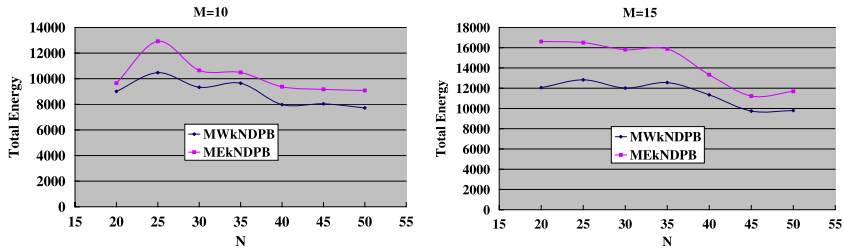


Fig. 3.  $r_{\max} = 1/2 \cdot (R)$  and  $k = 2$ .

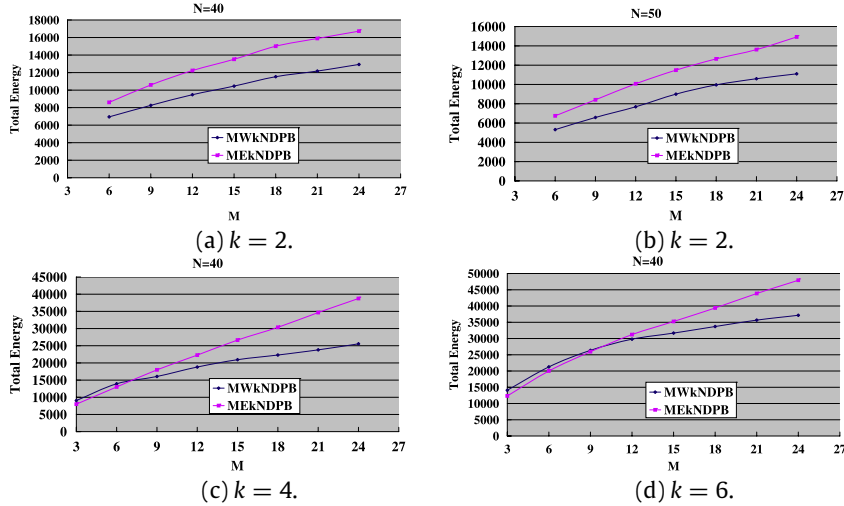


Fig. 4.  $r_{\max} = 1/2 \cdot (R)$ .

#### 4. Simulations

In the simulations, we focus on comparing our two approximation algorithms MWkNDPB and MEKNDPB, since the MEkinMRP problem has been little studied.

We study how the total energy cost is affected by varying three parameters over a wide range: the total number of nodes in the network ( $N$ ), the number of source nodes group ( $M$ ), the maximum transmission range of all nodes ( $r_{\max}$ ).

The simulation is conducted in a  $100 \times 100$  2-D free-space by randomly allocating  $N$  nodes. The unit of  $r_{\max}$  is respect to the length of one side in the square region, i.e., when  $r_{\max} = \sqrt{2}R$ , a node's transmission range using  $r_{\max}$  covers the whole region. The power model is:  $P = r^2$ , where  $P$  is the transmission power and  $r$  the radius that the signal can reach. Each node has the same maximum transmission range  $r_{\max}$  and the transmission range of each node can be any value between zero and  $r_{\max}$ .

We present averages of 100 separate runs for each result shown in the following figures. In each run of the simulations, for given  $N, M, r_{\max}$ , we randomly place  $N$  nodes in the square, and randomly select  $M$  source nodes and a destination. Then we assign each node with the maximum transmission range  $r_{\max}$ , and any topology that is not  $k$  node-connected is discarded. Then we run the two algorithms on this network.

In Fig. 3, we fix  $M = 10$  and  $M = 15$ ,  $r_{\max} = 1/2 \cdot (R)$  while varying  $N$ . As we can see, the total energy cost of the subgraph decreases as the growth of  $N$  approximately. It is comprehensible that the number of hops between source and destination increases with  $N$  increasing and, as we know, more hops between source and destination implies smaller total energy cost approximately. It is also shown that the MWkNDPB algorithm is better than MEKNDPB algorithm all the time in this condition for  $k = 2$ .

In Fig. 4, we fix  $N = 40$  and  $N = 50$ ,  $r_{\max} = 1/2 \cdot (R)$  while varying  $M$ . As we can see, the total energy cost of the subgraph increases as the growth of  $M$ . It is intelligible that, since there must be  $k$  node-disjoint paths between each source and destination, some nodes in the subgraph should increase their transmission range or some nodes not in the subgraph should assign a transmission range to provide more paths. It is also shown that the MWkNDPB algorithm is better than MEKNDPB algorithm all the time for  $k = 2$ . And for  $k = 4$  or  $6$ , the the MWkNDPB algorithm is better than MEKNDPB algorithm as  $M$  increases some value.

In Fig. 5, we fix  $M = 15$ ,  $N = 40$  and  $N = 50$  while varying  $r_{\max}$ . In this simulation, we keep the location of all the nodes invariably when varying  $r_{\max}$ . As we can see, the total energy cost of the subgraph is invariably. It is because the graph with all nodes having smaller  $r_{\max}$  is subgraph of the graph with bigger  $r_{\max}$ , the subgraph obtained from MWkNDPB and MEKNDPB algorithms for smaller  $r_{\max}$  must be a feasible subgraph for bigger  $r_{\max}$ . Considering that the total energy cost is

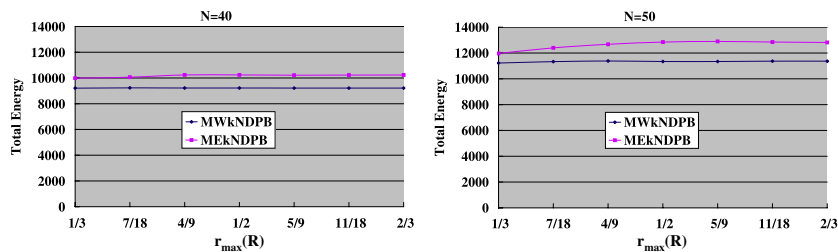


Fig. 5.  $M = 15$  and  $k = 2$ .

not the sum of weights of edges in the subgraph, the total energy cost of the subgraph changes a little. It is also shown that the MWkNDPB algorithm is better than MEkNDPB algorithm.

## 5. Conclusion

We have studied the minimum energy  $k$ -inconnected many-to-one routing problem in wireless networks. Two approximation algorithms MWkNDPB and MEkNDPB for this problem have been proposed. Extensible simulations have been conducted to compare the two approximation algorithms.

## Acknowledgments

D. Li was supported in part by NSFC under grant 10671208. And Y. Wang was supported in part by the National Basic Research Program of China Grants 2007CB807900 and 2007CB807901, NSFC under Grant 60604033, and the Hi-Tech Research Development Program of China Grant 2006AA10Z216.

## References

- [1] M. Bahrangiri, M. Hajiaghayi, V. Mirrokni, Fault-tolerant and 3-dimensional distributed topology control algorithms in wireless multi-hop networks, in: Proc. 11th IEEE Int'l Conf. Computer Comm. and Networks, ICCCN '02, 2002.
- [2] G. Calinescu, P.-J. Wan, Range assignment for high connectivity in wireless ad hoc networks, in: Proc. Second Int'l Conf. Ad-Hoc Networks and Wireless, ADHOC-NOW '03, 2003.
- [3] M. Hajiaghayi, N. Immerlica, V.S. Mirrokni, Power optimization in fault-tolerant topology control algorithms for wireless multi-hop networks, in: Proc. ACM MobiCom, 2003.
- [4] D.S. Hochbaum, Approximation Algorithms for NP-Hard Problems, PWS Publishing Company, 1995.
- [5] S. Khuller, B. Raghavachari, Improved approximation algorithms for uniform connectivity problems, in: Proc. 27th Ann. ACM Symp. Theory of Computing, STOC '95, 1995.
- [6] X. Jia, D. Kim, P. Wan, C. Yi, Power assignment for  $k$ -connectivity in wireless ad hoc networks, in: Proc. IEEE INFOCOM, 2005.
- [7] D. Li, X. Jia, H. Liu, Energy efficient broadcast routing in static ad hoc wireless networks, IEEE Transactions on Mobile Computing 3 (2004) 144–151.
- [8] D. Li, Q. Liu, X. Hu, X. Jia, Energy efficient multicast routing in ad hoc wireless networks, Computer Communications 30 (2007) 3746–3756.
- [9] D. Li, L. Liu, Q. Hui, Minimum connected  $r$ -hop  $k$ -dominating set in wireless networks, Discrete Mathematics, Algorithms and Applications 1 (2009) 45–57.
- [10] D. Li, Q. Zhu, H. Yang, Fault-tolerant routing:  $k$ -inconnected many-to-one routing in wireless networks, in: Proc. COCOA, 2009.
- [11] L. Li, J.Y. Halpern, P. Bahl, Y.-M. Wang, R. Wattenhofer, Analysis of a cone-based distributed topology control algorithm for wireless multi-hop networks, in: Proc. 20th Ann. ACM Symp. Principles of Distributed Computing, PODC '01, 2001.
- [12] Z. Li, D. Li, Minimum energy broadcast routing in ad hoc and sensor networks with directional antennas, Discrete Mathematics, Algorithms and Applications 1 (2) (2009) 205–218.
- [13] E.L. Lloyd, R. Liu, M.V. Marathe, R. Ramanathan, S.S. Ravi, Algorithmic aspects of topology control problems for ad hoc networks, in: Proc. ACM MobiHoc, 2002.
- [14] E.L. Lloyd, R. Liu, M.V. Marathe, R. Ramanathan, S.S. Ravi, Algorithmic aspects of topology control problems for ad hoc networks, Mobile Network Application 10 (2005) 19–34.
- [15] N. Li, J.C. Hou, FLSS: A fault-tolerant topology control algorithm for wireless networks, in: Proc. ACM MobiCom, 2004.
- [16] R. Ramanathan, R. Hain, Topology control of multihop wireless networks using transmit power adjustment, in: Proc. IEEE INFOCOM, 2000.
- [17] A. Srinivas, E. Modiano, Minimum energy disjoint path routing in wireless ad-hoc networks, in: Proc. ACM MobiCom, 2003.
- [18] J.W. Suurballe, Disjoint paths in a network, Networks 4 (1974) 125–145.
- [19] F. Wang, My T. Hai, Yingshu Li, Xiuzhen Cheng, Ding-Zhu Du, Fault-tolerance topology control for all-to-one and one-to-all communication in wireless networks, IEEE Transaction on Mobile Computing 7 (2008) 322–331.
- [20] R. Wattenhofer, L. Li, P. Bahl, Y.-M. Wang, Distributed topology control for wireless multihop ad-hoc networks, in: Proc. IEEE INFOCOM, 2001.