PEV-based Reactive Power Compensation for Wind DG Units: A Stackelberg Game Approach

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***Abstract*—There has been a growing interest recently towards integrating more renewable energy resources, in particular wind power, in form of distributed generation (DG) units. However, one important challenge with wind DG units is to provide low-cost and fast-responding reactive power compensation of the wind turbine's inductive load to ensure a stable voltage profile in the system. Since reactive power can only be compensated locally, we consider a scenario where a wind DG unit is co-located with a plug-in electric vehicle (PEV) charging station or a parking lot, and we investigate how to align incentives to encourage PEV owners to participate in reactive power compensation for wind DG units. For this purpose, in this paper, we introduce a two-stage Stackelberg game between the wind DG unit and the PEV owners. We use backward induction to analyze the formulated game and derive the optimal pricing scheme. We assess the performance of our proposed scheme using field data and make suggestions for the size of the charging stations.**

***Keywords*—Distributed generation, wind power integration, plug-in electric vehicles, reactive power compensation, real time pricing, Stackelberg game, subgame perfect equilibrium.**

I. Introduction

To achieve a larger penetration of renewable power, renewable distributed generation (DG) units are becoming increasingly popular [1]. In contrast to the conventional approach of a few large-scale generators located far from the load centers, DG units will speed up the revolution of providing power on site with little reliance on the distribution and transmission network. The most popular renewable DG scheme is wind turbines. Therefore, in this paper, we focus on the analysis of wind power integration. There are several challenges to integrate these DG units. Above all, there exists no accurate long term wind power prediction method, which makes it very difficult for independent system operators to perform capacity planning. To address this challenge, in [2], Neely *et al.* used Lyapunov theory to obtain a *centralized* optimal queueing system for allocating renewable energy to delay-tolerant consumers. In [3], He *et al.* proposed a multiple timescale dispatch for smart grid with integrated wind power. Wu *et al.* investigated how to utilize wind power integration into the power grid when aggregators use a linear pricing scheme in [4]. The same authors also and proposed a cost sharing game among end users to utilize available wind power in [5]. In [6], Li *et al.* used stochastic programming to obtain the optimal plug-in electric vehicle (PEV) power management solutions for renewable wind and solar energy integration.

In addition to the need for constantly balancing supply and demand for active power, we also need to compensate reactive power for each wind DG unit. Reactive power compensation is needed since the load is *not* pure resistive and may include inductors and capacitors that can may cause power oscillations. In particular, given the inductive load of wind turbines, we need reactive power compensation in order to stabilize the voltage profile. The common solution to tackle this problem is to install a combination of shunt capacitors, switchable capacitors, and static var compensators (SVCs) to be constantly adjusted according to the reactive power compensation needs of the wind DG unit [7]. However, with the increasing penetration rate of wind power generation, the amount of reactive power to be compensated is growing. In most cases, such amount is a fluctuating stochastic variable. Since the response time for SVCs is relatively slow, engineers are investigating the use of static synchronous compensators (STATCOMs), which have faster response time [8]. However, STATCOM devices are usually expensive and may not be a feasible solution for large-scale deployment of wind DG units.

An alternative approach to compensate reactive power of wind DG units is to utilize certain types of load that participate in demand response. In particular, some recent studies have shown that the power electronics AC-DC inverter circuits used for charging PEVs can potentially contribute in to reactive power compensation using appropriate P-Q control [9]. In [10], Cvetkovic *et al.* presented the structure and capabilities of a small, grid-interactive, distributed energy resource system using PEVs to perform frequency and voltage regulation. In [11], Turitsyn *et al.* considered the possibility of reactive power control by distributed photovoltaic generators. In [12], Wu *et al.* addressed a joint PEV-based optimization of frequency and voltage regulation for smart grid. In [13], Farag *et al.* proposed a two-way communication distributed control scheme for voltage regulation in distribution feeders.

Following the results in [9]–[13], in this paper, we consider a scenario where a wind DG unit is co-located with a PEV charging station. We aim in answering the following question: How can we provide incentives for PEV owners to participate in reactive power compensation in order to optimally utilize the reactive power compensatio potential of PEV AC-DC inverters in vehicle-to-grid systems? We use tools and techniques from game theory [14] to answer this question. Our contributions in this paper can be summarized as follows.

- *PEV-based Reactive Power Compensation:* Instead of

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installing SVCs or STATCOM, we consider an alternative *distributed* approach to utilize PEV-based reactive power compensation for wind DG units, which has lower cost than installing STATCOM and is more flexible and controllable than only using shunt and switchable capacitors.

- *Stackelberg Game Formulation:* We formulate the interactions between the wind DG unit and the PEV owners as a two-stage Stackelberg game, where the wind DG unit is the leader and the PEV owners are the followers. Our proposed model captures each player's selfish behavior and views each player as independent decision maker.
- *Equilibrium Analysis:* We use backward induction to analyze the formulated game, and to obtain the optimal pricing scheme. Subgame perfect equilibrium is investigated as the solution concept of the proposed game.
- *Simulation Assessment:* We use field data to assess our proposed approach in a two hours simulation. We investigate the desirable scale of the charging station for wind DG units when using different charging methods.

The rest of this paper is organized as follows. The system model is introduced in Section II. A centralized control approach for reactive power compensation is discussed in Section III. Our proposed distributed interactive approach based on the Stackelberg game model is presented in Section IV. We use backward induction to analyze the formulated Stackelberg game and to obtain the optimal pricing scheme in Section V. Simulation results are presented in Section VI. Concluding remarks and future work are discussed in Section VII.

II. System Model

As shown in Fig. 1, in our system, there is a wind DG unit equipped with several shunt and switchable capacitors. The wind DG unit is co-located with a PEV charging station or a parking lot, where several PEVs are parked and plugged in. Some of these PEVs choose to charge their batteries as quickly as possible, and thus, they are not willing to provide any service to the wind DG unit. On the other hand, some PEVs are available for a relatively longer periods of time and are willing to participate in reactive demand response programs, as long as they receive proper payments for the service they offer. Let $\mathcal{N}$, with cardinality N, denote the latter group of PEVs. We assume that the wind DG unit tries to use the shunt and switchable capacitors to compensate the reactive power due to the dominantly inductive impact of wind turbines. However, since such capacitors cannot achieve arbitrary values, the wind DG unit may choose to also utilize the reactive power compensation potential of the N available PEVs. Without loss of generality, we assume that time is divided into several equal-length time slots, *e.g.,* one minute per slot, and we focus on the analysis within one particular time slot. A multi-slot analysis will address the long term management of switchable capacitors to further reduce the operating costs for these capacitors while maintaining a stable voltage profile, and will be left to be studied as a future work.

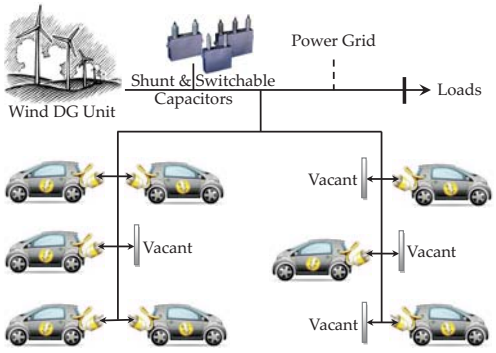


Fig. 1. The vehicle-to-grid system model considered in this paper.

III. Centralized Control Design

If the wind DG unit has full control over the PEVs, it can design a centralized algorithm to coordinate the operation of their chargers and inverters in order to achieve the desirable amount of reactive power compensation. This would be needed to stabilize voltage magnitude around its nominal value. At each time slot, the wind DG unit predicts the amount of reactive power compensation that it needs. We denote this amount as Q_d. As already mentioned in Section II, the wind DG unit prefers to maximally utilize its shunt and switchable capacitors. We denote $Q_c \in \mathcal{C}$ as the amount of reactive power injection provided by the shunt and switchable capacitors, with feasible set $\mathcal{C}$. Thus, the wind DG unit will select Q_c according to the solution of the following optimization problem:

$$\begin{aligned} \underset{Q_c}{\textbf{minimize}} \quad & |Q_d - Q_c| \\ \textbf{subject to} \quad & Q_c \in \mathcal{C}. \end{aligned} \tag{1}$$

We note that, in general, problem (1) is NP-complete due to the fact that the feasible set $\mathcal{C}$ is discrete. However, in practice, the size of $\mathcal{C}$ is very small and most capacitors are almost identical. Therefore, we can efficiently enumerate all the elements in $\mathcal{C}$, and thus determine a suitable Q_c efficiently.

Let $\boldsymbol{q} = \{q_n, \forall n \in \mathcal{N}\}$, where for each PEV $n \in \mathcal{N}$, q_n denotes the amount of reactive power provided by PEV n to the wind DG unit in the current time slot. We assume that the PEVs use trickle chargers [15]. Therefore, we can assume that the maximal reactive power that they can provide is $q_{\max} = S_{\max}$, where $S_{\max}$ is the maximal apparent power supported. As such, we impose the following constraint on q_n:

$$-q_{\max} \leq q_n \leq q_{\max}, \forall n \in \mathcal{N}. \tag{2}$$

When using level 1 charging, $q_{\max}$ is 1.44 kVar and when using level 2 charging, $q_{\max}$ is 7.68 kVar in US [16].

With a selected Q_c, the wind DG unit would like to select $\boldsymbol{q}$ to achieve the desirable Q_d, *i.e.,*

$$\begin{aligned} \underset{\boldsymbol{q}}{\textbf{minimize}} \quad & \left(\sum_{n \in \mathcal{N}} q_n + Q_c - Q_d \right)^2 \\ \textbf{subject to} \quad & -q_{\max} \leq q_n \leq q_{\max}, \quad \forall n \in \mathcal{N}. \end{aligned} \tag{3}$$

Problem (3) is a quadratic convex program and can be solved efficiently in a centralized fashion. However, in most practical cases, the wind DG unit does not have full control over

PEVs. Therefore, it cannot directly select the reactive power compensation profiles $\boldsymbol{q}$ for the PEVs. Given the fact that PEV owners are independent decision makers, next we use game theory to analyze their interactions with the DG unit.

IV. Stackelberg Game Design

We would like to design a proper pricing scheme to align incentives to the PEVs in $\mathcal{N}$. We denote $Q_{\min}$ as the minimal reactive power injection the wind DG unit needs, and $Q_{\max}$ as the maximal reactive power injection the wind DG unit can handle. The overall idea of designing the pricing scheme is that higher payments correspond to higher incentives. That is, if the total reactive power injection,

$$Q_c + \textstyle\sum_{n\in\mathcal{N}} q_n, \tag{4}$$

keeps the voltage profile in an acceptable range (*i.e.,* between $Q_{\min}$ and $Q_{\max}$), then each PEV will receive a higher payment than some base price p_b. However, if the total reactive power injection falls beyond the expected range, then each PEV will face a lower payment than the base price p_b.

If $Q_d < Q_c$, *i.e.,* the wind DG unit needs reactive power *consumption* from the PEVs in $\mathcal{N}$, then it expects a negative q_n, for all $n \in \mathcal{N}$. On the other hand, if $Q_d \geq Q_c$, *i.e.,* the wind DG unit needs reactive power *injection*, then it expects a positive q_n from all PEVs in $\mathcal{N}$. Therefore, we set up two different pricing schemes based on these observations. When $Q_d < Q_c$, we set the price as:

$$p^c(\boldsymbol{q}) = -p_b - \alpha\left(Q_c + \sum_{n\in\mathcal{N}} q_n - Q_{\min}\right). \tag{5}$$

Here, $\alpha > 0$ is the *incentive control parameter* which is adjusted to control PEVs reactive power compensation behavior. We note that, the price is negative because the wind DG unit expects a negative q_n, for all $n \in \mathcal{N}$ in this case. Furthermore, the price only relates to $Q_{\min}$ because the wind DG unit wants the PEVs to consume the extra reactive power provided by the shunt and switchable capacitors. Although, the DG unit does not expect the total reactive power injection to be less than $Q_{\min}$, *i.e.,* not adequate to ensure a stable voltage profile. As we will show later, by applying the pricing scheme in (5), the rational behaviors of all the PEVs will lead to a total reactive power injection within the desirable range.

On the other hand, when $Q_d \geq Q_c$, we set the price as:

$$p^i(\boldsymbol{q}) = p_b + \beta\left(Q_{\max} - Q_c - \sum_{n\in\mathcal{N}} q_n\right), \tag{6}$$

where $\beta > 0$ is another *the incentive control parameter* which is again adjusted by the wind DG unit. The design idea is similar to the idea for (5). We care about $Q_{\max}$ in (6) since though the wind DG unit expects the PEVs to inject reactive power, it doesn't want the total reactive power injection exceeds $Q_{\max}$.

Above all, the pricing scheme is:

$$p(\boldsymbol{q}) = \begin{cases} p^c(\boldsymbol{q}), & \text{if } Q_d < Q_c, \\ p^i(\boldsymbol{q}), & \text{otherwise.} \end{cases} \tag{7}$$

As we will show next, we aim to set incentive control parameters α and β to achieve the optimal overall system performance. We note that in our proposed pricing scheme, $\boldsymbol{q}$ is set by the PEV owners, while α, β, Q_c, $Q_{\max}$, and $Q_{\min}$ are set by the wind DG unit.

For the reactive power compensation that each PEV n provides, it receives the following payment from the wind DG unit:

$$f_n(\boldsymbol{q}) = p(\boldsymbol{q}) q_n. \tag{8}$$

Due to space limitation, here we only analyze the more common scenario when the wind DG unit needs reactive power *injection*. The analysis for the case when the wind DG unit needs reactive power consumption is very similar. As such, when $Q_d < Q_c$, we can simplify the pricing scheme in (7) as

$$p(\boldsymbol{q}) = p_b + \beta\left(Q_{\max} - Q_c - \textstyle\sum_{n\in\mathcal{N}} q_n\right). \tag{9}$$

Based on (9), we next introduce our game-theoretic model.

A. Wind DG Unit's Payoff

As explained in Section III, the wind DG unit is interested in constantly performing reactive power compensation to maintain a stable voltage profile. Therefore, we can model the DG unit's payoff function based on the mismatch between the needed and achieved reactive power compensation:

$$g(\beta, Q_{\max}, Q_c) = -\left(\sum_{n\in\mathcal{N}} q_n(\beta, Q_{\max}, Q_c) + Q_c - Q_d\right)^2. \tag{10}$$

Here, we denote q_n in the function form of $q_n(\beta, Q_{\max}, Q_c)$ to emphasize that each PEV n will make its own decision given parameters β, $Q_{\max}$, and Q_c in the pricing scheme in (9).

B. PEVs' Payoff

For each individual PEV owner, we formulate the payoff functions as the payment that the user would obtain in (8). That is, for each PEV $n \in \mathcal{N}$, the payoff function becomes

$$\begin{aligned} f_n(q_n; q_{-n}) &= p(\boldsymbol{q}) \cdot q_n \\ &= \left(p_b + \beta\left(Q_{\max} - Q_c - \sum_{n\in\mathcal{N}} q_n\right)\right) q_n, \end{aligned} \tag{11}$$

where q_{-n} is the set of all PEVs' reactive power compensation profiles other than the reactive power compensation profile of PEV n, *i.e.,* $q_{-n} = \{q_s | s \in \mathcal{N} \setminus \{n\}\}$. Therefore, for each PEV, the payoff function depends on not only that PEV's operation, but also *all other* PEVs' operations. This leads to the formulation of our proposed two-stage Stackelberg game.

C. Stackelberg Game Formulation

As shown in Fig. 2, in the proposed two-stage Stackelberg game, the wind DG unit is the Stackelberg leader. It first decides the pricing parameters β and $Q_{\max}$ in Stage I. Then, the PEVs act as Stackelberg followers and choose their reactive power compensation profiles to maximize their own payoffs in Stage II. For the case $Q_d \geq Q_c$, we can formulate a similar two-stage Stackelberg game, where the leader chooses the pricing parameters α and $Q_{\min}$.

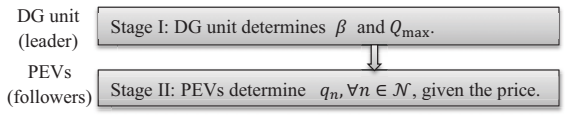


Fig. 2. A Stackelberg game formulation for reactive power compensation.

V. Backward Induction of The Two-stage Game

The Stackelberg game falls into the general class of dynamic games. Its common solution concept is the subgame perfect equilibrium (SPE). A powerful technique to obtain SPE is backward induction [17]. To derive the SPE, our analysis starts in Stage II and captures the PEVs' behaviors given the wind DG unit's pricing scheme. Then, it moves backward in time to Stage I to analyze how the wind DG unit may choose the pricing parameters β and $Q_{\max}$, given the expected response of followers in Stage I. That is, backward induction captures the sequential dependence of the decisions in the two stages.

A. Reactive Power Compensation Subgame in Stage II

The subgame in Stage II is the reactive power compensation game among PEVs. We can formally define it as follows:

Reactive Power Compensation Subgame (RPC Game):

- *Players:* The set $\mathcal{N}$ of all the PEVs;
- *Strategies:* For each PEV $n \in \mathcal{N}$, based on the given pricing scheme set by the DG unit, it chooses its own reactive power compensation profile $q_n \in [-q_{\max}, q_{\max}]$.
- *Payoffs:* For each PEV $n \in \mathcal{N}$, its payoff function is defined as its payment from the DG unit as in (11).

To analyze the equilibrium of the above game, first, we obtain the model for *best response*, which is a PEV's best choice to maximize its own payoff assuming that all other PEVs' strategies are fixed.

Definition 1: For a PEV $n \in \mathcal{N}$, its best response is:

$$q_n^{best}(q_{-n}) = \underset{q_n \in [-q_{\max}, q_{\max}]}{\arg\max} f_n(q_n; q_{-n}). \tag{12}$$

Since the payoff for each PEV is a quadratic function, we can further represent the PEV n's best response as follows:

$$q_n^{best}(q_{-n}) = \underset{q_n \in [-q_{\max}, q_{\max}]}{\arg\min} |q_n - p_b/2\beta - (Q_{\max} - Q_c - \textstyle\sum_{s \in \mathcal{N} \setminus \{n\}} q_s)/2|. \tag{13}$$

Next, we investigate the solution concept of SPE.

Definition 2: A strategy profile $\{q_n^*, \forall n \in \mathcal{N}\}$ is a subgame perfect equilibrium of the Stackelberg game if given any $\beta > 0$, $Q_{\max} \geq Q_c \geq 0$, the restricted strategy profile $\{q_n, \forall n \in \mathcal{N} | \beta, Q_{\max}, Q_c\}$ is a Nash equilibrium for the RPC subgame. That is, for any pricing parameter $\beta > 0$, $Q_{\max} \geq Q_c \geq 0$, *no* PEV $n \in \mathcal{N}$ can increase its payoff $f_n(\cdot)$ by *unilaterally* changing its own reactive power compensation profile q_n^*.

Based on the definition of SPE, we can obtain each PEV's strategy by computing the Nash equilibrium of the RPC subgame. Note that the Nash equilibrium is a fixed point of all PEVs' best responses, *i.e.,* $q_n^{best}(q_{-n}^*) = q_n^*$ for all $n \in \mathcal{N}$. Next, we prove the existence and the uniqueness of Nash equilibrium of the RPC subgame.

Theorem 1: Given $\beta > 0$, $Q_{\max} \geq Q_c \geq 0$, there always exists a unique Nash equilibrium for the RPC game.

Proof: For any $\beta > 0$, each PEV player's payoff function is concave, and the strategy space is composed by a set of linear constraints (*i.e.,* $q_n \leq q_{\max}, \forall n \in \mathcal{N}$), forming a convex set. Thus, we can conclude Theorem 1 directly from [18]. ■

Based on the PEVs' best responses given in (13), we can next move to Stage I of the Stackelberg game, where we will investigate how to design the optimal pricing parameter β.

B. Determining β and $Q_{\max}$ in Stage I when $Q_d \geq Q_c$

The wind DG unit should choose β such that at Nash equilibrium $(q_n^*, n \in \mathcal{N})$ of the RPC game, it can maximize its own payoff (10). To achieve this goal, if $0 \leq Q_d - Q_c \leq N q_{\max}$, then the wind DG unit wants each PEV n's strategy at Nash equilibrium to be $(Q_d - Q_c)/N$. From (13), In order to have

$$q_n^* = \frac{Q_d - Q_c}{N}, \ \forall n \in \mathcal{N}, \tag{14}$$

It is required that we actually have

$$q_n^* = \frac{p_b + \beta(Q_{\max} - Q_c - \sum_{s \in \mathcal{N} \setminus \{n\}} q_s)}{2\beta}. \tag{15}$$

This is achieved if we choose parameter β as

$$\beta = \frac{N p_b}{(N+1)Q_d - N Q_{\max} - Q_c}. \tag{16}$$

Since parameter β has to be positive, it is required that

$$(N+1)Q_d - N Q_{\max} - Q_c > 0. \tag{17}$$

That is,

$$Q_{\max} < Q_d + \frac{1}{N}(Q_d - Q_c). \tag{18}$$

Note that, parameter $Q_{\max}$ is naturally upper-bounded by the reactive power injection that causes the voltage profile higher than the upper bound of an accepted stable voltage profile. Let $Q_{\max}^V$ denote such upper bound. We have

$$Q_{\max} < \min\{Q_d + \frac{1}{N}(Q_d - Q_c), Q_{\max}^V\}. \tag{19}$$

We will show in Section V-D that this pricing parameter setup can achieve the desirable SPE in all cases when $Q_d \geq Q_c$.

C. Selecting α and $Q_{\min}$ when $Q_d < Q_c$

Following a similar analysis, we can obtain the optimal choice for parameter α, for the reactive power consumption case, as

$$\alpha = \frac{N p_b}{N Q_{\min} + Q_c - (N+1)Q_d}. \tag{20}$$

To maintain $\alpha > 0$, we need

$$Q_{\min} > Q_d + \frac{1}{N}(Q_d - Q_c). \tag{21}$$

Again, we note that parameter $Q_{\min}$ is naturally lower-bounded by the reactive power injection that causes the voltage

profile lower than the lower bound of an accepted stable voltage profile. Let $Q^V_{\min}$ denote such lower bound. We have

$$Q_{\min} > \max\{Q_d + \frac{1}{N}(Q_d - Q_c), Q^V_{\min}\}. \quad (22)$$

We will show in Section V-D that this pricing parameter setup can achieve the desirable SPE in all cases when $Q_d < Q_c$.

D. Optimality of the Pricing Scheme

With β as in (16), and α as in (20), we can obtain that

$$q_n^{best}(q_{-n}) = \underset{q_n \in [-q_{\max}, q_{\max}]}{\arg\min} |q_n - (Q_d - Q_c)/2N - (Q_d - Q_c - \textstyle\sum_{s \in \mathcal{N}\setminus\{n\}} q_s)/2|. \quad (23)$$

From (23) and Theorem 1, the following theorem is resulted.

Theorem 2: We can show that:

- If $(Q_d - Q_c)/N < -q_{\max}$, then at Nash equilibrium of the RPC game, we have
$$q_n = -q_{\max}, \forall n \in \mathcal{N}. \quad (24)$$
- If $-q_{\max} \leq (Q_d - Q_c)/N \leq q_{\max}$, then at Nash equilibrium of the RPC game, we have
$$q_n = (Q_d - Q_c)/N, \forall n \in \mathcal{N}. \quad (25)$$
- If $(Q_d - Q_c)/N > q_{\max}$, then at Nash equilibrium of the RPC game, we have
$$q_n = q_{\max}, \forall n \in \mathcal{N}. \quad (26)$$

From Theorem 2, the Nash equilibrium in all cases is the solution to the centralized optimization problem (3).

VI. Simulation Results

As shown in Section V, the proposed pricing model in (7) can achieve optimal performance for the explained distributed PEV-based reactive power compensation system. In this section, we investigate the required size of the PEV charging station to support reactive power compensation for wind DG units of different sizes. First, we consider a wind DG unit with 15 MW peak generation capacity. We assume the wind DG unit predicts the upcoming wind power P and use the P-Q characteristic curve to obtain Q_d. For one minute time slot, the prediction error is usually less than 2% [19]. To better characterize the interaction between the wind DG unit and the PEVs in $\mathcal{N}$, we assume that the one-time slot ahead predictions are perfect with no error. Its P-Q characteristic curve is as in Fig. 3. Note that the maximal reactive power required is around 6 MVAR. We assume that the DG unit is equipped with one 1 MVAR, one 2 MVAR, and one 3 MVAR switchable capacitors. Thus, $\mathcal{C}$ is $\{1, 2, 3, 4, 5, 6\}$. We use the scaled NOAA ASOS wind data with one minute resolution [20]. The wind DG unit will first try to use the switchable capacitors to perform reactive power compensation by solving optimization problem (1). During the two hours simulation, the reactive power requests and corresponding Q_c are set as in Fig. 4. The difference between the reactive power request and Q_c is supplemented by the set of PEVs that participate in reactive power compensation, which is shown in Fig. 5(a).

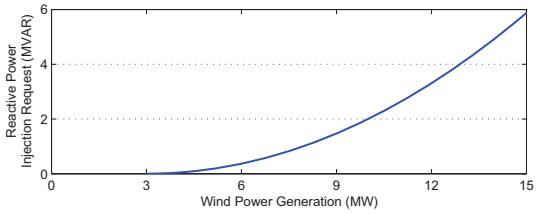


Fig. 3. P-Q characteristic of a typical wind farm from [7].

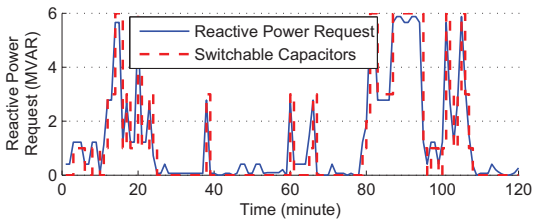


Fig. 4. The reactive power request and corresponding Q_c.

Fig. 5(b)-(f) show the reactive power mismatch reduction for various numbers of PEVs' participation when using level 2 charging. The simulation results are quite intuitive. More PEVs correspond to less fluctuations. The statistical features of the reactive power compensation mismatch are shown in Fig. 6. Fig. 6(a) shows that when using level 1 charging, to compensate all the reactive power mismatch, we need at least 326 PEVs' participation. On the other hand, when using level 2 charging, to compensate all the reactive power mismatch, we only need 62 PEVs' participation. We are also interested in the mismatch variance reduction. As shown in Fig. 6(b), when using level 1 charging, to reduce the mismatch variance by 80%, we need 172 PEVs' participation. To reduce the variance by 90%, we need 204 PEVs' participation. On the other hand, when using level 2 charging, to reduce the mismatch variance by 80%, we only need 33 PEVs' participation. To reduce the variance by 90%, we need 39 PEVs' participation.

By using level 2 charging, as shown in Fig. 5(f) and Fig. 6, the wind DG unit will need roughly 50 PEVs for a maximal 500 kVAR reactive power mismatch. We note that there is also a huge potential to electrify all the public transportation systems (*e.g.,* school buses), to have 50 PEVs' participation in the PEV charging station or a parking lot at each moment could be quite possible. Hence, the simulation results here suggest that our proposed framework could effectively reduce the reactive power mismatch when the wind DG unit is not equipped with SVCs or STATCOM. Furthermore, the scale of charging station it needs is reasonably small when we use level 2 charging given proper incentives to the PEV owners. Nevertheless, in big cities such as New York, a charging station of scale 300 PEVs is quite possible and using level 1 charging could be a desirable choice in such cases since the infrastructure cost for level 1 charging is lower than level 2 charging. As such, our proposed framework is promising.

VII. Conclusions

In this paper, we proposed a Stackelberg game-theoretic model to encourage PEV participation in reactive power com-

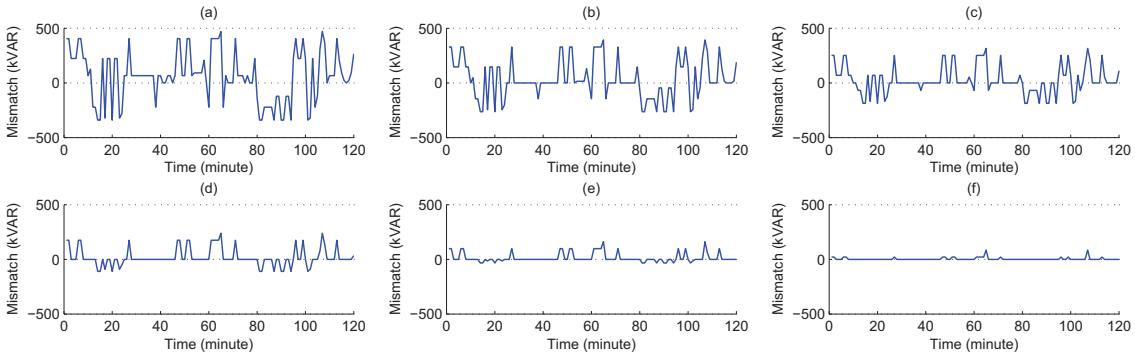


Fig. 5. Reactive power mismatch reduction when using level 2 charging based on different participation levels: (a) without PEVs' participation; (b) with 10 PEVs' participation; (c) with 20 PEVs' participation; (d) with 30 PEVs' participation; (e) with 40 PEVs' participation; (f) with 50 PEVs' participation.

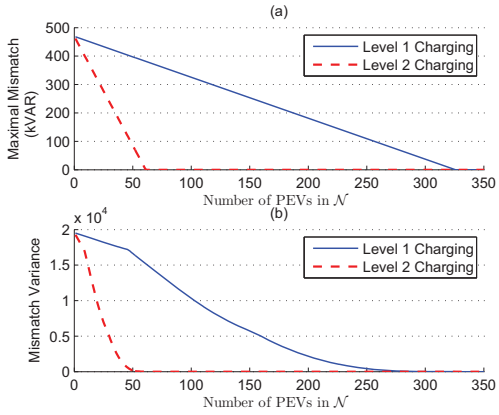


Fig. 6. Reactive power compensation mismatch versus the number of PEVs participating: (a) maximal mismatch; (b) mismatch variance.

pensation for wind DG units. Using backward induction, we first analyzed the proposed pricing scheme to set the pricing parameters such that an optimal reactive power compensation performance is achieved. Then, we used field data to assess our approach in a two hours simulation setting. Simulation results confirm that our approach can effectively perform reactive power compensation for wind DG units. The results also suggest that when using level 2 charging, the desirable scale of the charging station, co-located with the wind DG unit, can be as small as only 50 PEVs.

This work can be extended in several directions. First, we can consider the simultaneous active and reactive power compensation for the wind DG units to also tackle the fluctuations in real power generation. This will introduce new design trade-offs as the active and reactive power are restricted and coupled by the maximal apparent power constraint. We can also take into account the stochastic availability of the PEVs when it comes to decide on optimal operation of the charging stations to offer reactive and active power compensation. In addition, we may want to assess the impact of PEVs on the network stresses in our future work.

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