Optimizing Trading Assignments in Water Right Markets[*]

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Abstract

Over the past two decades, water markets have been successfully fielded in countries such as Australia, the United states, Chile, China, etc. Water users, mainly irrigators, have benefited immensely from water markets. However, the current water market design also faces certain serious barriers. It has been pointed out that transaction costs, which exists in most markets, induce great welfare loss. For example, for water markets in western China discussed in this paper, the influence of transaction costs is significant. Another important barrier is the locality of trades due to geographical constraints. Based on the water market at Xiying Irrigation, one of the most successful water market in western China, we model the water market as a graph with minimum transaction thresholds on edges. Our goal is to maximize the transaction volume or welfare. We prove that the existence of transaction costs results in no polynomial time approximation scheme (PTAS) to maximize social welfare (MAX SNP-hard). The complexities on special graphs are also presented. From a practical point of view, however, optimal social welfare can be obtained via a well-designed mixed integer linear program and can be approximated near optimally at a large scale via a heuristic algorithm. Both algorithms are tested on data sets generated from real historical trading data. Our study also suggests the importance of reducing transaction costs, for example, institutional costs in water market design. Our work opens a potentially important avenue of market design within the agenda of computational sustainability.

Introduction

Water markets provide an efficient approach for water reallocation (Rosegrant and Binswanger 1994). The discussion on water markets dates back to 1980s, followed by heated debates (Howe, Schurmeier, and Shaw 1986; Saliba 1987). Whereafter, the advantages of water markets have been proved worldwide, such as in Australia, United States, South Africa, Chile and China (Bjornlund and McKay 2002; Grafton et al. 2011). Previous study found that in an active water market, agents with higher marginal product tend to buy water from lower ones(Bjornlund and McKay 1998). On the one hand, the existence of opportunity costs encourage water users to improve water use efficiency. (Nieuwoudt and Armitage 2004; Turral et al. 2005). On the other hand, in dry years, high-marginal-product agents can buy water from markets to fulfill their demand and low-marginal-product ones earn money by selling water. In this way, agents suffer lower risks in drought years. (Ashton et al. 2009)

However, water markets typically do not work as well as expected. One major cause is transaction costs which significantly reduce the incentive for trading. (Carey, Sunding, and Zilberman 2002; Archibald and Renwick 1998) Different countries' water markets meet different restrictions because of local policies and cultures. Our study focuses on the water market in Xiying Irrigation in Northwestern China, which was established in 2008 and is one of the most successful water markets in China. Some properties of the studied area are as follows.

- Water is allocated to villages for each 2-month period.

- Water trades are conducted between villages.

- Most villages are only willing to trade with whose close to them because the agents from close villages are usually familiar with and trust each other.

- Villages are not willing to trade a tiny amount of water because of transaction costs (time costs, transportation costs, human costs).

- Because of short trading period, high transaction costs and small price differences, straddle has never been observed.

- Villages are not willing to abdicate the rights for deciding prices, making it impractical to propose a mechanism with pricing rule in the current market.

- Based on all the above properties, currently, villages propose their bids or asks to the center for trading and the center recommends sellers to buyers. When a buyer and a seller reach an agreement on volume and price, they need to sign a contract.

In light of these observations, we model a water right market (water market for short) as a directed bipartite graph con-

sisting of a set of buyers and a set of sellers. An arc is oriented from a buyer to a seller if and only if they are compatible (geographically connected and not hard to make the trade approved) and the buyer's bid price is above the seller's ask price. On each arc, there is volume threshold indicating the lowest transaction volume on this arc, if there is any. Each vertex has a price and a capacity that specify the demand or supply of the vertex.

A trading assignment is simply a feasible *flow*. The value for each arc is its trading volume times the price differences on the end vertices. The *social welfare* is the sum of values for all arcs. Our goal is to compute a trading assignment that maximizes (1) total trading volume; or (2) social welfare.

Our study on maximizing total trading volume and social welfare is motivated by (Shapley and Shubik 1971), which shows that any social welfare maximizing assignment corresponds to a stable matching outcome. In other words, our study, similar as (Shapley and Shubik 1971), concern about the property of stability instead of truthfulness.

Our contribution

- Transaction costs and locality feature induce computational barriers to maximizing social welfare or volume. Denote the problem of maximizing trading volume by MAX-VOLUME and the other by MAX-WELFARE. We prove that MAX-VOLUME is NP-HARD and MAX-WELFARE is MAX SNP-HARD. Even on simple graphs, the computational complexity is still high. MAX-WELFARE is NP-HARD even on some "sparse" graphs[1] and binary trees. MAX-WELFARE can be solved in polynomial time on line graphs and cycle graphs but with a high order.

- We overcome the computational barrier perfectly by well-designed optimization and a heurstic algorithm. We designed a novel mixed linear integer program to solve the MAX-WELFARE problem. The performance of both algorithms are proved to be efficient on data sets generated based on historical trading data.

- We compare the running time and social welfare with different transaction thresholds, which suggests the importance of reducing transaction costs.

Additional related work

Matching market has a long history in economic literature (see (Myerson and Satterthwaite 1983; Barbera and Jackson 1995; Shapley and Shubik 1971) and (Roth and Sotomayor 1992, Chapter 8)). The computation problem on matching market has also been considered in various settings (Kalagnanam, Davenport, and Lee 2000; Sandholm and Suri 2001; 2002; Blume et al. 2009; Li et al. 2014; Liu, Tang, and Fang 2014; Luo and Tang 2015). Our work differs from them in that we have thresholds on arcs and we consider special structures of graphs. Our problem is also related to a variant of the maximum flow problem where each edge has a minimum flow requirement. This version of maximum flow can be solved in polynomial time (Ford and Fulkerson 2010). The difference is that, in our model, even

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[1]It is defined later as k connected graph.

an edge is labeled with a transaction threshold, the trading volume can still be 0; while in the maximum flow model, this volume has to be above the minimum flow in any feasible solution.

Erfani et al (2014) build a model for water market in England via optimization. Our work is significantly different from that because of local policies. In addition, our work is quite involved in computation issues. Water right market design has in fact been considered in the multiagent system literature as well (Giret et al. 2011; Almajano et al. 2012). They consider the problem of computer assistance system, rather than from an optimization lens.

Preliminaries

A water right market can be modeled by a directed bipartite graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \mathcal{V}_\mathcal{B} \cup \mathcal{V}_\mathcal{S}$. $\mathcal{V}_\mathcal{B}$ denotes the set of buyers, while $\mathcal{V}_\mathcal{S}$ denotes the set of sellers. Agent v_i has a price $p_i \in R^+$ for each unit of water and the maximum amount of water $q_i \in R^+$ it demands/supplies. Each directed arc $v_i v_j \in \mathcal{E}$ denotes that seller v_i can sell water right to buyer v_j. v_i and v_j are connected if and only if three properties must hold in reality: (1) v_i and v_j are not distant and familiar with each other; (2) v_i is a seller and v_j is a buyer; (3) $p_i < p_j$.

The first constraint is particularly interesting as it distinguishes our market from a common marketplace and endows our market with graph properties.

For each arc $v_i v_j$, there is a minimum amount of transaction volume th_{ij} imposed, if a transaction were to occur between v_i and v_j. This transaction threshold ensures that each transaction covers its cost that is not reflected in the current model.

A trading assignment of the market is a *flow* from sellers to buyers. We denote the trading volume on $v_i v_j$ by f_{ij}. According to local policies, it is always the case that the center shall not specify any trading price but to leave the price bargaining process to the buyer and seller that are involved[2]. After the bargaining phase, a price $p_{ij} \in [p_i, p_j]$ is derived.

We aim to optimize two objectives in this paper: trading volume and social welfare. Trading volume is defined to be

$$VO = \sum_{v_i v_j \in \mathcal{E}} f_{ij} \tag{1}$$

While, social welfare[3] is defined to be

$$
\begin{aligned}
SW &= \sum_{v_i \in \mathcal{V}_\mathcal{S}} \sum_{v_i v_j \in \mathcal{E}} f_{ij} * (p_{ij} - p_i) + \\
&\quad \sum_{v_i \in \mathcal{V}_\mathcal{B}} \sum_{v_j v_i \in \mathcal{E}} f_{ji} * (p_i - p_{ji}) \\
&= \sum_{v_i v_j \in \mathcal{E}} f_{ij} * (p_j - p_i) \tag{2}
\end{aligned}
$$

For an instance in which the price differences between any buyer and any seller are the same, social welfare is reduced

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[2]the price is even fixed in some markets since 2013. However, we also care about the case without this policy.

[3]The same concept also appears in related work like (Brooks and Harris 2008)

to trading volume. Therefore, maximizing social welfare is a harder problem than maximizing trading volume.

To sum up, a feasible trading assignment satisfies the following constraints:

- Transaction threshold constraints: $\forall v_i v_j \in \mathcal{E}$, if $f_{ij} > 0$, then $f_{ij} \geq th_{ij}$.

- Supply/demand feasibility constraints: $\forall v_i \in \mathcal{V}_\mathcal{S}$, $\sum_{v_i v_j \in \mathcal{E}} f_{ij} \leq q_i$ and $\forall v_i \in \mathcal{V}_\mathcal{B}$, $\sum_{v_j v_i \in \mathcal{E}} f_{ji} \leq q_i$

Remarks: In reality, the marginal value of a single unit water may change along with the amount of water they own, such as piecewise linear. This can be solved by adding some constraints to our model and won't affect our theoretical results. As the curve of the valuation for single unit water is hard to estimate, we only consider the case of fixed marginal valuation.

Water right trading assignments

In this section, we investigate the computation problems for the two objectives: trading volume and social welfare. Denote the problem of maximizing trading volume by MAX-VOLUME and the other by MAX-WELFARE.

Maximizing total trading volume

As mentioned before, MAX-VOLUME can be regarded as a special case of MAX-WELFARE. So, the NP-HARDNESS of MAX-VOLUME implies the NP-HARDNESS of MAX-WELFARE.

In this subsection, we prove the decision version of MAX-VOLUME is NP-COMPLETE, that is, deciding whether the total trading volume can reach a fixed value X. We do this by a reduction from the well-known NP-COMPLETE problem PARTITION

Definition 1. *Given a set of n objects $O = \{o_1, o_2, \ldots o_n\}$, the weight of o_i is w_i. The PARTITION PROBLEM is to decide whether there is a subset S of O satisfying $\sum_{o_i \in S} w_i = \sum_{o_i \in O \setminus S} w_i$. It is a well-known NP-COMPLETE problem.*

Theorem 1. *Deciding whether the total trading volume can reach a fixed value X is NP-COMPLETE.*

Proof. Given a solution, we can easily verify whether the total trading volume reaches x. So, this problem is in NP.

Given an instance of PARTITION PROBLEM, $O = \{o_1, o_2, \ldots o_n\}$, where o_i has weight w_i. We construct an instance of water market as follows. We set $X = \sum_{o_i \in O} w_i$. The graph of the water market is $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. $\mathcal{V}$ contains n sellers $v_1, v_2, \ldots, v_n$ and two buyers v_{n+1} and v_{n+2} For each seller $v_i (i = 1, 2, \ldots, n)$, $p_i = 0$, $q_i = w_i$. For each buyer $v_i (i = n + 1, n + 2)$, $p_i = 1$, $q_i = X/2$. $\forall i = 1, 2, \ldots, n, \forall j = n+1, n+2, v_i v_j \in \mathcal{E}$ with $th_{ij} = q_i$.

We claim that the total trading volume can reach X if and only if the PARTITION PROBLEM has a feasible partition. Given a feasible trading assignment, for $i = 1, 2, \ldots, n$, $f_{i,n+1} > 0$ if and only if we put o_i into S. In this way, we have constructed a feasible partition. Given a feasible partition, for $i = 1, 2, \ldots n$, if $o_i \in S$, set $f_{i,n+1} = q_i$, $f_{i,n+2} = 0$; else set $f_{i,n+2} = q_i$, $f_{i,n+1} = 0$. In this way,

we obtain a feasible trading assignments. Both directions then follow directly. So a feasible trading assignment corresponds to a feasible partition. Therefore, we have proved the theorem. □

Inapproximability

We have known that MAX-WELFARE is NP-HARD. In addition, we show that MAX-WELFARE is not even approximative with $o(1)$ error ratio in polynomial time. We need to introduce a complexity class called MAX SNP (Papadimitriou and Yannakakis 1991). It is known that MAX SNP-COMPLETE problems do not have polynomial algorithm to achieve an approximation ratio[4] more than g (unless P=NP), where g is a constant less than 1.

Papadimitriou and Yannakakis (1991) list 10 known problems that are MAX SNP-COMPLETE, including MAX CUT, MAX 3-SAT and MAX 2-SAT.

Theorem 2. MAX-WELFARE *is* MAX SNP-HARD.

Proof. Our proof is based on the fact that MAX IS-3 (maximum independent set with degree of each vertex bounded by 3) is a MAX SNP-COMPLETE problem (Berman and Fujito 1995).

Given an instance I of MAX IS-3 on graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, we construct an instance I' of water right assignment problem as follows. Without loss of generality, $\mathcal{G}$ can be considered connected. $\mathcal{G}' = (\mathcal{V}', \mathcal{E}')$ denotes the graph of I'. For each vertex $v_i \in \mathcal{V}$, seller a_i and buyer b_i are in $\mathcal{V}'$, $q_{a_i} = q_{b_i} = |\mathcal{V}|$, $p_{a_i} = 0$, $p_{b_i} = 1/|\mathcal{V}|$ $a_i b_i \in \mathcal{E}'$ with $th_{a_i b_i} = |\mathcal{V}|$. For each edge $v_i v_j \in \mathcal{E}$, $e_{ij} \in \mathcal{V}'$ denotes a buyer in the graph with $p_{e_{ij}} = q_{e_{ij}} = 1$. $a_i e_{ij}, a_j e_{ij} \in \mathcal{E}'$, $th_{a_i e_{ij}} = th_{a_j e_{ij}} = 1$. We have the maximum social welfare $OPT(I') = |\mathcal{E}| + OPT(I)$, where $OPT(I)$ denotes the optimal solution for the MAX IS-3 instance, $OPT(I')$ denotes the optimal social welfare in the water right assignment problem. In fact our instance is constructed for the vertex cover problem. Given the optimal solution for I', we have that there is an optimal solution where all e_{ij}'s are assigned with sellers. In this specific solution, If a_i is assigned to b_i, it indicates v_i is not in the minimum vertex cover of $\mathcal{G}$, thus in the maximum independent set.

To finish this proof, we need the so-called L-reduction[5]. Specifically for our problem, c denotes a solution for I, which can be constructed from a solution c' of I' in polynomial time. If we can prove $OPT(I') \leq \alpha OPT(I)$ and $OPT(I) - c \leq \beta(OPT(I') - c')$ (α and β are positive constants), it implies our problem is harder to be approximated.

On the one hand, We notice that $|\mathcal{V}'| = |\mathcal{V}| * 2 + |\mathcal{E}| \leq |\mathcal{V}| * 2 + |\mathcal{V}| * 3/2 = 3.5|\mathcal{V}|$. The size of a maximum independent set is at least $|\mathcal{V}|/4$. We have $OPT(I') \leq 3.5|\mathcal{E}| \leq 14 OPT(I)$.

On the other hand, we claim $OPT(I) - c \leq OPT(I') - c'$. Given a solution s' of I' with social welfare c', we can construct a solution s for I with cost c as follows. (1) for every unassigned e_{ij}, force seller a_i to sells to an amount e_{ij}. This step will not decrease social welfare. (2) if a_i and

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[4]the cost of a solution divided by the cost of an optimal solution
[5]described in (Papadimitriou and Yannakakis 1991)

b_i are assigned to trade, put v_i into the independent set. In this way, we construct a feasible solution for the MAX IS-3 problem. If step (1) is not performed, $OPT(I) - c = OPT(I') - c'$. Since step (1) will not increase $OPT(I') - c'$, so $OPT(I) - c \leq OPT(I') - c'$ holds.

So, max-welfare is MAX SNP-HARD. $\qquad\square$

A mixed integer linear program for MAX-WELFARE

We have already known that MAX-WELFARE is NP-HARD (because of the NP-HARDNESS of MAX-VOLUME) and cannot be approximated efficiently. However, this difficulty can be resolved, to a certain degree, by a concise formulation of mixed integer linear programming (MIP). We will show in the experiment section that, with MIP solvers such as CPLEX [6], we can scale this problem to considerably large instances.

A key observation here is that the flow on any edge is either 0 or no less than the transaction threshold. We can then use a 0/1 indicator x_{ij} to denote the flow be 0 or no less than the transaction threshold. Let f_{ij} denote the trading volume on the arc $v_i v_j$.

Optimization 1

Maximize
$$\sum_{v_i v_j \in \mathcal{E}} f_{ij} * (p_j - p_i)$$

Subject to
$$f_{ij} \geq th_{ij} * x_{ij}, \forall v_i v_j \in \mathcal{E}$$
$$f_{ij} \leq q_j * x_{ij}, \forall v_i v_j \in \mathcal{E}$$
$$\sum_{v_i v_j \in \mathcal{E}} f_{ij} \leq q_i, \forall v_i \in \mathcal{V}_\mathcal{S}$$
$$\sum_{v_i v_j \in \mathcal{E}} f_{ij} \leq q_j, \forall v_i \in \mathcal{V}_\mathcal{B}$$

Theorem 3. *Optimization 1 always returns the solution of* MAX-WELFARE

Intuitively, when we focus on an edge $v_i v_j$ independently, it needs to satisfy the first two constraints. x_{ij} can only be 0 or 1. When $x_{ij} = 0$, the first two constraints become $0 \leq f_{ij} \leq 0$, indicating $f_{ij} = 0$, denoting the case where v_i and v_j don't trade. When $x_{ij} = 1$, the first two constraints become $th_{ij} \leq f_{ij} \leq q_j$. From the last constraint and the range of f_{ij}, $f_{ij} \leq q_j$ can always be satisfied. $th_{ij} \leq f_{ij}$ described the transaction threshold. So, we found when $x_{ij} = 1$, the first two constraints require exactly satisfying the transaction threshold. In one word, $x_{ij} = 0$ covers the case where v_i and v_j don't trade with each other, while $x_{ij} = 1$ covers the case then they trade. x_{ij} is independent of all the others and represents two separated intervals.

Lines and cycles

As shown above, the computational problem for our model is NP-HARD and even MAX SNP-HARD. We are curious to

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explore the complexities on special graphs, such as lines, cycles, trees and some other sparse graphs. These results can also be helpful to markets with geographic constraints and interesting as theoretical problems. First, we show MAX-WELFARE is polynomial for line graphs and cycle graphs.

Definition 2. *A line is a graph* $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, *where* $\mathcal{V} = \{v_1, v_2, \ldots, v_n\}$, $v_i v_j \in \mathcal{E}$ *only if* $|(i - j)| = 1$.

Definition 3. *A cycle is a graph* $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, *where* $\mathcal{V} = \{v_1, v_2, \ldots, v_n\}$, $v_i v_j \in \mathcal{E}$ *only if* $|(i - j)| = 1 \bmod n$.

Algorithm 1 provides a polynomial algorithm for solving MAX-WELFARE on a line graph. In a line, if two neighbors do not conduct a trade, the graph can be divided into two independent lines. If two neighbors conduct a trade, the transaction threshold constraints take effect. Given that all the transaction thresholds on a line graph take effect, we know that we can solve the MAX-WELFARE via linear programming (Optimization 2). So if we divide the line graph properly into several small line graphs, this problem can be solved efficiently. With the analysis above, we show that dynamic programming can be implemented to solve this case.

Optimization 2

Maximize
$$\sum_{v_i v_j \in \mathcal{E}} f_{ij} * (p_j - p_i)$$

Subject to
$$f_{ij} \geq th_{ij}, \forall v_i v_j \in \mathcal{E}$$
$$\sum_{v_i v_j \in \mathcal{E}} f_{ij} \leq q_i, \forall v_i \in \mathcal{V}_\mathcal{S}$$
$$\sum_{v_i v_j \in \mathcal{E}'} f_{ij} \leq q_j, \forall v_j \in \mathcal{V}_\mathcal{B}$$

Based on Optimization 2 and Algorithm 1, we can further compute MAX-WELFARE for a cycle. The idea is as follows. (1) If each pair of neighbors conduct trade with each other, the maximum social welfare can be solved by linear programming. (2) If there exists two neighbors who do not conduct trade with each other, the graph is reduced to a line graph.

Tree graph and k-connected graphs

We finally consider some simple cases where MAX-WELFARE is NP-HARD. Surprisingly, we prove that even if the graph is a binary tree or the maximum degree is as small as 4, MAX-WELFARE can still be NP-HARD. We formally state the two NP-HARD results as follows. Both proofs are non-trivial but omitted because of the page limit.

Theorem 4. *For general binary tree graphs, deciding whether social welfare can reach X is an* NP-COMPLETE *problem.*

Proof. Given a solution, we can easily verify whether the total social welfare on a tree reaches X. So, this problem is an NP problem.

Given an instance I_1 of PARTITION PROBLEM, $O = \{o_1, o_2, \ldots, o_n\}$, o_i's weight is w_i. We construct an instance of water market to solve it as follows. Without loss of generality, we assume $n = 4^k$, k is an integer. We construct

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Algorithm 1 Dynamic programming algorithm for MAX-WELFARE on a line graph

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Input: Given a line graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, $\forall v_i$, price p_i, demand/supply q_i. $\forall v_i v_j \in \mathcal{E}$, transaction threshold th_{ij}.

Output: maximum social welfare and the corresponding flow assignments.

1: In this algorithm, We call Optimization 2 via $solve(input(i,j))$, where $input(i,j)$ denotes all the inputs on the line interval between v_i and v_j.
2: $ans(-1) \leftarrow 0, ans(0) \leftarrow 0$, $ans(i)$ denotes the maximum social welfare on the subgraph between v_1 and v_i.
3: $F(-1) \leftarrow \{\}, F(0) \leftarrow \{\}$, $F(i)$ denotes the trading volume assignment corresponding $ans(i)$
4: **for** $i \leftarrow 1, 2, \ldots, n$ **do**
5:    $ans(i) \leftarrow 0, F(i) \leftarrow \{\}$
6:    **for** $j \leftarrow 0, 1, \ldots, i$ **do**
7:       $tempans, tempF \leftarrow solve(input(j,i))$
8:       **if** $ans(i) < tempans + ans(j-1)$ **then**
9:          $ans(i) \leftarrow tempans + ans(j-1)$
10:          $F(i) \leftarrow F(j-1) \bigcup tempF$
11:       **end if**
12:    **end for**
13: **end for**
14: Output $ans(n)$ as maximum social welfare, $F(n)$ as the corresponding assignments.

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a complete binary tree with n leafs. Obviously, the height of this tree is an odd number $2k + 1$, with the same k as before. All the agents with an odd height are sellers, while the others are buyers. In our tree, leafs and root have special properties. For all the others, they satisfy: (1) the quantity of a vertex v_i is $\sum_{l_j \in descendant(v_i)} w_j$, where $descendant(v_i)$ is the set of v_i's descendants; (2)the price of a seller is 0, while the price of a buyer is 2. For a leaf l_i, $q_{l_i} = w_i$, $p_{l_i} = 1$, the transaction threshold between l_i and its father is w_i. All the other arcs' transaction thresholds are 0. We define $S = \sum_{o_i \in O} w_i$, which is the total weight of O. For the root r, $p_r = 0$, $q_r = S/2$.

Let $X = (2k - \frac{1}{2})S$. Social welfare can reach X if and only if I_1 has a solution. Buyers can buy at most kS, the seller other than leafs can sell at most $(k - 0.5)S$. Each unit traded between them brings 2 unit welfare. Each unit sold by leafs can bring 1 unit. So the maximum possible social welfare is X, which can only be reached when leafs sell exactly $S/2$ units.

Thus, this problem is NP-COMPLETE. □

Definition 4. *We call a bipartite graph k-connected graph, if*

1. all the vertices are allocated on a line,

2. the distance between two adjacent vertices is 1,

3. there can be an arc between two vertices only if the distance between the two vertices is no more than k.

Theorem 5. *For $k \geq 2$, deciding whether social welfare can reach Y on a k-connected graph is* NP-COMPLETE.

The proof for Theorem 5 can be found in the full version in the supplemental materials. It is proved by a reduction from PARTITION PROBLEM. The rough idea is to construct two kinds of sellers with different properties such that the maximum revenue can only be reached when the answer of PARTITION PROBLEM is true.

Experimental evaluation

In this section, we develop a data generator based on historical data. Then, we characterize a model to simulate the current assignment algorithm as our baseline. Finally, we compare the efficiency and scalability among our MIP algorithm, our modified LP algorithm and the currently utilized algorithm.

Data generator

As the reported supplies/demands are not well maintained during the preceding years, we can only make use of the conducted trades during these years. We make use of the fact that about 80 percent of all the trades are between two villages within 10 kilometers. For simplification, we assume two villages with distance smaller than 10 km are compatible. We use the actual locations of villages in the real-world map of Xiying irrigated area, one of the irrigated areas at Shiyang River Basin. Each time, one agent appears at the position of one village in Xiying irrigated area and its supply or demand is generated according to the historical distribution. The thresholds are set to be one third of one's supply or demand. The valuations (asks and bids) on each unit of water are uniformly distributed between 0.15 and 0.25 yuan per m^3.

Additional algorithms used in experiments

We introduce two additional algorithms for experiments: (1) The greedy algorithm currently in use. (2) a *modified LP* algorithm based on Optimization 2.

The greedy algorithm is presented as follows.[7] We suppose that villages come to the market in a random order. When an agent x comes to the market, the conductor looks up the current market for one or more partners. If someone can trade with x, the conductor recommends them to trade with each other. If more than one agent in the market can trade with x, let them trade with x one by one until x is no longer able to trade with anyone else in the market. The volume of a trade in this algorithm is always the minimum of the left supply of the seller and the left demand of the buyer.

The modified LP algorithm composes of two stage. First, without considering thresholds, run the Optimization 2 on the graph with th_{ij}'s set to be 0. Second, remove all the trades whose volumes are below thresholds from the output of LP. Finally, conduct the remaining trades.

We implement both algorithms and compare their performances to the MIP algorithm.

Experimental results

All our experiments are conducted on a desktop with a quad-core CPU and 4G RAM. We run our algorithm on generated

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[7]We cannot give a formal version due to page limit.

graphs with different number of agents (asks and bids) and compare it with the simulated greedy algorithm. Figure 1 compares the social welfare yields by the three algorithms (the greedy algorithm, the modified LP and the MIP) and Figure 2 compares the volumes.

From the results, we get the following observations and conclusion:

- The social welfares or volumes of MIP and modified LP are close.

- The greedy algorithm performs badly on social welfare and the gap becomes larger with the increasing of market size.

- When the number of agents is more than 100, the greedy algorithm gets good results on trading volume but poor results on social welfare.

Currently, only very few trades occur in each period in the same irrigated area, which is often less than 50. So the greedy algorithm does not loss much in social welfare. An intuitive reason for the good performance of the greedy algorithm is that only few pairs can be matched and the results of the greedy algorithm are not very far from the optimal. However, the water right market in China is rapidly growing. If allowing each village to report orders at different prices, the number of agents[8] in our model will increase dramatically. After the number of bids has increased, the greedy algorithm will induce significant welfare loss. So it is important to replace it with our optimized algorithm.

Take the possibility of permitting multi-bid for single agent into consideration, the number of agents in our model can be quite large. So we test the time scalability of our MIP. The results shown in figure 3 indicates that MIP performs well on a graph with no more than 600 nodes. However, modified LP can handle graphs with 2000 nodes easily.

To sum up, (1) when the market has less than 50 few agents, the greedy algorithm won't cause much loss but MIP is surely better; (2) when the number of agents is no more than 700, MIP yields optimal solution; (3) when the number of agents is greater than 700, modified LP performs good on computation and social welfare.

Conclusion

In this paper, We investigate China's village-level water right market from both theoretical and experimental perspectives.

For the theoretical part, we show most related computational problems are NP-HARD, or cannot be efficiently approximated. We use MIP to solve the most general case and propose polynomial time algorithms when the underlying graphs are lines or cycles.

For the experimental part, our results show that when the market is large Our MIP algorithm can play a significant role in increasing social welfare. At the same time, though our algorithm is NP-HARD in theory, it performs quite well even on a graph with hundreds of nodes – which is important, because the more intuitive greedy algorithm that might

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[8]In our model, one agent only proposes one bid or one ask. So, a village may corresponds to multiple agents in our model.
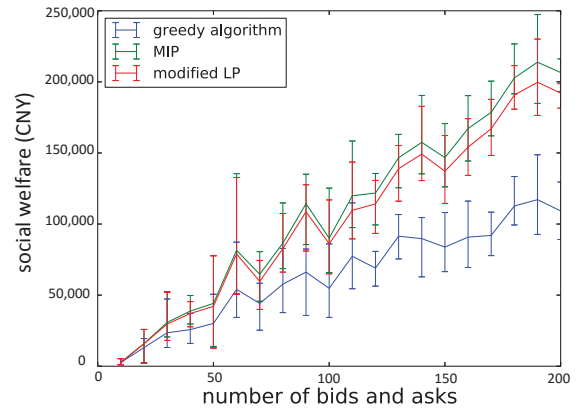

Figure 1: Comparison betweem the social welfares of optimal MIP, modified LP and the greedy algorithm. The lines connect medians and the best/worst cases are labelled.
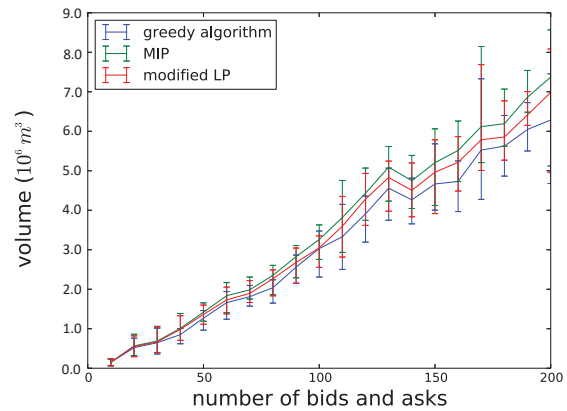

Figure 2: Comparison betweem the trading volumes of optimal MIP, modified LP and the greedy algorithm.
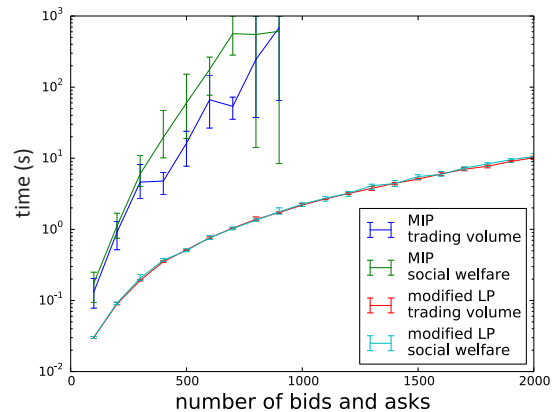

Figure 3: Running time of MIP and modified LP on markets with different sizes in log range. Memory-limit-exceeded instances are labeled as 1000 seconds.

develop "naturally" in a market results in a less thick market and, perhaps more critically, significant decreases in overall social welfare compared to our optimization-based algorithm. When the graph has thousands of agents, modified LP is a better trade off.

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