Path-Loss Fluctuations Towards Robust Scheduling Algorithms in the SINR Model

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***Abstract*—The SINR model has attracted much attention in the field of wireless networks. The path loss exponent α in the model is generally treated as a constant between two and six. However, in real scenarios, the path loss is influenced by many factors such as environment (vegetation and barriers), propagation medium (dry or moist air), the distance between the transmitter and the receiver, etc. Therefore, the exact value of α is hard to detect in real scenarios and the attenuation of signal powers transmitted through different areas varies, which causes the value of α to ebb and flow among all wireless requests. In this paper, we initiate the study about the impact of α that fluctuates on the SINR model. We prove that for any given α and the fluctuation δ, a specific topology can always be constructed which is extremely vulnerable to the small change in α and all the existing algorithms dealing with wireless network problems perform dramatically poorly with the inaccurate α value. We call algorithms that can still perform well despite the fluctuating α "α-Robust" algorithms and we propose the first α-Robust algorithm for the Connectivity Problem which generates a link schedule with size of $O(\log n \log \Delta)$ even in the worst case, where Δ is the ratio between the longest and the shortest links in a nearest neighbor tree.**

***Keywords*-SINR model; fluctuation; α-Robust; connectivity.**

I. Introduction

Ever since it was first discovered that radio waves could be used to send telegraph messages in the 1800s, using radio for communication purposes have been adopted by more and more people and developed by many intellects. Nowadays, there are millions of cell phone subscribers worldwide. In addition to new types of wireless devices, wireless sensor networks, have been attracting vast amount of attention and have been providing an easy approach to monitor the environment, detect physical events and so on.

Recently, the Wireless Links Scheduling (WLS) Problem in wireless networks has attracted many researchers due to its more realistic communication model, particularly the SINR model (also called physical model, defined in [8]), which describes the ratio between the desired signal strength and all other signal strengths plus ambient noise. Only when this ratio is above some threshold β, the receiver can decode the signal successfully. In the early days of wireless sensor network research, multi-hop wireless networks were modeled as graphs. The nodes of this communication graph represent the physical devices, two nodes being connected by an edge if and only if the respective devices are within mutual transmission range. In this graph-based model, a node is assumed to receive a message correctly if and only if no other node in physical proximity transmits at the same time. It is foreseeable that in graph theory, interference-free concurrent transmissions just boil down to solving variants of coloring or independent set problems. Compared with the tremendously simplified graph-theoretic model, the SINR model is a more accurate description of reality. The advantage and robustness of the SINR model are analyzed in [13], [17], which mainly focus on the effect of the SINR threshold. In technical, the threshold β has been discussed to show the robustness of the SINR model in [12]. However, in real scenarios, the exact value of α is harder than that of β to find; in addition the attenuation of signal powers transmitted through different areas varies, which may cause the value of path loss exponent α to ebb and flow among all wireless requests. There is almost no previous work considering this real factor and its influence. Thus, we initiate the study of robustness of the physical model with fluctuating α value in this paper.

In order to best investigate the fundamental possibilities and limitations of robustness in view of fluctuations and uncertainty of the path-loss exponent, we consider one of the fundamental scheduling problems in wireless networks, the connectivity problem: given some deployment of nodes in the plane, the goal is to compute the power assignment and time slot to transmit for all the nodes that forms a connected communication structure spanning all nodes without violating the SINR model. This problem, while simple enough to lend itself to concise theoretical analysis and lower bound proofs, is nevertheless a key building block for other more complex and practically more important problems. One example, the maximum capacity problem, which can be solved by our α-Robust algorithm with several small modifications.

Our contributions are summarized as follows: We explore how the path loss fluctuation will affect the results in wireless networks and enquire robust algorithm design if the impact is indeed large. In this paper, we discuss the α-sensitivity of the SINR model, concluding the dramatic impact of a fluctuating α such that no two links can transmit concurrently for the topology we construct. Taking one of the simplest problems: connectivity problem (i.e., minimize the amount of time slots required until links in a connected wireless network can be realized) as an example, we give an α-Robust algorithm that finds a schedule which completes all transmissions in $O(\log n \log \Delta)$ time slots, where Δ is the ratio between the longest link length and the shortest link length in a nearest neighbor tree of the node deployment.

II. Related Works

A. Power Assignment in the SINR Model

In the early theoretical research, the scheduling signals in wireless networks resort to protocol model (e.g., [1], [20]). Two nodes in the network are connected by an edge in a communication graph if and only if they are in each other's transmission range. So the transmitting power level of any node is characterized as the radius of this communication range. However, this modeling approach ignores the fact that radio signal and interference do not have the exact boundary of influence.

Later the physical (SINR) model is well-accepted in the networking and engineering community, which assumes that the strength of a signal attenuates with the increasing distance from the source. Since then, the *power assignment* has become a significant subproblem of the wireless scheduling problem. At first, most literature focuses on the *uniform power assignment*, in which all the nodes transmitted at the same power level (e.g., [9], [19], [21]). In some other studies, the very intuitive *linear power assignment* is adopted, in which the power level is chosen proportional to the transmission distance or the path loss (transmission distance to the power of the path loss exponent). As an example, Fanghänel et al. [2] proposed the *square root power assignment* (one kind of the *oblivious power assignment* they defined) setting the power level for a transmission equal to the square root of the transmission distance. Different from the oblivious power assignment, Moscibroda and Wattenhofer [17] presented the first analysis of the directed interference scheduling problem using a non-oblivious power assignment. They discussed the problem of how many time slots are needed to schedule a set of wireless communication requests ensuring connectivity among n points placed arbitrarily in two-dimensional Euclidean space. They show that there are scheduling instances that either using uniform or linear power assignments results in $\Theta(n)$ scheduling complexity. However, all of these methods assume that the path loss exponent is a constant and all the signal power attenuation is the same for any point in the network area. The path loss fluctuation is not taken into consideration and the ideas behind these power schemes can not be fully realized with this fluctuation.

B. Wireless Scheduling Problems

Ever since the SINR model has been adopted in the community and scheduling complexity was first defined by Moscibroda et al [17] in 2006, a mass of studies on the WLS Problem have appeared (see, e.g., [3], [6], [7], [11], [14], [18].) The first algorithm for the Connectivity Problem achieves $O(\log^4 n)$ scheduling complexity in [17], which is improved to $O(\log^2 n)$ later in [16]. Later, Gu et al. [7] improved this scheduling complexity to $O(\log n)$. Gradually deepening the study on scheduling problem, more interesting and fancy works are done. Halldórsson et al prove that One-Shot scheduling with uniform power assignment is APX in [13]. After that, a breakthrough is achieved by Thomas Kesselheim et al. in [14] giving the first constant approximation algorithm with power control. Very recently, based on Kesselheim's work, Magnús M. Halldórsson and Pradipta Mitra proposed an $O(\log n)$ algorithm to achieve connectivity in [10]. Apart from this problem, the capacity problem tries to find the maximum feasible subset of a given set of links is another fundamental one which attracts many researchers' attention. This problem is proved to be NP-Complete in [19] by Olga Goussevskaia et al. in conjuction with an approximation algorithm that grows linearly with the network size in the worst case. Katz, Vöker and Wagner extended the NP-hardness proof in [22]. They also established the first rigorous result that guarantee a constant approximation when finding the Maximum Independent Set, i.e. the largest set of links that can be scheduled concurrently in one time slot in [6]. Some other results such as constant approximation for capacity using oblivious power assignments is given in [11]. Some other problems have also been extended, such as topology control, routing and network coding in [4], [5], [18].

To the best of our knowledge, there is no previous studies on scheduling problem with path loss fluctuation under the SINR model. This paper is the first attempt to analyze the influence of this exponent in the SINR model and to propose the "α-Robust" scheduling algorithm. We believe that the theoretical scheduling algorithms can have more impact on the real MAC layer protocol design in real scenario if the path loss is described more accurately and treated carefully.

III. Model and Notations

We are given a set of nodes $X = \{x_1, x_2, \ldots, x_n\}$ located arbitrarily in the Euclidean plane. We divide time into time slots, defined to be the unit of time required to transmit once for any link. All nodes can be both a sender and receiver, but only in different time slots. The distance between any two nodes x_i, x_j is denoted by $d(x_i, x_j)$. Each edge $l_{ij} = (x_i, x_j)$ represents a communication request from a sender v_i to a receiver v_j. The length of link l_{ij} is denoted by $d_{ij} = d(x_i, x_j)$, where $d_{gj} = d(x_g, x_j)$ designates the distance between the sender of link l_{gh} and receiver of link l_{ij}.

Formally, the SINR model is defined as follows. The signal power $P_i(j)$ received at x_j from sender x_i depends on the transmission power P_{ij} of x_i and distance d_{ij}. The path loss radio propagation model for the reception of signals says the signal strength that x_j receives degrades at $d_{ij}^{-\alpha}$, where α denotes the path-loss exponent, i.e. $P_i(j) = P_{ij}/d_{ij}^{\alpha}$. In previous works, α is usually considered as a constant between two and six in previous works involved with the SINR model. In this paper, we adopt the definition of inaccurate α, which means the exact path loss is unknown and transmissions in different areas may be affected by different α valued. We assume the varying range of this exponent is $[\alpha - \delta, \alpha + \delta]$, where δ denotes the fluctuation bound. Every sender x_g (with corresponding receiver x_h) that transmits concurrently with another sender x_i (with corresponding receiver x_j) causes an interference $I_g(j) = P_g(j) = P_{gh}/d_{gj}^{\alpha}$ at receiver x_j.

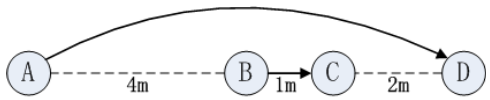

Fig. 1. Transmissions from node A to D and from B to C can simultaneously be scheduled successfully, whereas taking fluctuation into consideration, they will conflict.

All interferences accumulate. The total interference $I(x_j)$ experienced by receiver j is given as the sum of all interferences caused by other concurrently sending nodes, i.e. $I(x_j) = \sum_{l_{gh} \neq l_{ij}} I_g(j)$. A receiver x_j receives a message from its sender x_i successfully if and only if it suits the following SINR constraint:

$$SINR_S(x_j) = \frac{P_i(j)}{\sum_{l_{gh} \in S \backslash l_{ij}} I_g(j) + N} \geq \beta$$

where N is the ambient noise, $\beta \geq 1$ denotes the minimum SINR (Signal-to-interference-plus-noise-ratio) required for a message to be successfully received, and S is the set of concurrently transmitting links.

In this paper, we focus on the connectivity problem in wireless networks: Given some deployment of nodes X in the plane, what is the minimal amount of time required until a connected graph is realized based on the constructed links of these nodes, which describes the theoretically achievable efficiency of MAC layer protocols. Simply speaking, just like the capacity of a wireless network which expresses the maximum amount of information that can be transmitted through the network, the scheduling complexity indicates the minimum amount of time required to finish transmitting over a given set of communication links (these links should make all the nodes are connected and all the transmissions are successful without violating the SINR constraint, with the path loss exponent varying between $[\alpha - \delta, \alpha + \delta]$).

IV. α-SENSITIVITY OF THE SINR MODEL

In order to exemplify the impaction of fluctuating α, consider the simple four nodes deployment depicted in Fig. 1. As indicated by the arrows, node A wants to transmit to node D and node B wants to transmit to node C. Assume $\alpha = 3, \beta = 3$ and the ambient noise $N = 0.01\mu W$. If no fluctuation exists, the two transmissions can be scheduled simultaneously by setting power $P(A) = 1.26mW, P(B) = 31.6\mu W$ and the SINR values at receivers C and D can be computed as:

$$SINR_C = \frac{31.6\mu W/(1m)^3}{0.01\mu W + (1.26mW/(5m)^3)} \approx 3.13 > \beta$$

$$SINR_D = \frac{1.26mW/(7m)^3}{0.01\mu W + (31.6\mu W/(3m)^3)} \approx 3.11 > \beta$$

However, it's hard to measure the precise α value in practical situation. When a small fluctuation exists (such as $\alpha = 3.1$), can the two transmissions still be successful with no foresee of the fluctuation? The answer is NO because the SINR value at the receiver D is:

$$SINR_D = \frac{1.26mW/(7m)^{3.1}}{0.01\mu W + (31.6\mu W/(3m)^{3.1})} \approx 2.86 < \beta$$

Small fluctuation may cause simultaneous transmissions conflict even we take the fluctuations for different links the same as the example above. It's hard to image how badly it can be if the fluctuations are different for all transmissions. This is why we present α-Sensitivity of the SINR model in this section, which points out the deficiency of existing power assignment schemes that fail to take into account the inaccuracy of the path loss exponent and it demands robust algorithms against the fluctuant parameter from the algorithm designer.

Theorem 1: For any given α and $\delta > 0$, there exists a topology in which any two simultaneous transmissions using P_i and P_j cannot be both successful with two different path loss exponents α and $\alpha - \delta$ for any power assignment P_i, P_j, where $\alpha, \alpha - \delta \in (2, 6]$.

This theorem claims all the existing algorithms under the SINR model without involving the fluctuation will crash in some worse case topology. Robust algorithm design becomes so important in this field and we will present the first robust algorithm to the famous Connectivity Problem in the next section.

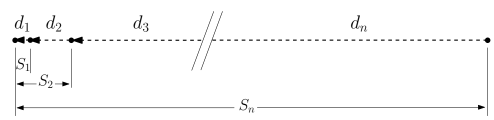

Fig. 2. Example of inductively constructed chain.

Proof: Consider the chain constructed in Fig. 2, all nodes are placed on a straight line with "super" exponentially increasing distances between any successive two. (Here, by "super" exponentially increasing, we mean that distances between nodes in this line grow much faster than the common exponential chain.) These nodes compose the tree topology by connecting the nearest neighbor (as the left arrow illustrated in Fig. 2) and the left-most one is the leaf.

The lengths of these links $l_1, l_2, \ldots, l_n$ in the topology are denoted by $d_1, d_2, \ldots, d_n$ in Fig. 2 . The following equations hold for all d_i:

$$d_1 = 1$$

$$S_i = \sum_{k=1}^{i} d_i$$

$$d_i^\alpha = S_{i-1}^\alpha \cdot (d_i + S_{i-1})^{\alpha - \delta} \tag{1}$$

First of all, we verify the existence of such an inductively constructed topology.

Lemma 4.1: In the inductive step, d_i defined in Eq. (1) satisfies $d_i > 0$ and $d_i \geq d_{i-1}$, i.e. the constructed topology is actually a nearest neighbor tree as depicted.

Proof: Define a constant C s.t.

$$C^{\alpha} = (C+1)^{\alpha-\delta}$$

Since α and δ are given before the construction, it is obvious that such a constant C exists and $C > 1$. Actually we can find that $C = d_2$.

Define function $f(x) = x^{\alpha} - S_{i-1}^{\alpha} \cdot (x + S_{i-1})^{\alpha-\delta}$. It's easy to verify $f(d_{i-1}) < 0$ based on the observations $S_i > d_i$ and $d_i > 1$ $(i = 2, 3, \dots)$. Consider the following two cases:

1. If $\alpha \geq \delta$, then

$$\begin{aligned} f(C \cdot S_{i-1}^{\frac{\alpha}{\delta}}) &= C^{\alpha} \cdot S_{i-1}^{\frac{\alpha^2}{\delta}} - S_{i-1}^{\alpha}(C \cdot S_{i-1}^{\frac{\alpha}{\delta}} + S_{i-1})^{\alpha-\delta} \\ &\geq C^{\alpha} \cdot S_{i-1}^{\frac{\alpha^2}{\delta}} - (C+1)^{\alpha-\delta} \cdot S_{i-1}^{(\frac{\alpha}{\delta}(\alpha-\delta)+\alpha)} = 0 \end{aligned}$$

2. If $\alpha < \delta$, then

$$\begin{aligned} f(C \cdot S_{i-1}^{2-\frac{\delta}{\alpha}}) &= C^{\alpha} \cdot S_{i-1}^{(2-\frac{\delta}{\alpha})\alpha} - S_{i-1}^{\alpha}(C \cdot S_{i-1}^{2-\frac{\delta}{\alpha}} + S_{n-1})^{\alpha-\delta} \\ &\geq C^{\alpha} \cdot S_{i-1}^{(2-\frac{\delta}{\alpha})\alpha} - (C+1)^{\alpha-\delta} \cdot S_{i-1}^{2\alpha-\delta} = 0 \end{aligned}$$

Thus there exists some value $x^* = C \cdot S_{i-1}^{\frac{\alpha}{\delta}} > d_{i-1}$ or $x^* = C \cdot S_{i-1}^{2-\frac{\delta}{\alpha}} > d_{i-1}$ such that $f(x^*) \geq 0$. Since this function is continuous, there exists some value $d_{i-1} < x \leq x^*$ satisfying $f(x) = 0$, and the solution is d_i. (Note: In most cases, we consider $\alpha > \delta$ where δ is a small constant, so d_i's value is in the range $[d_{i-1}, C \cdot S_{i-1}^{\frac{\alpha}{\delta}}]$). ■

Next, we prove the chain constructed above is the topology we want, i.e. it is extremely vulnerable to δ. Suppose there are two links $l_i, l_j (i < j)$ can be scheduled concurrently with path loss exponent α, which means (without loss of generality, we ignore the ambient noise here):

$$\frac{\frac{P_i}{d_i^{\alpha}}}{\frac{P_j}{d_{ji}^{\alpha}}} \geq \beta \quad \text{and} \quad \frac{\frac{P_j}{d_j^{\alpha}}}{\frac{P_i}{d_{ij}^{\alpha}}} \geq \beta$$

Derived from these, we get the following range:

$$\beta \cdot (\frac{d_i}{d_{ji}})^{\alpha} \leq \frac{P_i}{P_j} \leq \frac{1}{\beta} \cdot (\frac{d_{ij}}{d_j})^{\alpha} \tag{2}$$

Similarly, if we hope that links l_i, l_j can also transmit concurrently under a slightly decreased path loss exponent $\alpha - \delta$, the following condition must be satisfied:

$$\beta \cdot (\frac{d_i}{d_{ji}})^{\alpha-\delta} \leq \frac{P_i}{P_j} \leq \frac{1}{\beta} \cdot (\frac{d_{ij}}{d_j})^{\alpha-\delta} \tag{3}$$

However, the "super" exponentially increasing chain has the following property:

$$\begin{aligned} (\frac{d_{ij}}{d_j})^{\alpha} &= (\frac{S_{j-1} - S_i}{d_j})^{\alpha} < \frac{S_{j-1}^{\alpha}}{S_{j-1}^{\alpha} \cdot (d_j + S_{j-1})^{\alpha-\delta}} \\ &\leq \frac{1}{S_j^{\alpha-\delta}} < \frac{d_i^{\alpha-\delta}}{(S_j - S_{i-1})^{\alpha-\delta}} = (\frac{d_i}{d_{ji}})^{\alpha-\delta} \end{aligned}$$

A reasonable and acceptable assumption is $\beta \geq 1$, so it is obvious that $\frac{1}{\beta} \cdot (\frac{d_{ij}}{d_j})^{\alpha} < \beta \cdot (\frac{d_i}{d_{ji}})^{\alpha-\delta}$, which means Eq. (2) and Eq. (3) are contradict with each other. We call this "sensitive contradiction".

Thus, any P_i and P_j fail to make simultaneous transmissions successful in both cases when path loss exponent is α and $\alpha - \delta$. ■

Remark 4.1: We omit the ambient noise N during the proof to the theorem because N is a fixed constant which can be omitted by scaling to suit the equivalent condition.

Remark 4.2: We consider all the links have the same α value in the construction as a special case that the real α value for each link can vary between $[\alpha-\delta, \alpha+\delta]$. If different links have different α values, the impaction will be much more serious.

This theorem leads to an unexpected deficiency of all existing scheduling algorithms under the SINR model without considering the inaccuracy of α and we generate following corollaries:

Corollary 1: For any scheduling algorithm $\mathcal{A}$ without consideration of path-loss fluctuation, and for any given α and $\delta > 0$, real α values for all links vary between $[\alpha - \delta, \alpha + \delta]$, the schedules generated by $\mathcal{A}$ could fail, and all transmissions could face dramatic, unexpected interference.

Corollary 2: For any scheduling algorithm $\mathcal{A}$ without consideration of path-loss fluctuation, and for any given α and $\delta > 0$, real α values for all links vary between $[\alpha - \delta, \alpha + \delta]$, any power assignment for transmissions generated by $\mathcal{A}$ could lead to an $\Omega(n)$ scheduling complexity.

V. α-ROBUST ALGORITHM

Robust algorithm design becomes important since no existing algorithms for scheduling problems could be stable under inaccurate α value.There are many fundamental problems in wireless sensor networks under the SINR model and we give an improved α-Robust algorithm for the Connectivity Problem.

Definition 5.1: An algorithm $\mathcal{F}$ is defined to be α-Robust to problem P, if the solution (schedule) generated by $\mathcal{F}$ can hold for any $\alpha^* \in [\alpha - \delta, \alpha + \delta]$ under the SINR model. (here α^* can be different for different transmissions even they transmit in the same time slot).

A. Connectivity Problem

Connectivity is the most basic and important property in wireless networks. The goal of the connectivity problem is to minimize the amount of time slots required until a connected structure can be scheduled. This problem has been fully researched in [7], [10], [15], [16], [17]. However, all the existing algorithms can't work when fluctuation exists. Consider the nodes $X = \{x_1, x_2, \cdots x_n\}$ to be located arbitrary in the plane. We say node x_i is connected to x_j if the transmission between (x_i, x_j) is successful. The Connectivity Problem demands how quickly can they form a (weak) connected network under the SINR model? Usually we construct a topology based on the nodes deployment and schedule all the links in as minimum time slots as possible. This method has been used in [7], [16] and the best known result to the problem is $O(\log n)$ time slots ([7], [10]). In this paper, we propose the first α-Robust algorithm based on [7], [16] to achieve $O(\log n \log \Delta)$

scheduling complexity for the connectivity problem, where Δ is the ratio between the largest link length and the smallest link length of the topology constructed. The real α value for each link in the topology varies between $[\alpha-\delta, \alpha+\delta]$ and different links may suffer from different α values.

B. α-Robust Algorithm Design

The algorithm given in this part contains two main steps: topology construction and link scheduling.

Algorithm 1 Topology Construction on node set V

```
Tree := ∅;
while |V| > 1 do
  for each node v_i ∈ V do
    Find v_j ∈ V \ {v_i} minimizing d(v_i, v_j);
    if l_ji ∉ Tree then
      Tree := Tree ∪ {l_ij};
    end if
  end for
  for each link l_ij ∈ Tree do
    V := V \ {v_i};
  end for
end while
return Tree;
```

Algorithm 1 uses the nearest neighbor tree(NNT) method to construct the topology, which has been used in many previous papers such as [10], [16]. Some properties can result from this topology:

Property 5.1: Consider any two nodes $v_i, v_j \in V$, at most one link of l_{ij} and l_{ji} can be in $Tree$.

Property 5.2: Consider any link $l_{ij} \in Tree$, if there exists another node k with $d_{ik} < d_{ij}$, then $l_{ik} \in Tree$.

Before link scheduling, we scale all links and the ambient noise in Algorithm 2, in which we find the minimum length link and scale all the distances (between any sender and receiver) by a factor of d_{min}. After the scale, all links' length are greater than or equal to one in the nearest neighbor tree (NNT).

Algorithm 2 Scale All links in $Tree$

```
d_min := min_{l_ij ∈ Tree}{d_ij}
for any two nodes v_i, v_j ∈ V do
  d̃_ij = d_ij / d_min;
end for
N* = N d_min^{α+δ}
```

We prove the equivalence scheduling between the original instance and the scaled one. Suppose we have generated a schedule L_t for scaled links under ambient noise $N^* = Nd_{min}^{\alpha+\delta}$. Thus $\forall l_{ij} \in L_t$:

$$\frac{\frac{P_{ij}}{\tilde{d}_{ij}^{\alpha^*}}}{N^* + \sum_{\forall l_{gh} \in L_t, l_{gh} \neq l_{ij}} \frac{P_{gh}}{\tilde{d}_{gj}^{\alpha^*}}} \geq \beta$$

Apply the solution (including the power assignments) to the original instance. It holds that:

$$\begin{aligned} & \frac{\frac{P_{ij}}{d_{ij}^{\alpha^*}}}{N + \sum_{\forall l_{gh} \in L_t, l_{gh} \neq l_{ij}} \frac{P_{gh}}{d_{gj}^{\alpha^*}}} \\ = & \frac{\frac{P_{ij}}{d_{ij}^{\alpha^*}} \cdot d_{min}^{\alpha^*}}{N \cdot d_{min}^{\alpha^*} + \sum_{\forall l_{gh} \in L_t, l_{gh} \neq l_{ij}} \frac{P_{gh}}{d_{gj}^{\alpha^*} \cdot d_{min}^{\alpha^*}}} \\ \geq & \frac{\frac{P_{ij}}{\tilde{d}_{ij}^{\alpha^*}}}{N^* + \sum_{\forall l_{gh} \in L_t, l_{gh} \neq l_{ij}} \frac{P_{gh}}{\tilde{d}_{gj}^{\alpha^*}}} \\ \geq & \beta \end{aligned} \tag{4}$$

Thus, we only need to solve the problem under scaled ambient noise N^* and scaled link length. For simplicity, we use the original notations such as N, d_{ij} in the following part.

Algorithm 3 Set Division on links in $Tree$

```
1:  d_max := max_{l_ij ∈ Tree}{d_ij}, d_min := min_{l_ij ∈ Tree}{d_ij};
2:  Δ := d_max / d_min, Λ := ⌈log Δ⌉, a_1 := 4δ log(3βΔ) / (α−2−δ)
3:  for each l_ij ∈ Tree with Δ / 2^{a_1 m} < d_ij ≤ Δ / 2^{a_1 (m−1)} do
4:    A_m := A_m ∪ {l_ij}
5:  end for
6:  for each A_m, 1 ≤ m ≤ Λ / a_1 do
7:    d' := max_{l_ij ∈ A_m}{d_ij};
8:    for each l_ij ∈ A_m with d' / 2^k < d_ij ≤ d' / 2^{k−1} do
9:      A_mk := A_mk ∪ {l_ij}
10:   end for
11: end for
12: B_k := ⋃_{m=1}^{Λ} A_mk, 1 ≤ k ≤ a_1;
```

Algorithm 3 divides all links into subsets according to their length in two steps. In the first step, the links are divided into $\frac{\Lambda}{a_1}$ subsets according to their length where the subsets are denoted by A_m. In the second step, all links in each set A_m are further divided into smaller subsets according to the links' length relative to increasing power of 2 (line 8 of Algorithm 3). Each k-th subset from A_m is combined to construct a set B_k where $1 \leq k \leq a_1$. Fig. 3 illustrates how the algorithm processes.

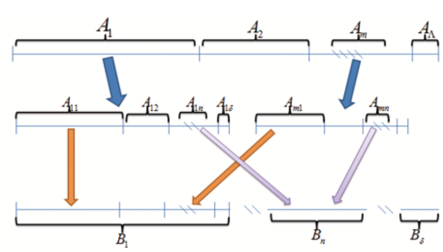

Fig. 3. An example of set division

Property 5.3: Consider any two links l_{ij} and l_{gh} in the same subset B_k. If l_{ij} and l_{gh} come from different A_{mk}

link sets (e.g. A_{m_1k} and A_{m_2k}, where $m_1 < m_2$) then $d_{ij} \geq 2^{(m_1-m_2)\cdot a_1} \cdot d_{gh}$.

Property 5.4: Consider any two links l_{ij} and l_{gh} in the same subset B_k, their lengths are either very similar (differ by a factor 2) or vastly different.

Proof: If the two links come from the same subset A_{mk}, it holds $\frac{1}{2} \leq \frac{d_{ij}}{d_{gh}} \leq 2$ from line 8 in Algorithm 3. Otherwise, they are vastly different by Property 5.3. ■

After the set division subroutine, each link is mapped to a subset in B_k according to its length. In the link scheduling phase, schedule each subset B_k separately taking advantage of the link length relation from Property 5.4. Before we describe the link scheduling phase, the definition *conflicting link* will be useful.

Definition 5.2: Link l_{gh} is a conflicting link to l_{ij} if any one of the following properties is satisfied: (Suppose l_{ij} is from subset A_{m_1k}, l_{gh} is from subset A_{m_2k} and c_1 is a large enough constant)

- $d_{ig} < c_1 \cdot d_{ij}$ when $m_1 = m_2$ which means they are from the same subset in the first layer division;
- $d_{gj} < d_{gh}$ when $m_2 < m_1$;
- $d_{hi} < c_1 \cdot d_{gh}$ when $m_1 - m_2 \in (0, \frac{(1+\log b)\log n}{\alpha a_1}]$;
- $d_{hi} < n^{\frac{1}{\alpha-\delta}} \cdot d_{ij} \cdot b^{\frac{(\tau_{ij}-\tau_{gh})+1}{\alpha-\delta}}$ when $m_1 - m_2 > \frac{(1+\log b)\log n}{\alpha a_1}$;

The definition of *conflicting link* is used in the Link Schedule algorithm design and performance analysis.

Algorithm 4 Maximal Concurrent Link Scheduling on $Tree$

$T := Tree \setminus \{l_{v's}\}$;
Scale T using Algorithm 2;
Set division on T using Algorithm 3;
$b = 3\beta\Delta^{2\delta}, k = N, t_{max} = 1, L_1 = \emptyset$
for $k = 1$ to a_1 **do**
  for each $l_{ij} \in B_k$ in decreasing order of length **do**
    for $t = 1$ to t_{max} **do**
      if no conflicting link to l_{ij} exists in L_t **then**
        $L_t := L_t \cup \{l_{ij}\}$; break;
      end if
    end for
    if link l_{ij} is not scheduled **then**
      $t_{max} := t_{max} + 1, L_{t_{max}} := \{l_{ij}\}$;
    end if
    $P_i(l_{ij}) := k \cdot b^m \cdot d_{ij}^{\alpha-\delta}$;
  end for
end for
return all link sets L_t

Algorithm 4 lays most of the ground work for scheduling. It schedules each subset B_k separately. For each link l_{ij} in B_k, if there exists a time slot with no conflicting links, add it to the time slot; otherwise create a new time slot with link l_{ij} and continue the process until all subsets are scheduled.

Remark 5.1: All the links in the same time slot L_t are from the same subset B_k.

Remark 5.2: The power level assigned to each link l_{ij} is a non-linear function of d_{ij} with relation to the maximum fluctuation range δ.

C. Correctness, Complexity and Performance

In this part, we first prove that all the links scheduled in the same time slot from the α-Robust algorithm above can transmit concurrently under the SINR model; and then we show the robust algorithm can achieve scheduling complexity $O(\log n \log \Delta)$ even for the worst case topology. Moreover, the time complexity of the algorithm itself is bounded by $O(n^2(\log n + \log \Delta))$ of efficiency.

Theorem 2: Consider any link set L_t generated by the α-Robust Algorithm, $\forall l_{ij} \in L_t$ and $\forall \alpha^* \in [\alpha - \delta, \alpha + \delta]$, it holds:

$$\frac{\frac{P_{ij}}{d_{ij}^{\alpha^*}}}{N + \sum_{\forall l_{gh} \in L_t, l_{gh} \neq l_{ij}} \frac{P_{gh}}{d_{gj}^{\alpha^*}}} \geq \beta \tag{5}$$

Remark 5.3: In Theorem 2, α^* can be different value for different links, which demands a stronger condition for robust algorithm design.

In order to deduce Theorem 2, we provide the following lemmas:

Lemma 5.1: Consider link $l_{ij} \in L_t$ (suppose $l_{ij} \in A_{m_1k}$) scheduled in time slot t. The interference caused at v_j by other links $l_{gh} \in L_t$ and $l_{gh} \in A_{m_2k}$ where $m_2 < m_1$ is bounded by: $I_1(v_j) \leq N_1 \cdot k \cdot b^{m_1-1}$ where $N_1 = \frac{64 \cdot 2^{\alpha-\delta}(c_1+4)}{c_1^2} \frac{\alpha-\delta-1}{\alpha-\delta-2}$.

Proof: Link l_{gh} should be scheduled before link l_{ij} because $m_2 < m_1$ which implies $d_{gh} > d_{ij}$. We know that l_{gh} is not a conflicting link for l_{ij} since they are scheduled in the same time slot, thus $d_{gj} \geq d_{gh}$ from the second case of the *conflicting link* definition. Now, we bound the interference caused by the simultaneous scheduled links in A_{m_2k} for a fixed m_2 value. Divide the plane into rings $R_1, R_2 \cdots R_\infty$. For each link $l_{g'h'}$ in ring R_λ: $(2\lambda-1)d_{gh} \leq d_{g'j} < (2\lambda+1)d_{gh}$. Since l_{gh} and $l_{g'h'}$ are scheduled in the same time slot with same subset index m_2, $d_{gg'} \geq min\{c_1 d_{gh}, c_1 d_{g'h'}\} \geq \frac{d_{gh}}{2}$ (from the first case of *conflicting link* definition) which implies the disks of radius $\frac{d_{gh}}{4}$ centered at each link's sender do not overlap. The number of the senders, then, can be bounded by:

$$\begin{aligned} N_\lambda &\leq \frac{\pi(2\lambda+1+\frac{c_1}{4})^2 - \pi(2\lambda-1-\frac{c_1}{4})^2}{\pi(\frac{c_1}{4})^2} \\ &= 16 \cdot \frac{4\lambda(\frac{c_1}{2}+2)}{c_1^2} \leq \frac{32(c_1+4)}{c_1^2} \cdot \lambda \end{aligned}$$

The interference caused at v_j by all senders in ring R_λ is bounded by:

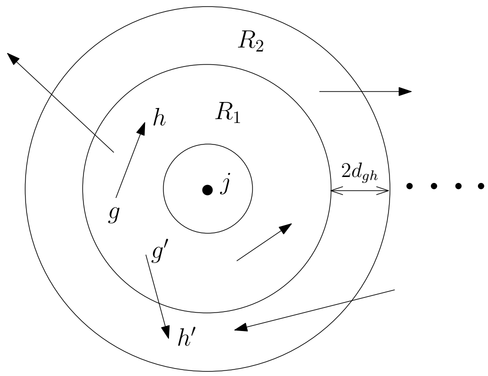

Fig. 4. An example of dividing the plane into rings, where for each link $l_{g'h'}$ in ring R_λ, $(2\lambda-1)d_{gh} \leq d_{g'j} < (2\lambda+1)d_{gh}$. We can bound the number of senders in each ring and thus bound the interference at the receiver node v_j.

$$
\begin{aligned}
I_{R_\lambda}(v_j) &\leq N_\lambda \cdot \frac{k \cdot b^{m_2} \cdot (2d_{gh})^{\alpha-\delta}}{[(2\lambda-1)\cdot d_{gh}]^{\alpha^*}} \\
&\leq N_\lambda \cdot \frac{k \cdot b^{m_2} \cdot (2d_{gh})^{\alpha-\delta}}{[(2\lambda-1)\cdot d_{gh}]^{\alpha-\delta}} \\
&= \frac{32(c_1+4)}{c_1^2} \cdot k \cdot 2^{\alpha-\delta} \cdot b^{m_2} \cdot \frac{\lambda}{(2\lambda-1)^{\alpha-\delta}} \\
&\leq \frac{32k2^{\alpha-\delta}(c_1+4)b^{m_2}}{c_1^2}\frac{1}{\lambda^{\alpha-\delta-1}}
\end{aligned}
$$

Naming the set of all links in L_t that are in A_{m_2k} as S_{gh} and combining all the rings, we can bound the total interference by senders in S_{gh} by:

$$
\begin{aligned}
I_{S_{gh}}(v_j) &= \sum_{\lambda=1}^{\infty} I_{R_\lambda}(v_j) \\
&\leq \frac{32k2^{\alpha-\delta}(c_1+4)b^{m_2}}{c_1^2}\sum_{\lambda=1}^{\infty}\frac{1}{\lambda^{\alpha-\delta-1}} \\
&\leq \frac{32k2^{\alpha-\delta}(c_1+4)}{c_1^2}\frac{\alpha-\delta-1}{\alpha-\delta-2}b^{m_2}
\end{aligned}
$$

Since $1 \leq m_2 \leq m_1$, if we sum up all the interference $I_{S_{gh}}(v_j)$, we get $I_1(v_j) = \sum_{m_2=1}^{m_1-1} I_{S_{gh}}(v_j) \leq C_1 \cdot k\sum_{m_2=1}^{m_1-1} b^{m_2} = C_1 \cdot k\frac{b(b^{m_1-1}-1)}{b-1} \leq 2C_1 \cdot kb^{m_1-1}$ where $C_1 = \frac{32\cdot 2^{\alpha-\delta}(c_1+4)}{c_1^2}\frac{\alpha-\delta-1}{\alpha-\delta-2}$ and $N_1 = 2C_1$, the lemma follows. ■

Remark 5.4: We assume $\alpha-\delta > 2$. This is reasonable since it is generally accepted under the standard SINR model that any pass-loss exponent lies between 2 to 6.

Using the same technique above and the method in the analysis of [16], we can obtain the following three lemmas easily:

Lemma 5.2: Consider link $l_{ij} \in L_t$ (suppose $l_{ij} \in A_{m_1k}$) scheduled in time slot t. The interference caused at v_j by other links $l_{gh} \in L_t$ and $l_{gh} \in A_{m_2k}$ where $m_1 = m_2$ is bounded by: $I_2(v_j) \leq N_2 \cdot k \cdot b^{m_1-1}$ where $N_2 = \frac{32b\cdot 2^{2\alpha-\delta}}{(c_1-2)^{\alpha-\delta}}\frac{\alpha-\delta-1}{\alpha-\delta-2}$

Lemma 5.3: Consider link $l_{ij} \in L_t$ (suppose $l_{ij} \in A_{m_1k}$) scheduled in time slot t. The interference caused at v_j by other links $l_{gh} \in L_t$ and $l_{gh} \in A_{m_2k}$ where $m_1 < m_2 \leq m_1 + \frac{(1+\log b)\log n}{\alpha a_1}$ is bounded by: $I_3(v_j) \leq N_3 \cdot k \cdot b^{m_1-1}$ where $N_3 = \frac{144\cdot 2^{\alpha-\delta}}{c_1^{\alpha-\delta}}\frac{\alpha-\delta-1}{\alpha-\delta-2}$.

Lemma 5.4: Consider link $l_{ij} \in L_t$ (suppose $l_{ij} \in A_{m_1k}$) scheduled in time slot t. The interference caused at v_j by other links $l_{gh} \in L_t$ and $l_{gh} \in A_{m_2k}$ is bounded by: $I_4(v_j) \leq k \cdot b^{m_1-1}$, where $m_2 \geq m_1 + \frac{(1+\log b)\log n}{\alpha a_1}$.

Based on the lemmas above, we prove the correctness of Theorem 2:

Proof: Combining Lemma 5.1, 5.2, 5.3 and 5.4, we calculate the SINR ratio for each link l_{ij} as:

$$
\begin{aligned}
&\frac{\frac{P_{ij}}{d_{ij}^{\alpha^*}}}{N + \sum_{\forall l_{gh}\in L_t, l_{gh}\neq l_{ij}} \frac{P_{gh}}{d_{gj}^{\alpha^*}}} \\
\geq\ &\frac{\frac{P_{ij}}{d_{ij}^{\alpha^*}}}{N^* + (I_1(v_j)+I_2(v_j)+I_3(v_j)+I_4(v_j))} \\
\geq\ &\frac{k\cdot b^{m_1}}{[N + (N_1+N_2+N_3+1)k\cdot b^{m_1-1}]\cdot\Delta^{2\delta}} \\
\geq\ &\frac{k\cdot b^{m_1}}{(N+2k\cdot b^{m_1-1})\Delta^{2\delta}} \geq \beta
\end{aligned}
$$

where $N_1+N_2+N_3 < 1$ can be verified when we take c_1 as a large enough constant. So Theorem 2 follows. ■

Remark 5.5: We have generated schedules for all scaled links. This result is also the solution to the original problem, since we have proved the correctness of their equivalence in Equation (4).

Theorem 3: The α-Robust algorithm can schedule each link $l_{ij} \in Tree$ in time slot $0 \leq t(l_{ij}) \leq C\log n\log\Delta$ for some constant C.

Proof: The sketch is to bound the number of conflicting links in the scheduling step. For each link $l_{ij} \in B_k$, if we figure out there are only $O(\log n)$ conflicting links before its schedule turn, the worst case would be allocating a new time slot for it, which means it can be also scheduled in time slot $O(\log n)+1 \in O(\log n)$. The number of conflicting links for each link l_{ij} can bounded by $O(\log n)$ in [7]. Since there are a_1 loops and a_1 is bounded by $O(\log\Delta)$, which implies all links can be scheduled in time slot $[0, C\log n\log\Delta]$ where C is certain constant. ■

Theorem 3 implies that the scheduling complexity of the α-Robust algorithm is bounded by $O(\log n\log\Delta)$ even for the worst case topology. In addition, we analysis the complexity of the algorithm as follows: the α-Robust algorithm takes $O(n^2\log n)$ time to construct the NNT topology, scaling and set division process are bounded by $O(n^2)$ and $O(n)$ separately. The core subroutine is link scheduling, which involves three nested iterations and this procedure can be also bounded by $O(n^2\log\Delta)$. Thus, the time complexity of the α-Robust algorithm is $O(n^2(\log n+\log\Delta))$ of efficiency.

D. Performance Evaluation

Since no work before this paper takes the fluctuation into consideration, we have to choose the best known result, which generates $O(\log n)$ time slot for the Connectivity Problem from [7], [10] for comparison, i.e. we evaluate our α-Robust algorithm by comparing the performance with the best known result in two scenarios:

1) Constant Distance Line Topology: Suppose all the n nodes are deployed in a line, and the distance between each two adjacent node is the same. We run the two algorithms on different size of nodes (from $512 = 2^9$ to $32768 = 2^{15}$) to see the efficiency tradeoff under two different α values: $\alpha = 3$ and $\alpha = 4$ where $\beta = 3, \delta = 0.5$.

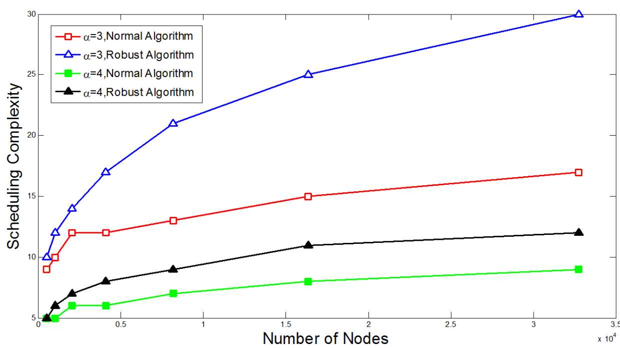

Fig. 5. The trade off between normal algorithm and the robust one under two different α values

Fig. 5 shows the extra time consumed in robust algorithm is very small compared with the $O(\log n)$ normal algorithm. However, when we verify the scheduled time slots by normal algorithm with $\alpha^* = \alpha - 0.1$, no one can still hold the SINR constraint, which means all time slots crash when small fluctuation exists. Through the comparison, we claim the robust algorithm is efficient and robust to the fluctuating α value for the constant distance line topology.

2) Random Topology: In this scenario, we first generate a sequence of random graphs with n nodes arbitrary deployed in the Euclidean plane, and then test the two algorithms on different size of nodes in order to find out the trade off of scheduling complexity and the ratio of failed transmission time slots generated by normal algorithm if fluctuation exists.

Fig. 6 and Fig. 7 implies the scheduling complexity of the robust algorithm is comparable to the $O(\log n)$ one. When the fluctuation is very small (such as 0.1 or 0.2), almost all time slots generated by the normal algorithm can be successful in the random graph, while the ratio of unsuccessful time slots becomes larger when the fluctuation increases from 0.3 to 0.5. When n is big enough, only half number of time slots can be successful when the fluctuation is maximal (in our simulation, it's 0.5). Thus, the α-Robust algorithm proposed in this paper achieves better performance in the random network topology with little extra cost in time consuming, compared with the best known result.

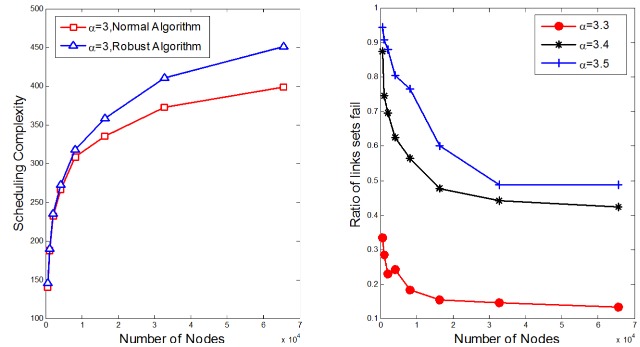

Fig. 6. The comparison between two algorithms when $\alpha = 3$ and the ratio of failed time slots for different fluctuation

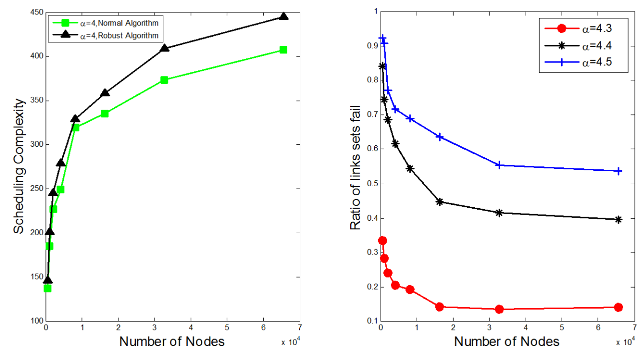

Fig. 7. The comparison between two algorithms when $\alpha = 4$ and the ratio of failed time slots for different fluctuation

VI. Conclusion

In this paper, we have initiated the study of an inaccurate α in algorithm design and analysis under the SINR model. Inaccurate α may destroy performance guarantees given by any algorithm under the standard SINR model, since there exists a topology in which no two links can transmit concurrently if the path loss exponent is allowed to vary between $[\alpha - \delta, \alpha + \delta]$, where δ is the maximum deviation. Thus α-Robust algorithm design becomes important to all the existing results. We give an improved α-Robust algorithm to the fundamental connectivity problem, achieving $O(\log n \log \Delta)$ time slots to compose a (weak) connected network even for the worst case topology. This result is also comparable to the best known result $O(\log n)$ time slots. Extensive simulation results on both constant distance line topology and random topology scenarios have shown little gap exists between the two algorithms, where the improved one is robust to the fluctuation, while the normal algorithm crashes.

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