# Accurate measurements of cold atomic cloud with area fit method 

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#### Abstract

We propose a scheme for accurately determining the number and the temperature of cold atomic cloud by the area fit, which is more accurate than former Gaussian fit method and can be used mostly when the cloud takes a shape that cannot be accurately captured using common Gaussian fit. We have realized measurements of cold atomic cloud with area fit. The number of trapped atoms is about $10^{8}$ and the temperature estimated with the analysis is about $328 \mu \mathrm{~K}$ after 2 s loading.


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The concept of the magneto-optical trap (MOT) was first suggested by Jean Dalibard [1], when pioneers of the field such as Pritchard and Chu were still struggling with dipole trapping of neutral atoms [2-5]. The MOT was shown to be able to trap a much higher number of atoms (on the order of $10^{7}$ [3]), making it a preferred choice as a source of cold atoms. The two properties of the MOT cloud that we are most interested in are the number and the temperature and how they behave in different regimes.

To measure these properties, we need methods to measure the cloud size, the number of atoms, and the expansion of the cloud. The most common method is using another weak laser beam as a probe which aligned to pass through the cloud and collected on a photodiode [6].

Here we propose the area fit method for accurately determining atom number and the temperature, which can be used mostly when the cloud takes a shape that cannot be accurately captured using former common Gaussian fit method [7]. Moreover it is more accurate than Gaussian fit method.

To estimate the temperature and the number of the trapped rubidium atoms in MOT, the light of 780 nm is switched on, and the slope of the B field is set to $50 \mathrm{G} / \mathrm{cm}$. In order to enhance the signal-to-noise ratio 780 nm light locked to $5^{2} S_{1 / 2}, F=2-5^{2} S_{3 / 2}, F=3$ transitions of rubidium is used to probe the MOT atoms, shown in Fig. 1 [8]. The diameter of the probe beam is expanded to be about 10 larger than the atomic cloud, which is covered by probe beam fully. And the light pulse is generated by controlling the on and off of AOM with the help of a time sequence controller.

Camera shutter is opened in advance, and the total exposure time of the CCD camera, usually, has been set at 200 microseconds. When the external trigger pulse from the time sequence controller has arrived, it will start the exposure of

[^0]

Fig. 1. Schematic of magneto-optical trap. The bronze rings represent the magnetic field coils in an anti-Helmholtz configuration. The red ball in the middle represents the atomic cloud. The pink arrows are the laser beams arranged in counter-propagating pairs of mutually orthogonal polarization. The pink beam is probe laser which radiates on CCD camera for measurements. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)


Fig. 2. Time sequence for atoms number and temperature measurements. Images are taken 3 times for subtracting the background.
the CCD. The diagram is illustrated in Fig. 2. The in-trap atoms cloud images can easily be taken by switching on the resonant probe light pulse at any time expected. In order to take the time of flight (TOF) image, atoms are released from the trap, and freely expand for period $t$.

We take Photos 3 times for images with background. The first image is TOF signal with CCD and probe laser background. The second is CCD and probe laser background. And the third is only CCD background. For images with background patterns, the image of the background is taken separately and subtracted from the cloud image before analysis. The center of the cloud is determined by summing up each column (for finding the center in the horizontal dimension) or row (gravity dimension). The expansion is recorded by taking pictures of the cloud after it is allowed to expand for a set period of time. We set expansion period t of TOF to $100 \mu \mathrm{~s}, 200 \mu \mathrm{~s}, 300 \mu \mathrm{~s}, 400 \mu \mathrm{~s}, 500 \mu \mathrm{~s}, 600 \mu \mathrm{~s}, 700 \mu \mathrm{~s}, 800 \mu \mathrm{~s}, 900 \mu \mathrm{~s}, 1 \mathrm{~ms}, 2 \mathrm{~ms}, 3 \mathrm{~ms}$, separately. For better understanding, hence also we can take different period $t$ as the processing moment of TOF.

We choose 5 moments of TOF process as a research, $0,600 \mu \mathrm{~s}, 1 \mathrm{~ms}, 2 \mathrm{~ms}, 3 \mathrm{~ms}$ respectively, which are shown in Fig. 3(a) where the first figure is the beginning point before TOF. The cloud radius in x dimension and y dimension is expanding in TOF process. The data collection is controlled by a program written in LabVIEW which also controls the camera operation mode and exposure, and then handled in MATLAB by the area fit method.

The expansion of the cloud width is under the assumption that the velocity distribution is not disturbed during the expansion. Hence, the trap, meaning both the magnetic field and the trapping light, has to be shut down during the expansion. The pictures for each shutdown interval are 0.2 s . The cloud radius is measured using the area fit method, and cloud radius in x dimension (horizontal in real setup) and y dimension (gravity in real setup) are expanding for TOF reason, illustrated in Fig. 3(b).

The appropriate quantity is the optical density with is for resonant light $D=\sigma_{0} \tilde{n}$. The cross section for resonant light is $\sigma_{0}=3 \lambda^{2} / 2 \pi$ and $\tilde{n}$ is the atomic density integrated along the propagation direction of the light [9,10]. Correspondingly we can calculate the number of atoms that is around $10^{8}$, shown in Fig. 4.

Also the temperature of atoms is needed to be measured. Since the transverse magnetic field is azimuthally symmetric, the force on the $x$ and $y$ dimensions should be equal for an optimized trap and therefore the transverse velocity components
should also be azimuthally symmetric. Therefore, the temperature of the transverse dimension of the cloud can be measured directly from the image by $x$ and $y$ direction separately. The temperature of a particular dimension can be measured by the rate of cloud expansion along the dimension without the influence of the trapping forces [10].


Fig. 3. (a) Images of in-trap atoms cloud and TOF atoms cloud after $600 \mu \mathrm{~s}, 1 \mathrm{~ms}, 2 \mathrm{~ms}, 3 \mathrm{~ms}$ points free expanding respectively. (b) The changing of cloud radius in x dimension and y dimension as time flight. The radius $R x$ (red circle) and $R y$ (blue circle) are calculated by profiling images. The lines are linear fitting of them. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)


Fig. 3. (Continued)


Fig. 4. Atoms number after atoms cloud free expanding.
To see how this works, we first assume the atoms to have a Maxwell-Boltzmann velocity distribution in one dimension of [10]

$$
\begin{equation*}
f\left(v_{i}\right)=\sqrt{\frac{m}{2 \pi k_{B} T_{i}}} \exp \left(-\frac{m}{2 \pi k_{B} T_{i}} v_{i}^{2}\right) \tag{1}
\end{equation*}
$$

The initial atomic distribution is taken to be a Gaussian function

$$
\begin{equation*}
n\left(r_{0}\right)=n_{0} \exp \left\{-r_{0}^{2} / R_{0}^{2}\right\} \tag{2}
\end{equation*}
$$

where $R_{0}$ is the initial cloud radius and $r_{0}$ is the position variable at time $t=0$. Without the influence of the trap, the atoms move to different positions according to the velocities they initially have. If we are observing at a position $r$ from the cloud center, once a time $t$ has passed the atoms which are at $r$ will be the ones that have the velocity and initial position such that $r=v \times t+r_{0}$. To find how many will reside at $r$, we have to integrate over all the initial positions of the atoms, $r_{0}$. The velocity distribution is assumed to be independent of the position. Therefore, the atomic distribution at time $t$ and position $r$ is the integration over the distribution of the initial atomic positions multiplied by the probability of an atom in position $r_{0}$ having the velocity $v=\left(r-r_{0}\right) / t[10]$ :

$$
\begin{equation*}
n(r, t)=\int_{-\infty}^{\infty} \sqrt{\frac{m}{2 \pi k_{B} T}} \exp \left(-\frac{m}{2 \pi k_{B} T} \frac{\left(r-r_{0}\right)^{2}}{t^{2}}\right) n_{0} \exp \left\{-r_{0}^{2} / R_{0}^{2}\right\} d r_{0} \tag{3}
\end{equation*}
$$

Eq. (3) is a convolution of two functions. Therefore, we can compute this using the Fourier transform property of the convolution which states that [10]

$$
\begin{equation*}
F\left[\int_{-\infty}^{\infty} G\left(x_{0}\right) H\left(x-x_{0}\right) d x_{0}\right]=F[G(x)] F[H(x)] \tag{4}
\end{equation*}
$$



Fig. 5. (a) In-trap atoms cloud fitted by the area fit before TOF. The blue are the fitting lines by Gaussian fit method. And the red is fitting line by area fit method. (b) The radius squared $R_{x}^{2}$ (red circle) and $R_{y}^{2}$ (blue circle) are calculated by area fit. The data fitting is for temperature measurement. By the fitting $T x$ is $234 \mu \mathrm{~K}$ and $T y$ is $175 \mu \mathrm{~K}$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The Fourier transformation is defined by

$$
\begin{equation*}
F\left[G\left(x_{0}\right)\right]=\int_{-\infty}^{\infty} G\left(x_{0}\right) \exp (-i 2 \pi k x) d x \tag{5}
\end{equation*}
$$

From this theorem, Eq. (3) can be computed much more easily by first performing the Fourier transformation on each distribution separately, joining the function in the momentum space, and performing an inverse Fourier transformation. Since the function for both atomic distribution and velocity distribution are Gaussian, the result of the calculation is also a Gaussian function. The width of the resulting Gaussian distribution is

$$
\begin{equation*}
R^{2}(t)=R_{0}^{2}+\frac{2 k_{B} T}{m} t^{2} \tag{6}
\end{equation*}
$$

Here $m$ is the mass of a rubidium atom, $k_{B}$ is Boltzman's constant, $t$ is the flight time, Rand $R_{0}$ are $1 / e$ Gaussian fitted radius of the TOF and in-trap atoms clouds respectively.

In experiments such as trap compression and cloud translation it is possible to observe a cloud of various distorted shapes as a result of changes in the trap condition that cannot be accounted for in the optimization process. Thank to the


Fig. 6. (a) The in-trap atoms cloud fitted by Gaussian fit before TOF. The blue are the real data collected by CCD. And the red is fitting line by Gaussian fit method. (b) The radius squared $R_{x}^{2}$ (red circle) and $R_{y}^{2}$ (blue circle) are calculated by Gaussian fit. The data fitting is for temperature measurement. Tx is $234 \mu \mathrm{~K}$ and $T y$ is $57 \mu \mathrm{~K}$ in x and y directions. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
area fit method, we can measure distorted shapes by counting pixels of the image with values higher than a cutoff. Using the model of a Gaussian cloud, the cutoff is set to $1 / \mathrm{e}$ of the highest value. Then the image of the cloud is averaged to remove large fluctuations. And the program searches for the highest value in the picture to calculate the cutoff, and then counts the number of pixels with values above this cutoff and finally converts it into an area. The image data is saved and then imported into Origin software for post-processing. The next step is summing 100 rows (or columns) around the center and fitting the profile to a Gaussian function. This is done for the two dimensions separately. The number of active pixels of the CCD is $1608 \times 1208$ and the size of each pixel is $7.4 \times 7.4 \mu \mathrm{~m}^{2}$. The width of the Gaussian function in pixels is then converted into millimeters by taking into account the magnification of the imaging system ( 1 in this case).

This ballistic expansion can be interrupted by other factors aside from the trap forces. One is the collisions with hot atoms which makes the cold atoms move randomly rather than in straight line as previously assumed. This limits the time where

Eq. (6) is valid during the expansion. It has been demonstrated that it is possible to observe the expansion up to 3 ms under appropriate conditions [11]. According to Eq. (6), the temperature of the particular dimension can be extracted from the slope. At the beginning, we calculate the $1 / e$ radius of the atoms cloud in horizontal and gravity directions to be 0.99 mm and 1.13 mm respectively from its area fit, as shown in Fig. 5(a). The temperature of the particular dimension can be extracted from the slope. Thus, the temperature of the atoms in the MOT is estimated to be approximately $234 \mu \mathrm{~K}$ and $175 \mu \mathrm{~K}$ in x and $y$ directions after 2 s loading according to Eq. (6). That is illustrated in Fig. 5(b).

We verify the area fit method by checking the figures of squared cloud radius $x$ and $y$ in Fig. 5(b), where these two linear fittings are almost parallel. It indicates that the temperature calculated by $x$ figure and $y$ figure are near equal. And that follows our reality. Therefore we conclude that the area fit method is accurate in calculating the temperature of atomic cloud.

Furthermore we get average radius of the atoms cloud by

$$
\begin{equation*}
R=\left(R_{x} R_{y} R_{z}\right)^{1 / 3} \tag{7}
\end{equation*}
$$

Here, $R_{x}, R_{y}, R_{z}$ are the $1 / e$ radius of the atoms cloud along the $x, y, z$ axes. If we assume that the cloud has rotation symmetry along the gravity, the radius of the atoms cloud is calculated to be 1.06 mm . In the same way, the radius of the TOF atoms cloud is calculated to be 1.30 mm . Thus, the temperature of the MOT atoms, after 2 s loading, is estimated to be about $328 \mu \mathrm{~K}$.

Besides we use former common Gaussian fit to calculate the temperature as a comparison. In this case, the Gaussian fit method will give a representation of the cloud size. The fitting procedure is as follows. The images of the cloud are taken as previously described. The position of the maximum is determined by a Gaussian fit to average out the fluctuations. The diagram is illustrated in Fig. 6(a). Comparing Fig. 5(a) and Fig. 6(a), we found that the fitting by the Gaussian fit has larger fluctuations with the real data.

From the Gaussian fit, we calculate the $1 / e$ radius of the atoms cloud in horizontal and gravity directions to be 0.97 mm and 1.09 mm respectively shown in Fig. 6(b). Therefore, it is estimated that the temperature of the atoms in the MOT is $234 \mu \mathrm{~K}$ in x direction and $57 \mu \mathrm{~K}$ in y direction after loading 2 s .

We find that the figures of squared cloud radius $x$ and $y$ in Fig. 6(b) are not parallel. It indicates that there is a big difference between $x$ temperature and $y$ temperature. That does not meet the reality. Therefore we conclude the Gaussian fit method is less accurate than the area fit.

## 1. Conclusion

In this paper, we have realized measurements of cold atomic cloud with area fit. Compared to the Gaussian fit method, area fit is more accurate and is used mostly when the cloud takes a shape that cannot be accurately captured using the Gaussian fit method. The number of trapped atoms is about $10^{8}$ and the temperature estimated with area fit analysis is about $328 \mu \mathrm{~K}$ after 2 s loading.

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