

# A Methodology for Designing the Control of Energy Harvesting Sensor Nodes

Neda Edalat, *Student Member, IEEE*, Mehul Motani, *Member, IEEE*,  
Jean Walrand, *Fellow, IEEE*, and Longbo Huang, *Member, IEEE*

**Abstract**—Sensor nodes equipped with renewable energy sources are capable of recharging their batteries and supporting data collection and transmission indefinitely. Energy and data management of these types of systems is challenging primarily due to the variability of renewable energy sources and transmission channels. This paper explores a methodology for designing the control of such systems. The goal is to jointly control the energy usage and data sampling rate to maximize the long-term performance of the system subject to the constraints imposed by the available energy and data. The design of this control is based on estimates of the large deviations of the energy stored in the battery and of the queued data. A low-complexity control policy is proposed that does not depend on the instantaneous charge of the battery and data backlog and almost maximizes the long-term data transmission rate. Moreover, the results show that one can decouple the analysis of the energy and of the data queue without much loss in performance.

**Index Terms**—Renewable energy, storage, data buffer, control, large deviations, Markov chain, effective rate.

## I. INTRODUCTION

AMBIENT energy harvesting is a solution to mitigate the typical finite energy supply of sensor nodes in wireless sensor networks (WSNs). Energy from renewable energy sources can recharge the sensor nodes' battery and extend the network's lifetime. However, designing the control of such systems that use renewable sources of energy presents new challenges because of the variability of these sources [1]. The energy usage should be carefully controlled to maximize system performance.

Consider a wireless sensor node equipped with a solar cell, a battery that stores the energy, and a data queue. The sensor node samples data and transmits it opportunistically depends on communication channel status and available energy and data. A complex control system would determine when to sample additional data and when to transmit on the basis of the backlog

in the data queue, the energy stored in the battery, and the state of the environment, such as the weather and the quality of the channel. However, one hopes that a simpler system that does not include the data backlog and the stored energy in its decisions might perform almost as well, provided that the battery and data queue capacities are not very small.

Several works address the power allocation problem in sensor networks equipped with renewable energy [2]–[10]. The work in [2] solved the transmission completion minimization problem for energy harvesting communication systems. They adaptively change the transmission rate according to the traffic load and battery charge. The authors in [3] proposed a solution for rate maximization for transmission over multiple fading channels, assuming that the future values of the available power and channel states are known. Under the same assumptions, [4] showed that the transmission time minimization problem and the transmission data maximization problem are the dual of each other and their solutions are identical for the same parameters. That problem was extended to the multiple access channel in [5] and the broadcast channel in [6]. The authors in [7] addressed the routing and power allocation problems as a standard convex optimization problem with energy constraints; they consider a finite horizon formulation and then relax it to derive an online algorithm. The authors in [8] designed a solution for high throughput data extraction from all nodes, guaranteeing fairness while maximizing the sampling rate and throughput. [10] proposed a joint data queue and battery buffer control algorithm, thus the long-term average sensing rate maximization subject to stability of data queue and desired data loss ratio could be achieved. They considered a static channel model and offline knowledge about the energy input. A policy with decoupled admission control and power allocation decisions was developed in [9] that achieves asymptotic optimality for sufficiently large battery capacity to maximum transmission power ratio (explicit bounds are provided). The information-theoretic approach [11] used a best effort transmit scheme which does not depend on the current size of the energy queue. Because it is assumed that the battery size is large enough, the data is transmitted with a Gaussian codebook whose average power is less than the average recharge rate.

Our paper proposes a novel approach for designing the control policy. That approach takes into account the size of the energy and data buffers and produces a policy that performs satisfactorily for medium value of those sizes. Also, the policy does not depend on the instantaneous charge of the battery and backlog of the data queue and it has a low complexity. A main feature of the approach is to estimate the outage probability

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N. Edalat and M. Motani are with the Department of Electrical and Computer Engineering, National University of Singapore, Singapore 117583 (e-mail: neda@nus.edu.sg; motani@nus.edu.sg).

J. Walrand is with Department of Electrical Engineering and Computer Sciences, University of California Berkeley, Berkeley, CA 94720-1770 USA (e-mail: wlr@eecs.berkeley.edu).

L. Huang is with Institute for Interdisciplinary Information Sciences (IIIS), Tsinghua University, Beijing 100084, China (e-mail: longbohuang@tsinghua.edu.cn).

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for the energy storage and the overflow probability for the data buffer and to design the policy to keep these probabilities acceptably small.

This methodology developed in this paper uses results from the theory of large deviations. This theory offers a collection of techniques to estimate the properties of rare events, such as their frequency and most likely manner of occurrence. Some references on large deviations include Bahadur (1971) [12], Varadhan (1984) [14], Deuschel and Stroock (1989) [16], and Dembo and Zeitouni (1998) [15]. The theory of large deviations has been applied to the analysis of Asynchronous Transfer Mode (ATM) networks [17] and [18]. ATM is a packet switching standard that aimed to limit the rate of cell losses due to buffer overflow to negligible values, comparable to losses caused by transmission errors. The novelty of our analysis is that the source and usage are both variable, in contrast to the theory of effective bandwidth where the transmission rate is constant [17] and [18].

In our paper, the main idea is to use a stochastic model of the system and estimate the likelihood that the data queue gets full or that the battery goes empty by studying the large deviations of functions of appropriate Markov chains. These functions are parametrized by design choices for the control policy. The result is an optimization problem where the constraints are on the exponents of the large deviation probabilities. The main advantage of this approach is that the control policy for the energy usage and data sampling rate do not involve the instantaneous amount of energy stored in the nodes and data backlog which significantly simplifies the implementation. We evaluate the proposed method on representative examples.

The paper is structured as follows. We introduce the system model and problem formulation in Sections II and III. In Section IV, we approximate the control policy by replacing the constraints on probabilities by constraints on large deviation exponents. Section V explains the large deviation properties and introduces the notions of effective power of a variable source and effective consumption rate of a variable load. Section VI provides numerical results for different examples. Section VII concludes and summarizes the paper.

## II. MODEL

The system model for a sensor node is sketched in Fig. 1. A discrete time model of the system is as follows. At time  $n \geq 0$ , the battery, with capacity  $B$ , has accumulated an amount  $X_n \in \{0, 1, \dots, B\}$  of energy. The finite data buffer at time  $n$  has  $W_n \in \{0, 1, \dots, D\}$  amount of data, where  $D$  is the capacity of data buffer. A battery stores energy it gets from a variable source and the node uses that energy to sample and transmit data. The more data the node transmits, the better. However, the node cannot transmit when it runs out of energy. In this model, there are three independent Markov chains  $Y_n^1$ ,  $Y_n^2$  and  $Y_n^3$ , with respective transition matrices  $P_1$ ,  $P_2$ , and  $P_3$ , that model the state of the sensed environment, the communication channel, and the renewable source, respectively. The amount  $A_n$  of data that the node samples is a function of  $Y_n^1$  and of a control variable  $S_n$ . For instance, the sensor may sample the environment at a slow rate under normal conditions and more

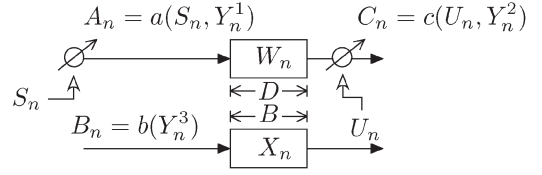


Fig. 1. The sensor node.

frequently when triggered by a proximity sensor or some other detection of an unusual condition. The sampling is controlled by the variable  $S_n \in \mathcal{S}$ , where  $\mathcal{S}$  is a finite set, to adjust its rate to a value compatible with the ability of the node to transmit the data it samples. Moreover,  $U_n \in \mathcal{U}$  is the amount of energy the node uses to transmit at time  $n$  where  $\mathcal{U}$  is a finite set. The amount  $C_n$  of data that the node transmits depends on the state  $Y_n^2$  of the communication channel and on the amount  $U_n$  of energy that the node uses to transmit under a given physical layer modulation and coding strategy. This model considers that the energy required to sample the environment is negligible compared to the energy required to transmit data. Finally, the amount  $B_n$  of energy that the renewable source produces at time  $n$  is a function of a Markov chain  $Y_n^3$ .

This model is representative of the type of system that the methodology in the paper can address. Many variations are possible. For instance, the energy that sampling requires might be non-negligible. Moreover, one might be able to move the solar cell by spending some energy. Our goal is to illustrate the methodology on a fairly simple, but non-trivial, example that captures the essence of the method but avoids complicating the setup.

## III. FORMULATION

The objective is to maximize the average rate at which the node gets to sample *and* transmit data. Note that these average rates must be equal if the system is stable. The node cannot sample when its data buffer is full and it cannot transmit when it runs out of energy.

One could formulate this objective as a Markov decision problem. The result would be control decisions of the form

$$S_n = \gamma_1(Y_n^1, Y_n^2, Y_n^3, X_n, W_n) \text{ and}$$

$$U_n = \gamma_2(Y_n^1, Y_n^2, Y_n^3, X_n, W_n).$$

The possible actions  $S_n$  and  $U_n$  at each time are constrained to avoid the battery underflows and data buffer overflows. That is, the problem is as follows:

$$\text{Maximize} \quad \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^N E(a(S_n, Y_n^1))$$

$$\text{over} \quad \gamma_1, \gamma_2$$

$$\text{s.t.} \quad X_{n+1} = \min\{X_n + b(Y_n^3) - U_n, B\},$$

$$W_{n+1} = W_n + a(S_n, Y_n^1) - c(U_n, Y_n^2),$$

$$U_n \leq X_n + b(Y_n^3),$$

$$c(U_n, Y_n^2) \leq W_n + a(S_n, Y_n^1),$$

$$a(S_n, Y_n^1) \leq D - W_n + c(U_n, Y_n^2).$$

The optimal policy is complicated to implement because it requires monitoring precisely the data backlog and the amount of energy stored in the battery. Finding the policy also requires solving a Markov decision problem with a large state space. For those reasons, we explore simpler policies.

#### IV. CONTROL POLICY

Intuition suggests that if the battery capacity is not too small, then the fluctuations in the amount of renewable power  $B_n$  average out and make it unnecessary to react in real time to them. Thus, one expects that the transmission and sampling decisions should not depend on the state  $Y_n^3$ . Similar considerations suggest that  $S_n$  should not depend on the fluctuations of the transmission channel  $Y_n^2$  nor of the weather  $Y_n^3$ . Moreover, one expects that the decisions should not depend on the instantaneous values of  $X_n$  and  $W_n$ . Thus, a good policy should be one where the sampling decision  $S_n$  depends only on the state  $Y_n^1$  of the sensed environment and the transmission decisions  $U_n$  depend only on the quality  $Y_n^2$  of the communication channel. Clearly this simplification of the control policies results in a loss of performance, but the conjecture is that this loss is negligible when  $D$  and  $B$  are not too small.

Accordingly, we formulate the problem as follows. We choose control policies of the form

$$S_n = \gamma_1(Y_n^1) \text{ and } U_n = \gamma_2(Y_n^2).$$

That is, at the outset we limit the complexity of the control policies. The sampling rate depends on the state of the sensed environment and the transmission rates depend on the state of the channel. The intuitive justification for the structure of these policies is that the effectiveness of sampling is related instantaneously to the state  $Y_n^1$  and that of transmission to  $Y_n^2$ . The constraints appear indirectly through the battery and data buffer, and are thus decoupled in time from the instantaneous states of the Markov chains.

More generally, we consider randomized policies specified as follows:

$$P[S_n = s | Y_n^1 = y^1] = \sigma(y^1, s) \text{ and} \\ P[U_n = u | Y_n^2 = y^2] = \tau(y^2, u).$$

If the data queue or the battery is full or empty, the transitions of  $W_n$  and  $X_n$  are then modified accordingly. For instance, if one attempts to transmit when the data queue is empty, it remains empty and the transmission energy is wasted.

Thus, a policy is completely specified by the two stochastic matrices  $\sigma$  and  $\tau$ . The randomization enlarges the set of strategies to make it possible to meet the performance constraints with equality.

Fig. 2 shows the resulting system.

The goal of the paper is to explore a methodology for designing good policies of the form indicated above.

A brute force approach would be to formulate the problem of optimizing  $\sigma$  and  $\tau$  as a linear optimization problem of the following form:

$$\text{Maximize } E_{\sigma, \tau}(A) \quad (1)$$

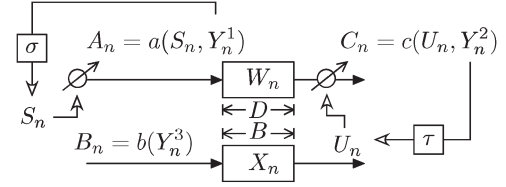


Fig. 2. The sensor node with its control policies.

over  $\sigma$  and  $\tau$

$$\text{s.t. } P_{\sigma, \tau}(X = 0) \leq \epsilon, P_{\sigma, \tau}(W = D) \leq \epsilon. \quad (2)$$

In this formulation,  $P_{\sigma, \tau}(\cdot)$  and  $E_{\sigma, \tau}(\cdot)$  correspond to the invariant distribution of  $(Y_n^1, Y_n^2, Y_n^3, X_n, W_n)$  that results from the policy  $(\sigma, \tau)$ . This problem is linear in  $\sigma$  and  $\tau$  because the expectation and the probability constraints are linear in the invariant distribution, and the latter solves linear equations in the transition probabilities and these are linear in  $\sigma$  and  $\tau$ . See Theorem 3.1 of [20] for a discussion of this formulation of average cost dynamic programming as a linear program.

Also,  $\epsilon$  is a very small value. The justification for this formulation is that, since the control decisions  $S_n$  and  $U_n$  are not based on  $X_n$  nor  $W_n$ , one must make sure that the policies do not violate the physical constraints. This constraint is relaxed by specifying that the probability that they are violated is very small.

Instead of pursuing this large optimization problem, we use the structure of the problem to express the constraints in terms of large deviations of  $W_n$  and  $X_n$ . The central idea is that (see Appendix A)

$$P_{\sigma, \tau}(X = 0) \approx \exp\{-\alpha(\tau)B\} \text{ and} \\ P_{\sigma, \tau}(W = D) \approx \exp\{-\beta(\sigma, \tau)D\}.$$

Here,  $\alpha$  and  $\beta$  are the large deviations exponents for the random processes  $X_n$  and  $W_n$ , respectively. Also, the first expression means precisely that

$$\lim_{B \rightarrow \infty} \frac{1}{B} \log\{P_{\sigma, \tau}(X = 0)\} = -\alpha(\tau), \quad (3)$$

and similarly for the second approximation. Thus, the approximations are asymptotic in the relevant size and they ignore the factors of the exponentials. We use these approximations to obtain estimates of the small probabilities.

Using the approximations, we rewrite the problem as follows:

$$\text{Maximize } E_{\sigma, \tau}(A) \quad (4)$$

over  $\sigma$  and  $\tau$

$$\text{s.t. } \alpha(\tau) \geq -\log(\epsilon)/B \text{ and} \quad (5)$$

$$\beta(\sigma, \tau) \geq -\log(\epsilon)/D. \quad (6)$$

In the next section, we explain the method for calculating  $\alpha(\tau)$  and  $\beta(\sigma, \tau)$ . The problem is then solved by first choosing  $\tau$  such that  $\alpha(\tau) = -\log(\epsilon)/B$  and then choosing  $\sigma$  such that  $\beta(\sigma, \tau) = -\log(\epsilon)/D$ . This solution is justified by the fact that, in our problem, the data transmission rate is maximized by using the most energy possible, given the constraint on the probability that the battery is empty. Thus, one can first choose

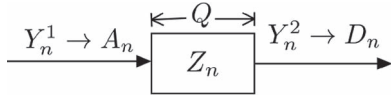


Fig. 3. The queue with Markov-modulated arrivals and departures.

$\tau$  to meet that objective and then choose the value of  $\sigma$  to meet the constraint on the data buffer occupancy.

When  $D$  and  $W$  become large, the solutions of the formulations (1), (2) and (4)–(6) become identical. The reason is that, in that case, the constraints (6) become  $\alpha(\tau) \geq 0$  and  $\beta(\sigma, \tau) \geq 0$ , which are satisfied if  $E(A_n) \leq E(C_n)$  and  $E(B_n) \geq E(U_n)$ , and these are the same as constraints (2).

## V. LARGE DEVIATIONS FOR MARKOV CHAINS

In this section, we explain a method for calculating the large deviation exponents  $\alpha$  and  $\beta$ . The backlog in the data queue and the amount of energy stored in the battery are both modeled as the content of a queue whose arrivals and departures are determined by the states of two independent Markov chains.

Accordingly, we need to study systems such as the one shown in Fig. 3 where we have a buffer with the capacity  $Q$  and accumulated content of the buffer  $Z_n$ . In this system, there are two independent Markov chains  $Y_n^1$  and  $Y_n^2$  and random variables for arrival  $A_n$  and departure  $D_n$  such that

$$P[A_n = a | Y_n^1 = y^1, Y_n^2 = y^2] = p(y^1, a),$$

$$P[D_n = d | Y_n^1 = y^1, Y_n^2 = y^2] = q(y^2, d).$$

One then defines the backlog  $Z_n$  in a queue with arrivals  $A_n$  and departures  $D_n$  as follows:

$$Z_{n+1} = [Z_n + A_n - D_n]_0^Q$$

where  $Q$  is the capacity of the queue. In the last expression, the notation is  $[z]_0^Q = \max\{0, \min\{z, Q\}\}$ .

One expects that if

$$E(A_n) > E(D_n),$$

then the stationary probability  $P(Z_n = 0)$  that the queue is empty is very small when  $Q$  is large. In fact, we explain in Appendix A that

$$\lim_{Q \rightarrow \infty} \frac{1}{Q} \log \{P(Z_n = 0)\} = -\psi(p, q) \quad (7)$$

for some  $\psi(p, q) > 0$ . Specifically,

$$\psi(p, q) = \inf_{d > a} \frac{\phi_A(a, p) + \phi_D(d, q)}{d - a}. \quad (8)$$

In this expression,

$$\phi_A(a, p) = \sup_{\theta < 0} \{\theta a - \log(\lambda_A(\theta, p))\} \quad (9)$$

where  $\lambda_A(\theta, p)$  is the largest eigenvalue of the matrix  $G_{\theta, p}^A$  defined by

$$G_{\theta, p}^A(y^1, \tilde{y}^1) = \left[ \sum_a e^{\theta a} p(y^1, a) \right] P_1(y^1, \tilde{y}^1).$$

where  $P_1(y^1, \tilde{y}^1) = P_1[Y_n^1 = \tilde{y}^1 | Y_{n-1}^1 = y^1]$ . Similarly,

$$\phi_D(d, q) = \sup_{\theta > 0} [d\theta - \log(\lambda_D(\theta, q))] \quad (10)$$

where  $\lambda_D(\theta)$  is the largest eigenvalue of the matrix  $G_{\theta, q}^D$  defined by

$$G_{\theta, q}^D(y^2, \tilde{y}^2) = \left[ \sum_d e^{\theta d} q(y^2, d) \right] P_2(y^2, \tilde{y}^2).$$

Using this method, we can calculate the decay rates  $\alpha(\tau)$  and  $\beta(\sigma, \tau)$  for the battery and the data buffer shown in Fig. 2.

To calculate  $\alpha(\tau)$ , one notes that the battery system is identical to the system in Fig. 3 since its arrivals  $B_n$  and departures  $U_n$  are functions of two independent Markov chains  $Y_n^3$  and  $Y_n^2$ . Similarly, to calculate  $\beta(\sigma, \tau)$ , one observes that if one considers a queue with arrivals  $C_n$  and departures  $A_n$ , then its occupancy  $Z_n$  behaves like  $D - W_n$ , so that the probability that it is empty is the probability that  $W_n = D$ . Accordingly,  $P(W_n = D) = P(Z_n = 0)$  can be calculated using the method of this section.

## VI. EFFECTIVE POWER AND EFFECTIVE CONSUMPTION RATE

In this section, we adapt the notion of *effective bandwidth* (see e.g., [17], [18]) to variable sources and variable loads, as a summary of their statistical characteristics. The novelty of our analysis is that we consider queues where the arrivals and departures are both variable.

The result of this section is that one can define the *effective power* of a variable renewable source and the *effective consumption rate* of a variable load in a way that the battery is almost never empty if and only if the former is larger than the latter. The effective power and effective consumption rate depend on the battery capacity. If the capacity is infinite, the effective power and consumption rate are simply the mean values. For a finite capacity, the effective power is less than the average renewable power and the effective consumption rate is larger than the mean value of the load. In such a system, the battery is rarely empty if there is a sufficient gap between the average power and the average consumption rate, and the effective values determine the required gap.

The results provided in this section correspond to the model shown in Fig. 3 where the queue is the battery and  $A_n$  and  $D_n$  represent the energy arrival and consumption, respectively.

**Definition 1:** Consider a random sequence  $A_n$  that is a function of a Markov chain, as in Fig. 3. For a given  $\delta > 0$ , the *effective power* of the sequence  $A_n$  is the maximum value of  $c$  such that the sequence  $Z_n$  defined by

$$Z_{n+1} = [Z_n + A_n - c]_0^Q, \quad n \geq 0$$

has a stationary distribution  $P(\cdot)$  such that

$$\lim_{Q \rightarrow \infty} \frac{1}{Q} \log \{P(Z_n = 0)\} \leq -\delta.$$

□



Thus, the effective power of the sequence  $A_n$  is the maximum constant departure rate  $D_n = c$  from a queue with arrivals  $A_n$  in the model shown in Fig. 3 so that the probability that the queue is empty decays exponentially fast in  $Q$  with rate at least  $\delta$ .

Intuitively, if the effective power of a random source  $A_n$  is  $c$ , then the source equipped with a battery can deliver a constant power  $c$ . The effective power is a value that decreases from the mean value  $E(A_n)$  to 0 as  $\delta$  increases. Thus, if the battery size is  $Q$  and we want the probability that the battery is empty to be of the order of  $\epsilon \ll 1$ , then we need  $\exp\{-Q\delta\} \approx \epsilon$ , so that  $\delta \approx -\log(\epsilon)/Q$ . The effective power for a given  $\epsilon \ll 1$  then increases from 0 to  $E(A_n)$  as the battery size increases. For instance, the effective power of a solar panel is its average power if it is equipped with a very large battery. If the battery is small, the effective power is considerably smaller.

Using the results of Appendix A, one can show that

$$\lim_{Q \rightarrow \infty} \frac{1}{Q} \log \{P(Z_n = 0)\} = -\inf_{a < c} \frac{\phi_A(a, p)}{c - a}. \quad (11)$$

Thus, the effective power is the maximum value of  $c$  such that

$$\inf_{a < c} \frac{\phi_A(a, p)}{c - a} \geq \delta. \quad (12)$$

Using the argument introduced in [17], one then has the following result.

**Theorem 1:** The effective power of a sequence  $\{A_n, n \geq 1\}$  is given by

$$\frac{\phi_A(\delta, p)}{\delta}$$

where

$$\phi_A(\delta, p) = \sup_{\theta > 0} \{\theta\delta - \log(\lambda_A(\theta, p))\}.$$

■

We also have the following definition.

**Definition 2:** Consider a random sequence  $D_n$  that is a function of a Markov chain, as in Fig. 3. For a given  $\delta > 0$ , the *effective consumption rate* of the sequence  $D_n$  is the minimum value of  $a$  such that the sequence  $Z_n$  defined by

$$Z_{n+1} = [Z_n + a - D_n]_0^Q, \quad n \geq 0$$

has a stationary distribution  $P(\cdot)$  such that

$$\lim_{Q \rightarrow \infty} \frac{1}{Q} \log \{P(Z_n = 0)\} \leq -\delta.$$

Thus, the effective consumption rate of the sequence  $D_n$  is the minimum constant power that one needs to provide the battery with output  $D_n$  so that it is empty only with a small probability. This value is an equivalent constant power, after being smoothed out by the battery. The effective consumption rate for a given small probability of the battery being empty decreases from the peak value of  $D_n$  to the average value  $E(D_n)$  as the battery increases.

Using the results of Appendix A, one can show that

$$\lim_{Q \rightarrow \infty} \frac{1}{Q} \log \{P(Z_n = 0)\} = -\inf_{d > a} \frac{\phi_D(d, q)}{d - a}. \quad (13)$$

Thus, the effective power consumption is the smallest value of  $a$  such that

$$\inf_{d > a} \frac{\phi_D(d, q)}{d - a} \geq \delta. \quad (14)$$

Once again using the argument of [17], one has the following result.

**Theorem 2:** The effective consumption rate of the sequence  $\{D_n, n \geq 1\}$  is given by

$$\frac{\phi_D(\delta, q)}{\delta}$$

where

$$\phi_D(\delta, q) = \sup_{\theta > 0} \{\theta\delta - \log(\lambda_D(\theta, q))\}.$$

■

We then have the following intuitive result for the system model of Fig. 3 where the queue is energy buffer. This result is a two-sided version of the corresponding result in [17] in that it applies to queues with random arrivals and departures.

**Theorem 3:** For some  $\delta > 0$ , the stationary probability  $P(\cdot)$  of the queue is such that

$$\lim_{Q \rightarrow \infty} \frac{1}{Q} \log \{P(Z_n = 0)\} \leq -\delta \quad (15)$$

if and only if there is some  $c$  such that the effective power of the arrivals  $A_n$  is at least  $c$  and the effective consumption rate of the departures  $D_n$  is less than  $c$ .

*Proof:*

- (a) **Sufficiency.** Assume that the effective power of  $A_n$  is larger than  $c$ . That implies, by (12), that

$$\frac{\phi_A(a, p)}{c - a} \geq \delta, \quad \forall a < c,$$

so that

$$\phi_A(a, p) \geq (c - a)\delta, \quad \forall a < c.$$

Similarly, assuming that the effective consumption rate of  $D_n$  is less than  $c$ , we get that

$$\phi_D(d, q) \geq (d - c)\delta, \quad \forall d > c.$$

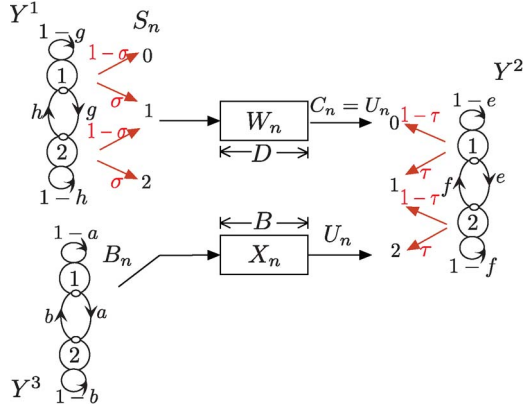
Consequently,

$$\phi_A(a, p) + \phi_D(d, q) \geq (d - a)\delta, \quad \forall d > a.$$

Hence,

$$\frac{\phi_A(a, p) + \phi_D(d, q)}{d - a} \geq \delta, \quad \forall d > a,$$

which implies, from (8), that (15) holds.

Fig. 4. Model of the system with 2-state Markov chains  $Y^1, Y^2, Y^3$ .

- (b) Necessity. Assume that the queue has the indicated property. Let  $a^*$  and  $d^*$  be the minimizers of

$$\frac{\phi_A(a, p) + \phi_D(d, q)}{d - a}$$

with  $a^* > d^*$  and let  $\delta$  be the minimum value. The first order conditions are

$$\phi'_A(a^*, p) = -\phi'_D(d^*, q) = \delta.$$

Now, choose  $c$  so that

$$\frac{\phi_A(a^*, p)}{c - a^*} = \delta.$$

Then we see that

$$\phi'_A(a^*, p)(c - a^*) = \phi_A(a^*, p),$$

so that  $a^*$  minimizes

$$\frac{\phi_A(a, p)}{a - c}$$

and the minimum is  $\delta$ . Similarly,  $d^*$  minimizes

$$\frac{\phi_D(d, q)}{c - d}$$

and the minimum is also  $\delta$ , which proves the claim.  $\square$

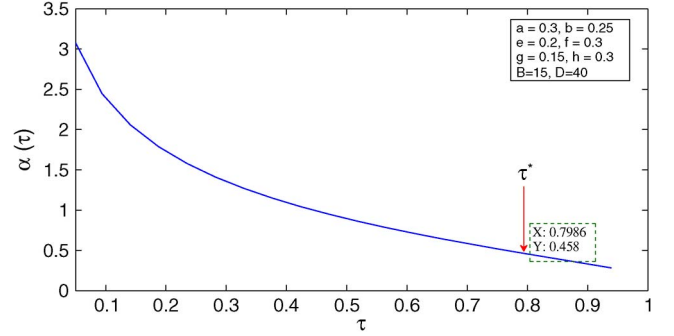
## VII. NUMERICAL RESULTS

In this section, we present numerical results for the system shown in Fig. 4. In this example, the two-state Markov chains  $Y_n^1, Y_n^2, Y_n^3 \in \{1, 2\}$  represent the sensed environment, communication channel and harvested energy per unit time, respectively.

We consider randomized policies specified as follows:

$$P[S_n = y | Y_n^1 = y] = \sigma, \forall y = 1, 2.$$

$$P[S_n = y - 1 | Y_n^1 = y] = 1 - \sigma, \forall y = 1, 2.$$

Fig. 5. Exponential rate of decay as a function of  $\tau$  with parameters from Fig. 4.

Similarly,

$$P[U_n = y | Y_n^2 = y] = \tau, \forall y = 1, 2.$$

$$P[U_n = y - 1 | Y_n^2 = y] = 1 - \tau, \forall y = 1, 2.$$

We set  $C_n = U_n$ . To use the results of the previous section, we compute the largest eigenvalue of the matrix

$$G_\theta(y^2, y^3, y'^2, y'^3) = h(y^2, y^3)P(y^2, y^3, y'^2, y'^3),$$

where

$$\begin{aligned} h(y^2, y^3) &= E[\exp\{\theta(U_0 - B_0)\} | Y_0^2 = y^2, Y_0^3 = y^3] \\ &= e^{\theta(y^2 - y^3)} [\tau + (1 - \tau)e^{-\theta}]. \end{aligned}$$

Fig. 5 shows the exponential rate of decay  $\alpha(\tau)$  as a function of  $\tau$ . To calculate the  $\tau^*$ , we need to find the lower bound for  $\alpha(\tau)$  which is  $-\log(\epsilon)/B$  from (4). We set the probability that battery goes empty ( $\epsilon$ ) to be 0.02 and  $B$  battery capacity to be 15 units. Hence,  $\tau^*$  is determined as shown in Fig. 5.

Considering  $\tau^*$  and analyzing the data buffer, we can compute  $\beta(\sigma, \tau^*)$ . The value of  $h(y^1, y^2)$  is calculated as follows

$$\begin{aligned} h(y^1, y^2) &= E[\exp\{\theta(S_0 - C_0)\} | Y_0^1 = y^1, Y_0^2 = y^2] \\ &= \tau^* e^{\theta(y^1 - y^2)} [\sigma + (1 - \sigma)e^{-\theta}] \\ &\quad + (1 - \tau^*) e^{\theta(y^1 - y^2)} [\sigma e^{\theta} + (1 - \sigma)]. \end{aligned}$$

Fig. 6 shows the exponential rate of decay  $\beta(\sigma, \tau^*)$  as a function of  $\sigma$ . We can determine  $\sigma^*$  by finding the lower bound  $-\log(\epsilon)/D$  where  $\epsilon = 0.01, D = 40$ . Then, the average sampling rate can be calculated as follows

$$\pi(1)\sigma^* + \pi(2)(\sigma^* + 1).$$

Numerical examples are provided which shows that this method works for different system parameters. This is tabulated in Table I, which shows that the values of  $a$  and  $b$  have an effect on both control policies  $\tau^*$  and  $\sigma^*$ . Not surprisingly, increasing  $b$  or decreasing  $a$  causes the values of the control policy to decrease.

Table II shows the effect of varying  $g$  and  $h$  on the control policies.

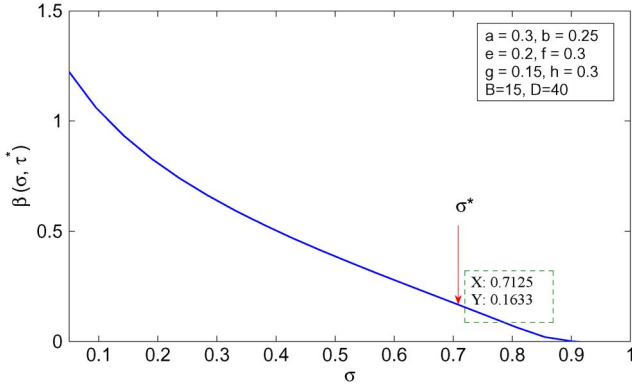


Fig. 6. Exponential rate of decay as a function of  $\sigma$  with parameters from Fig. 4.

TABLE I  
VALUES OF CONTROL POLICY OBTAINED USING THE  
DIRECT METHOD, GIVEN  $g = 0.15, h = 0.3$

$a$	$b$	$\tau^*$	$\sigma^*$	$\alpha(\tau^*)$	$\beta(\sigma^*, \tau^*)$
0.15	0.1	0.65	0.71	0.46	0.15
0.3	0.2	0.82	0.94	0.44	0.14
0.45	0.3	0.94	1	0.42	0.13
0.15	0.2	0.39	0.13	0.81	0.12
0.15	0.35	0.16	0.08	1.5	0.15

TABLE II  
VALUES OF CONTROL POLICY OBTAINED USING THE  
DIRECT METHOD, GIVEN  $g = 0.3, h = 0.2$

$g$	$h$	$\tau^*$	$\sigma^*$	$\alpha(\tau^*)$	$\beta(\sigma^*, \tau^*)$
0.15	0.3	0.79	0.70	0.45	0.16
0.25	0.3	0.79	0.61	0.45	0.14
0.45	0.3	0.79	0.46	0.45	0.16
0.15	0.2	0.79	0.37	0.45	0.17
0.45	0.4	0.79	0.56	0.45	0.13

As can be seen in Table II, increasing  $g$  causes the policy for sampling rate  $\sigma$  to act more conservatively by decreasing  $\sigma$ . Not surprisingly,  $\tau^*$  and  $\alpha(\tau^*)$  are constant by changing the values of  $g$  and  $h$ .

#### A. Stochastic Dynamic Programming

To demonstrate the effectiveness of the solution approach, we compare it with the results obtained by stochastic dynamic programming. Bellman's equation for the long-term average reward are as follows (see e.g. [19]):

$$v^* + \mathcal{H}(x) = \max_{u \in \mathcal{U}} \left[ r(x, u) + \sum_{x'} P(x, x'; u) \mathcal{H}(x') \right]. \quad (16)$$

In this expression,  $v^*$  is the optimum reward,  $\mathcal{H}(x)$  is the differential reward starting from state  $x$  and  $r(x, u)$  is the reward of taking action  $u$  in state  $x$ .

The value iteration for average reward dynamic programming works as follows: we choose the value of  $\mathcal{H}^0(x) = 0$  for all states  $x = 1, \dots, n$ . For each time step  $k = 1, \dots, K$ , the Bellman's equation  $Th^k(x) := \max_{u \in \mathcal{U}} [r(x, u) + \sum_{x'} P(x, x'; u) \mathcal{H}^k(x')]$  is updated for all states. We choose an arbitrary reference state  $x_0$  which is constant for all time steps. Then, we set  $v^k = Th^k(x_0)$  and update the differential reward function as  $\mathcal{H}^{k+1}(x) = Th^k(x) - v^k$ . The value of  $v^k$  for

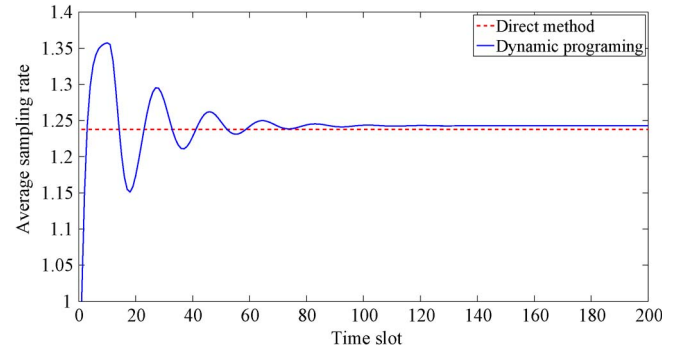


Fig. 7. Comparison of dynamic programming and direct method with same settings.

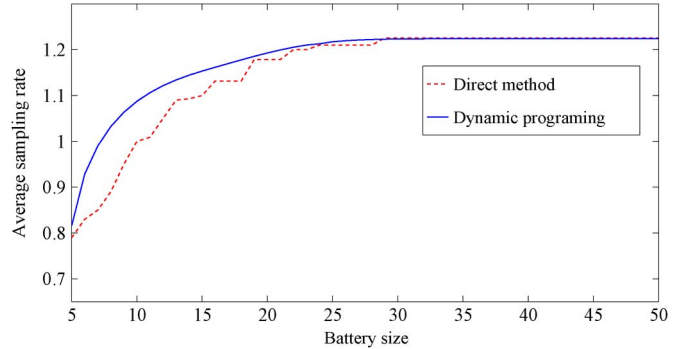


Fig. 8. Comparison of sampling rate results from dynamic programming and direct method for different battery sizes.

the last step is the optimum average reward. For our example shown in Fig. 4, the system states are  $(X_n, W_n, Y_n^1, Y_n^2, Y_n^3)$ . We use the same setting applied for developing our approach. Fig. 7 shows the optimal average reward that the algorithm calculates, as a function of the number of iterations. As can be seen, the algorithm converges to the essentially the same average value as determined from our control policy with same setting. This shows that our solution method results in essentially the optimal transmission rate achieved by dynamic programming. Note that in dynamic programming, we observe the state of the battery and the data buffer at the start of every time slot for making decisions.

Another set of simulation is conducted using different battery sizes. Fig. 8 shows that by increasing battery sizes, the average sampling rate increases and our proposed direct method is relatively near to the result from dynamic programming. It also shows that they converge for larger battery sizes. This is quite promising as we claim our methodology results in optimum solution for medium and large battery sizes. For smaller battery sizes, since the state space is relatively small, dynamic programming is less complex and can be a viable solution approach. Similar results for different data buffer sizes are shown in Fig. 9. By increasing the size of the buffer (data or energy), the dynamic programming alters to a linear programming with constraints related to average arrival and departure. For example, the constraint for energy queue is to drain the energy at the rate less than the average arrival rate. For our approach, given the constant  $\epsilon$ , by increasing the size of the buffer, the large deviation exponent is approaching to zero. This point, for

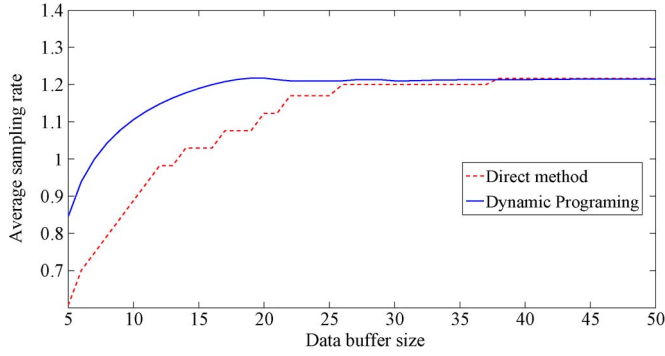


Fig. 9. Comparison of sampling rate results from dynamic programming and direct method for different data buffer sizes.

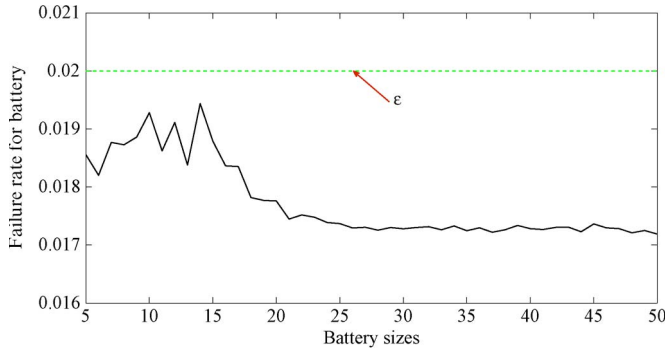


Fig. 10. Failure rate for battery applying direct method, given  $\epsilon$  for battery is 0.02.

example in energy buffer, means that the drain rate should be less than the expected value of arrival rate (for more details see Theorem 3.1. in [13]). This is the reason that for the very large battery size, the result from both approaches converges. The key point of this result is the advantage of our approach for the medium battery size, which for our proposed approach is fairly near to the optimum result.

Figs. 10 and 11 show representative results measured by simulating  $X_n$  and  $W_n$  for  $10^6$  steps for different value of battery sizes. The failure rate for the battery is calculated as the number of times that the battery goes empty over the number of steps ( $10^6$  in the example here). The failure rate for the buffer is calculated as the number of times that the buffer becomes full over the number of steps. The results show that for different battery sizes, the constraints for the battery and data buffer are not violated, since they are always less than  $\epsilon$ . This is because in our simulation for calculating  $\sigma^*$  and  $\tau^*$ ,  $\alpha(\tau)$  and  $\beta(\sigma, \tau)$  are bounded by  $-\log(\epsilon)/B$  and  $-\log(\epsilon)/D$  respectively.

### VIII. CONCLUSION

This is a methodological paper that presents a technique for state space dimension reduction for solving optimal control of systems with coupled data buffer and energy storage. The technique consists of relaxing the underflow constraint on the energy level in the battery and overflow constraint on the data backlogs in data buffer by replacing them with a bound on the probability of energy underflow and data overflow as derived via large deviation theory. The considered policies do

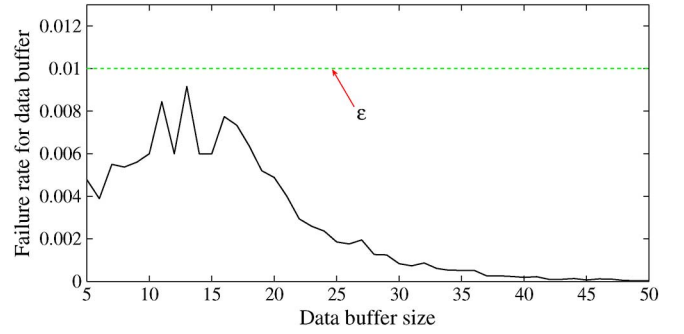


Fig. 11. Failure rate for buffer applying direct method, given  $\epsilon$  for data buffer is 0.01.

not depend on the state of the battery and data buffer, but rather only of the state of the environment. We demonstrated the use of the approach for the control of a wireless sensor node equipped with a solar panel and compared the results with the solution of the Markov decision problem.

It should be noted that the analysis assumes a detailed knowledge of the statistics of the renewable source and the environment. The methodology suggests adaptive schemes that do not require such knowledge and the analysis of such schemes is left for further investigation.

Our approach assumes a knowledge of statistical models of the renewable energy, of the channel, and of the data collection. Hopefully, these models can be fitted over time. Also, although the numerical complexity of the approach grows with the complexity of the model, the method may be practical even for fairly large models.

### APPENDIX A

To determine  $\psi$ , one argues as follows. Consider the backlog process  $\{Z_n, n \geq 0\}$  and fix  $Q \gg 1$ . Define a cycle as a time interval between two successive times when  $Z_n = Q$ . Since the arrival rate into the queue is larger than the service rate, the backlog does not reach the value 0 during most cycles.

Consider a cycle of such that  $Z_m$  reaches the value 0 in  $n$  steps. During these  $n$  steps, the departures occur at some rate

$$d = \frac{D_0 + \dots + D_{n-1}}{n},$$

and the arrivals occur at some rate

$$a = \frac{A_0 + \dots + A_{n-1}}{n}.$$

Thus, one has

$$Q = n(d - a).$$

Indeed, say that the cycle starts at time 0 with  $Z_0 = Q$  and is such that  $Z_n = 0$  and  $0 < Z_m < Q$  for  $m = \{1, 2, \dots, n-1\}$ . Then, since  $Z_m$  does not hit the boundaries 0 or  $Q$  during the cycle,

$$Z_{m+1} = Z_m + A_m - D_m, \quad m = 0, \dots, n-1.$$

In particular,

$$Z_n = Z_0 + (A_0 + \dots + A_{n-1}) - (D_0 + \dots + D_{n-1}),$$



so that

$$\begin{aligned} 0 &= Q + (A_0 + \cdots + A_{n-1}) - (D_0 + \cdots + D_{n-1}) \\ &= Q + n(a - d), \end{aligned}$$

which implies that  $Q = n(d - a)$ .

Thus, the probability that a cycle hits 0 is the probability that, for some  $d > a$ , and  $n = Q/(d - a)$ , the arrivals occur with average rate  $a$  and the departures with average rate  $d$  during  $n$  steps.

It is shown in [14] that, if  $a > E(A_m)$ , then

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{1}{n} \log \{P(A_1 + \cdots + A_n \geq na)\} \\ = -\inf_{\theta > 0} \{\theta a - \log(\lambda_A(\theta, p))\} \quad (17) \end{aligned}$$

where  $\lambda_A(\theta, p)$  is the largest eigenvalue of the matrix  $G_{\theta, p}^A$  defined by

$$G_{\theta, p}^A(y^1, \tilde{y}^1) = \left[ \sum_u e^{\theta u} p(y^1, u) \right] P_1(y_1, \tilde{y}^1)$$

where  $p_1(y^1, \tilde{y}^1) = P[Y_n^1 = \tilde{y}^1 | Y_{n-1}^1 = y^1]$ .

We need a similar result for the probability that the empirical rate is less than some value  $a$  smaller than the average rate.

Applying the previous result directly, we conclude that, if  $a < E(A_m)$ ,

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{1}{n} \log \{P(A_1 + \cdots + A_n \leq na)\} \\ = \lim_{n \rightarrow \infty} \frac{1}{n} \log \{P(-A_1 - \cdots - A_n \geq -na)\} \\ = -\inf_{\theta > 0} \{\theta(-a) - \log(\rho_{-A}(\theta, p))\}. \end{aligned}$$

Since

$$\begin{aligned} G_{\theta, p}^{-A}(y^1, \tilde{y}^1) &= \left[ \sum_u e^{-\theta u} p(y^1, u) \right] P_1(y_1, \tilde{y}^1) \\ &= G_{-\theta, p}^A(y^1, \tilde{y}^1), \end{aligned}$$

we find that

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{1}{n} \log \{P(A_1 + \cdots + A_n \leq na)\} \\ = -\inf_{\theta > 0} \{\theta(-a) - \log(\lambda_A(-\theta, p))\} \\ = -\inf_{\theta < 0} \{\theta a - \log(\lambda_A(\theta, p))\} =: \phi_A(a, p) \end{aligned}$$

Similarly, one has

$$\lim_{n \rightarrow \infty} \frac{1}{n} \log \{P(D_1 + \cdots + D_n \geq nd)\} = -\phi_D(d, q)$$

with

$$\phi_D(d, q) = \sup_{\theta > 0} [\theta d - \log(\lambda_D(\theta, q))],$$

where  $\lambda_D(\theta, q)$  is the largest eigenvalue of the matrix

$$G_{\theta, q}^D(y, y') = E[\exp\{\theta(D_1)\} | Y_1^2 = y] P_2(y, y').$$

Note the asymmetry between the definitions of  $\phi_A(a, p)$  and  $\phi_D(d, q)$ : in the former, the supremum is over  $\theta < 0$  whereas in the latter, it is over  $\theta > 0$ .

Accordingly, for any  $a > 0$ ,  $d > 0$ , one has

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{1}{n} \log \{P(A_1 + \cdots + A_n \leq na \text{ and } D_1 + \cdots + D_n \geq nd)\} \\ = -\phi_A(a, p) - \phi_D(d, q). \end{aligned}$$

As stated earlier, the probability that a cycle hits 0 is the probability that, for some  $d > a$ , and  $n = Q/(d - a)$ , the arrivals occur with average rate  $a$  and the departures with average rate  $d$  during  $n$  steps.

For  $Q \gg 1$ , one sees that, with  $n = Q/(d - a)$ ,

$$\begin{aligned} \lim_{Q \rightarrow \infty} \frac{1}{Q} \log P(A_1 + \cdots + A_n \leq na \text{ and } D_1 + \cdots + D_n \geq nd) \\ = -\frac{\phi_A(a, p) + \phi_D(d, q)}{d - a}. \end{aligned}$$

Consider the event  $\mathcal{E}$  that a busy cycle reaches the value 0 when

$$A_1 + \cdots + A_n \leq na \text{ and } D_1 + \cdots + D_n \geq nd$$

for some  $a, d, n$  where  $d > a$  and  $n = Q/(d - a)$ . The probability of that event is the sum over  $d > a$  of the probabilities that the event occurs for given values of  $a$  and  $d$ . Since each of these probabilities is approximately  $\exp\{-Qh(a, d)\}$  for some  $h(a, d) > 0$ , the sum of these terms, when  $Q \gg 1$ , is dominated by the term with the smallest value of  $h(a, d)$ . For instance,  $\exp\{-3Q\} + \exp\{-2Q\} + \exp\{-4Q\} \approx \exp\{-2Q\}$  for  $Q \gg 1$ . The precise justification of this result is the *contraction principle* for large deviations (see Theorem 2.4 in [13]).

Thus, one expects the probability of the event  $\mathcal{E}$  to be such that

$$\begin{aligned} \lim_{Q \rightarrow \infty} \frac{1}{Q} \log \{P(\mathcal{E})\} &= -\inf_{d > a} h(a, d) \\ &= -\inf_{d > a} \frac{\phi_A(a, p) + \phi_D(d, q)}{d - a}, \quad (18) \end{aligned}$$

So, as claimed

$$\psi(p, q) = \inf_{d > a} \frac{\phi_A(a, p) + \phi_D(d, q)}{d - a}. \quad (19)$$

Now, let  $p$  be the probability that a cycle hits 0,  $\alpha$  the average duration of a cycle that hits 0, and  $\gamma$  the average number of steps that the queue is 0 in a cycle that hits 0. Let also  $\delta$  be the average duration of a cycle that does not hit 0 (see Fig. 12). Thus, during  $n \gg 1$  cycles, there are approximately  $np$  cycles that hit 0 and the queue is empty during approximately  $np\gamma$  steps. These  $n$  cycles take approximately  $np\alpha + n(1 - p)\delta$ . Accordingly, the fraction of time  $\pi$  that the queue is empty is approximately given by

$$\pi(Q) = \frac{np\gamma}{np\alpha + n(1 - p)\delta} = \frac{p\gamma}{p\alpha + (1 - p)\delta}.$$

One can expect that  $\alpha = O(Q)$ ,  $\gamma = O(1)$ ,  $\delta = O(1)$ . Indeed, the time to hit 0, given that the cycle hits 0 is  $Q/(d - a)$  where  $a$  and  $d$  achieve the minimum in (18) and the average time from empty to  $Q$  is  $Q/(E(A_1) - E(D_1))$ .

Since  $p = O(\exp\{-\lambda Q\})$  with  $\lambda = \psi(p, q)$ , one sees that

$$\begin{aligned} \pi(Q) &\approx \frac{C_1 Q \exp\{-\lambda Q\}}{C_1 \alpha \exp\{-\lambda Q\} + (1 - \exp\{-\lambda Q\}) \delta} \\ &\approx C_2 Q \exp\{-\lambda Q\}, \end{aligned}$$

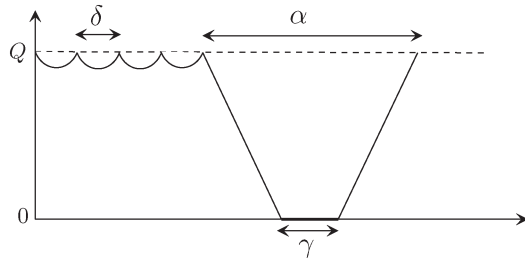


Fig. 12. This figure clarifies the notations used for cycles in Appendix A.

so that

$$\lim_{Q \rightarrow \infty} \frac{1}{Q} \log \{\pi(Q)\} \rightarrow -\lambda = -\psi(p, q).$$

Thus, the steady state probability that  $Z_n = 0$  is such that

$$\lim_{Q \rightarrow \infty} \frac{1}{Q} \log \{P(Z_n = 0)\} \rightarrow -\psi(p, q),$$

as we wanted to show.

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**Neda Edalat** (S'10) received the B.S. degree from Shiraz University of Technology in 2007 and the M.Eng. degree from National University of Singapore in 2010, both in electrical and computer engineering (ECE). Since January 2011, she has been pursuing the Ph.D. degree in ECE at the National University of Singapore. She has held a visiting scholar appointment in the Electrical Engineering and Computer Sciences department at the University of California at Berkeley from October 2013 to December 2014.



**Mehul Motani** (S'93–M'00) received the B.E. degree from Cooper Union, New York, NY, the M.S. degree from Syracuse University, Syracuse, NY, and the Ph.D. degree from Cornell University, Ithaca, NY, all in Electrical and Computer Engineering. He is currently an Associate Professor in the Electrical and Computer Engineering Department at the National University of Singapore (NUS). He was a Visiting Fellow at Princeton University, Princeton, NJ. He was a Research Scientist at the Institute for Infocomm Research in Singapore, for three years, and a Systems Engineer at Lockheed Martin in Syracuse, NY for over four years. His research interests are broadly in the area of wireless networks. Recently, he has been working on research problems which sit at the boundary of information theory, networking, and communications, with applications to mobile computing, underwater communications, sustainable development and societal networks. He was the recipient of the Intel Foundation Fellowship for his Ph.D. research, the NUS Faculty of Engineering Innovative Teaching Award, and the NUS Faculty of Engineering Teaching Honours List Award. He actively participates in the IEEE and the Association for Computing Machinery (ACM), and has served as the Secretary of the IEEE Information Theory Society Board of Governors. He has served as an Associate Editor for the IEEE TRANSACTIONS ON INFORMATION THEORY and an Editor for the IEEE TRANSACTIONS ON COMMUNICATIONS.



**Jean Walrand** (S'71–M'74–SM'90–F'93) received the Ph.D. degree in EECS from UC Berkeley, where he has been a Professor since 1982. He is the author of *An Introduction to Queueing Networks* (Prentice Hall, 1988) and *Communication Networks: A First Course* (2nd ed. McGraw-Hill, 1998) and co-author of *High Performance Communication Networks* (2nd ed, Morgan Kaufman, 2000), *Communication Networks: A Concise Introduction* (Morgan and Claypool, 2010), and *Scheduling and Congestion Control for Communication and Processing Networks* (Morgan and Claypool, 2010). His research interests include stochastic processes, queueing theory, communication networks, game theory, and the economics of the Internet. He has received numerous awards for his work over the years. He is a Fellow of the Belgian American Education Foundation and of the IEEE. Additionally, he is a recipient of the Lanchester Prize, the Stephen O. Rice Prize, the IEEE Kobayashi Award, and the ACM SIGMETRICS Achievement Award.



**Longbo Huang** (M'11) received the B.E. degree from Sun Yat-sen University, Guangzhou, China, in June 2003, the M.S. degree from Columbia University, New York, NY, USA, in December 2004, and the Ph.D. degree from the University of Southern California in August 2011, all in electrical engineering. He then worked as a Postdoctoral Researcher in the Electrical Engineering and Computer Sciences Department, the University of California at Berkeley, from July 2011 to August 2012. He is currently an Assistant Professor in the Institute for Interdisciplinary Information Sciences (IIIS) at Tsinghua University, Beijing, China.