An Algorithmic and Systematic Approach for Improving Robustness of TOA-based Localization

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Abstract—Indoor localization using Time-of-Arrival (TOA) of ultrasound is accurate, but remarkable errors may occur occasionally due to effects by indoor environment issues, such as when ultrasound propagates in Non-Line-Of-Sight (NLOS) paths, or synchronization signal is interfered by background signals. This paper presents an algorithmic and systematic approach to address these issues to improve robustness of ultrasound TOA positioning. We firstly show from an optimization point of view that NLOS detection problem is NP-hard. We propose a novel clustering and filtering (COFFEE) algorithm to conduct density-based clustering iteratively on a bipartite graph model, which enables accurate, robust positioning and efficient NLOS outlier detection. Then, we develop a systematic method to address the robust time synchronization problem, which is called first-falling-edge time synchronization. It guarantees robust time synchronization even in severe interference environments. Both the COFFEE algorithm and the robust synchronization scheme are developed and implemented in a ultrasound positioning prototype called Dragon. Extensively simulations and experiments in Dragon show that the proposed methodologies outperform the robustness performances of the state-of-the-art methods, which demonstrate great improvements in various interference scenarios.

I. INTRODUCTION

Fine-grained location-based services (LBS) has leaded to various designs and implementations of accurate indoor localization systems. Different kinds of signals and techniques have been explored in literatures, including infrared, radio frequency (RF), ultrasound (US), UWB, audible sound, and techniques such as Time-of-Arrival (TOA), Angle-of-Arrival (AOA), Time Difference Of Arrival (TDOA), which are thoroughly surveyed in [5] [11]. When comparing to other signals and techniques, ranging by TOA of ultrasound is a very competitive technique due to its high accuracy, low cost, safety and user-imperceptibility. Since the positioning accuracy of ultrasound TOA positioning can be generally in centimeter level even in 3D space, it is very fascinating in may indoor positioning systems, such as Bat [14], Cricket [12], LOSNUS [13] etc.

By measuring TOA of ultrasound to locate an active target (sender) by a set of receivers (reference points), three steps are generally required: 1) The sender broadcast RF and Ultrasound signals simultaneously; 2) the receivers measure Time-of-Arrival of ultrasound to estimate their distances to the sender; 3) the position of the sender is inferred by positioning algorithm such as Least Square estimation (LSQ), or Multiliteration [5]. Although this process generally provides accurate localization to the target, the steps of time synchronization and TOA measurement are highly sensitive to indoor environments issues, which may cause serious positioning errors occasionally:

1) In indoor environments, same band radio signals are hard to prevent, such as radio from micro-wave oven or WiFi. They may collide the synchronization RF signal, causing errors of TOA measurements;
2) Indoor objects such as furnitures, doors or people may block the direct paths from sender to receivers, which cause ultrasound propagate in the none-line-of-sight (NLOS) paths (reflection or refraction paths), resulting in large errors of TOA measurement because the receivers cannot justify the NLOS paths.

These inevitable impacts cause serious positioning error, but are hard to deal with. Previous studies mainly addressed the NLOS detection problem by geometric filters [15] [8] and statistical methods [1] [9]. But geometric NLOS filters are coarse-grained when noises of line-of-sight distances are considered; statistical methods need high computation costs and are not robust to the number of NLOS outliers. This paper presents a systematic approach to address not only the NLOS detection problem but also the robust synchronization problem. Different to existing approaches, it treats the NLOS outlier detection problem as an optimization problem. The objective is to find a set of *normal* (line-of-sight) distances, which minimizes the residue error of the estimated position. We prove this optimization problem is NP-hard and develop an efficient clustering and filtering algorithm (COFFEE), which conducts clustering and filtering iteratively on a bipartite graph model of this problem, until convergence of position estimation. We show COFFEE is efficient, locates target accurately, converges quickly, can successfully identify NLOS outliers, and is robust to the number of NLOS outliers. It outperforms the state-of-the-art algorithms.

In addition to COFFEE, robust time synchronization is addressed by a systematic method. A first-falling-edge time synchronization technique is developed by connecting receivers using sync-line, so that only if one receiver detects the Sync signal, all receivers will be synchronized. Above design methodologies leads to a robust positioning system, called Dragon, which implements COFFEE and robust time synchronization techniques. Dragon demonstrates great improvements of robustness than existing systems. Extensive simulations and

experiments validate not only the sole properties of COFFEE but also the overall performances of Dragon system.

The rest part of the paper is organized as follows. In section 2, we investigate the hardness of robust ultrasound positioning. In Section 3, we introduce COFFEE algorithm. Section 4 introduces First-Falling-Edge-Synchronization and the design of Dragon system. Experiments and performance evaluation results are presented in Section 5. Related works are introduced in Section 6, and the paper is concluded in Section 7.

II. CHALLENGES OF ROBUST ULTRASOUND TOA-BASED POSITIONING

We consider a snapshot of locating a target by a distance set measured from N receivers. Suppose $m < N/2$ outliers are hiding in the distance set, which maybe caused by NLOS effects of ultrasound or synchronization failures. An outlier distance has obvious large ranging error. Let's suppose the other $N - m$ measurements are *normal* (normal) distances, whose ranging errors are small. We denote the whole distance set by $\mathbf{D} = \{d_i, i = 1, ..., N\}$, and denote the coordinates of the reference points by $\mathbf{X} = \{\vec{x}_1, \vec{x}_2, \cdots, \vec{x}_N\}$.

Consider a generic localization function F, which calculates target position by $\theta = F(\mathbf{D_s}, \mathbf{X_s})$. $\mathbf{D_s}$ is a subset of distances selected from $\mathbf{D}$, and $\mathbf{X_s}$ is the coordinates of the selected reference points. The generic function F can be arbitrary, such as least square estimation (LSQ) or multilateration algorithms. The problem of robust positioning with outlier detection is to select $\mathbf{D_s}$ appropriately so that all the outliers are excluded from $\mathbf{D_s}$ to avoid generating big positioning error.

A. Hardness of Robust Positioning with Outlier Detection

It is nontrivial to exactly select the normal distances. Existing methods generally apply geometrical filters onto distance set to identify outliers before position calculation, which include triangle inequality filter [15] and graph embeddability filter [8]. But these geometric filters are rough when the ranging noises of the normal distances are considered. As shown in Fig.1, d_3 is an outlier distance. But d_3 satisfies the triangle inequality conditions whether in triangle $X_1 X_3 \theta$, $X_2 X_3 \theta$ or in $X_3 X_4 \theta$, which cannot be detected by triangle inequality filter. Using rigid graph embeddability method, since d_3 is embeddable in a rigid graph component $\{X_3 X_4 X_2 \theta\}$, it will not be detected as an outlier. Therefore, geometric methods provide coarse-grained outlier detection. Some outliers may escape the filters due to non-rigidity caused by the ranging noises of normal distances.

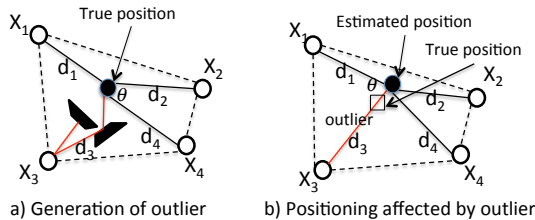a) Generation of outlier   b) Positioning affected by outlier

Fig. 1. Impacts of outlier in positioning algorithms

Other approaches detect outliers by checking the residue error of positioning result [9]. The residue error evaluates the difference between the selected distances and their corresponding *posterior distances* inferred by the estimated position of target.

$$R(\theta) = \frac{1}{|\mathbf{D}_s|} \sum_{i=1}^{|\mathbf{D}_s|} (\mathbf{D}_s(i) - \|\theta - \mathbf{X}_s(i)\|)^2 \qquad (1)$$

$|\mathbf{D}_s|$ is the size of set $\mathbf{D}_s$. The function $\|\theta - \mathbf{X}_s(i)\|$ returns the distance from the estimated position θ to the position of reference point $\mathbf{X}_s(i)$. If $\mathbf{D}_s$ contains outlier distances, the residue error will generally be large. Therefore, residue error is used as a metric to evaluate whether the selected set of distances contain outliers. The optimal positioning result is the position estimation which has the minimal residue error:

$$\theta^* = \arg\min_{\theta} \quad R(\theta) \qquad (2)$$

But the residue error function only tell which selected distance set may contain outliers, without the capability to directly identify outliers. To vote the distance set with the minimum residual error, enumeration over all distance combinations is needed, causing high computation cost when N is large.

Theorem 1 (Problem Hardness). *In d dimensional space with one target and N distance measurements, it is NP-hard to enumerate all combinations of distance measurements to find the set of distances which has the minimum residue error.*

Proof: In d-demential space, a position can be calculated by using at least $d + 1$ non-collinear measurements. Each position estimation is called a *potential position*. Therefore, N distance measurements can generate at most $\sum_{i=d+1}^{N} C_N^i$ potential positions by enumerating all combinations of the measurements. Since $\sum_{i=d+1}^{N} C_N^i$ is in the magnitude of $O(2^N)$, we cannot find a polynomial time algorithm to find the position point with the minimum residue error. Therefore, finding the position with the minimum residue error is NP-hard. ∎

B. Feasibility of Efficient Approach

Can we design efficient algorithms to address the optimal positioning problem while detecting the outliers? An efficient algorithm is proposed by reducing the size of the potential position set while keeping satisfactory positioning performances. Let's generate potential positions by selecting exactly $d+1$ (d is the dimension of space) measurements. Then N distances will generate at most C_N^{d+1} potential position points. We denote this set by $\{\theta_{d+1}\}$ and denote:

- $\{\theta_{d+1}^n\}$: *normal potential positions*, calculated by only normal distances;
- $\{\theta_{d+1}^o\}$: potential positions affected by distance outliers.

We show that the optimal position θ^* must be within the bounding region that covers $\{\theta_{d+1}^n\}$. We present the theorem in 2-D space, i.e., $d = 2$, which can be easily generalized into 3D space where $d = 3$.

Theorem 2. *let* $\mathbf{A}$ *be the bounding region covering the points in* $\{\theta_{d+1}^n\}$*, then* θ^* *which is the optimal positioning result that has the minimum residue error must also be covered by* $\mathbf{A}$*.*

Proof: Without losing of generality, let's suppose the ranging errors of normal distances are upper bounded by r, where r is determined by the accuracy of ranging techniques, and so that the distance from the ground truth position to a reference point i is bounded by $[d_i - r, d_i + r]$, which means that the target must locate in a circular ring centered at the reference point i, with inner radius $d_i - r$ and outer radius $d_i + r$. Since θ^* has the minimum residue error, it must be calculated by a set, i.e., $n \geq d+1$ normal distances, therefore θ^* must be in the intersected area of $n \geq d+1$ circular rings. We denote such an intersected region of n circular rings by $\mathbf{A}^*$. Because normal positions are calculated by normal distances, each point i in $\{\theta_{d+1}^n\}$ must be in the intersected region of $d+1$ circular rings, which are denoted by regions $\mathbf{A}^i$. Since $n \geq d+1$, if the n distance selected to calculate θ^* contains the $d+1$ distances selected for calculating point i, $\mathbf{A}^*$ must be within the region $\mathbf{A}^i$. Therefore, let $\mathbf{A} = \bigcup \mathbf{A}_i$ be the bounding region covering $\{\theta_{d+1}^n\}$; then $\mathbf{A}^*$ must be within $\mathbf{A}$. ∎

III. COFFEE: CLUSTERING AND FILTERING FOR OUTLIER DETECTION

Theorem 2 indicates that we can give a rather accurate estimation to θ^* if we can distinguish the point set $\{\theta_{d+1}^n\}$ from $\{\theta_{d+1}\}$. Since the number of normal distances is much more than the number of outliers, the normal positions, i.e., $\{\theta_{d+1}^n\}$ tend to form a dense cluster and the potential positions affected by outliers tend to be apart from the core cluster. Using such an idea, we present an efficient clustering and filtering algorithm (COFFEE) to distinguish the core cluster of $\{\theta_{d+1}^n\}$ from the potential positions, so as to accurately locate the target while identifying the outliers.

Inputs to COFFEE algorithm are: i) the distance set $\mathbf{D}$; ii) the coordinates of the reference points $\mathbf{X}$. Its outputs are i) the position estimation of target i.e., θ; ii) the set of detected distance outliers. The COFFEE algorithm contains an initializing phase and an online phase. In initialization phase, the potential positions $\{\theta^{d+1}\}$ are generated to form a Bipartite graph model to link the distance set to the potential positions. The online phase contains iterations of clustering, voting and outlier filtering.

A. Bipartite Graph Construction

In initialization phase, if feasible, each $d+1$ distances are used to calculated a potential position. The relationship between the distances and potential positions is modeled by a bipartite graph $G = \{\mathbf{D}, \mathbf{P}, \mathbf{E}\}$. $\mathbf{D}$ represents the distance set; $\mathbf{P}$ is the potential position set; and $\mathbf{E}$ denote the edges. $e_{i,j} = 1$ if the ith distance is used in calculating the jth potential position, otherwise $e_{i,j} = 0$. So that each potential position is linked to exactly $d+1$ distances. The potential position

generation process can be further optimized by selecting better geometries of $d+1$ reference points, such as selecting reference points with good geometric dilution of precision (GDOP) [16]. But we show by experiments that COFFEE is robust to the generation of potential positions. Even if the reference points with bad GDOP are selected to generate the potential positions, such potential positions will be filtered out during iterations of COFFEE and only pose little effect to the final positioning result.

B. Iterative Clustering, Voting and Filtering

The online phase of COFFEE conducts clustering, voting and filtering iteratively on the Bipartite graph.

Algorithm 1 COFFEE Algorithm

Require: $\mathbf{G} = (\mathbf{D}, \mathbf{P}, \mathbf{L})$, $\mathbf{X}_0 = \{\vec{x}_1, \vec{x}_2, \cdots, \vec{x}_N\}$;
Ensure: Valid distance set $\mathbf{D}_v$, Robust Position Estimation
    $\theta = F(\mathbf{D}_v, \mathbf{X}_v)$
1: ————Clustering and Weighting ————
2:   $\mathbf{D}_v = \mathbf{D}$
3: **while** $(sizeof(\mathbf{D}_v) > N_{min})$ **do**
4:     $[\mathbf{P}_{in}, \mathbf{P}_{out}] = DBSCAN(\mathbf{P}, MinPts, Eps)$;
5:     **for** $(j = 1; j \leq sizeof(\mathbf{P}_{out}); j++)$ **do**
6:       $y =$ the index of $\mathbf{P}_{out}[j]$ in $\mathbf{P}$;
7:       **for** $(k = 1; k \leq N; k++)$ **do**
8:         **if** $(\mathbf{L}(k, y) == 1)$ **then**
9:           $\mathbf{W}(d_k) = \mathbf{W}(d_k) + \omega$; //add doubting weight to d_k.
10:         **end if**
11:       **end for**
12:     **end for**
13:     —————-Filtering ————
14:   $i_{max} = \text{argmax } \{\mathbf{W}(i), i \subset [1, N]\}$;
15:   **if** $(\mathbf{W}[i_{max}] > Threshold)$ **then**
16:     $\mathbf{D}_v = \mathbf{D}_v \setminus d_{i_{max}}$, $\mathbf{X}_v = \mathbf{X}_v \setminus \vec{x}_{i_{max}}$;
17:     **for** $(j = 1; j \leq sizeof(\mathbf{P}); j++)$ **do**
18:       **if** $(\mathbf{L}(i_{max}, j) == 1)$ **then**
19:         $\mathbf{P} = \mathbf{P} \setminus \mathbf{P}[j]$;
20:       **end if**
21:     **end for**
22:   **else**
23:     break; //The loop stops to output $\mathbf{D}_v$ and p.
24:   **end if**
25: **end while**
26: —————Output Results—————
27: **Output** $\mathbf{D}_v$;
28: **Output** $\theta = F(\mathbf{D}_v, \mathbf{X}_v)$;

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1) Density-based clustering by DBSCAN

Initially all the distances and potential positions are labeled valid. Then in each iteration, COFFEE clusters the potential positions by a density-based clustering algorithm DBSCAN [4] with two user defined parameters: Eps and $MinPts$. Eps is a radius that delimitate the neighbourhood area of a point (Eps-neighbourhood). $MinPts$ is the minimum number of points required to be in the Eps-neighborhood to delimitate

the density requirement for clustering. By DBSCAN, potential positions with neighborhood density satisfying Eps and $MinPts$ will be classified into one cluster. The largest cluster is called core cluster and is denoted by $\mathbf{P}_{in}$. The points outside the core cluster are denoted by $\mathbf{P}_{out}$ which are called position outliers.

2) Voting by Doubting Weights

After DBSCAN, COFFEE traverses all the points in $\mathbf{P}_{out}$. For each point in $\mathbf{P}_{out}$, COFFEE assigned doubting weights to the distance linked to this point in the bipartite graph. If a distance has links to k position outliers, its doubting weight will be assigned k.

3) Filtering Distance and Position Outliers

After doubting weight assignment in each iteration, the distance with the maximum doubting weight will be detected as distance outlier and is removed from the valid distance set $\mathbf{D}_v$. The potential positions which are linked to the distance outlier in the bipartite graph are also removed from the valid position set. Note that although only 1 distance is removed in each iteration, all the potential positions connected to this outlier distance will be removed. The number of potential positions removed in one iteration can be up to C_{N-1}^d, which guarantees the fast convergence of the algorithm.

4) End Condition and Output

The iteration ends if all remained valid distances have doubting weight smaller than a *Threshold*. Then, the valid distance set $\mathbf{D}_v$ will be output to calculate the position of the target by applying F to the valid distance set $\theta = F(\mathbf{D}_v, \mathbf{X}_v)$. The pseudocode of COFFEE is shown in Algorithm 1.

C. Parameter Setting and Algorithm Properties

In COFFEE, Eps, $MinPts$ and $Threshold$ are key parameters, which affect the algorithm performances. $MinPts$ can be set to $C(N/2, d+1)$, which is the minimum number of valid positions, because the number of distance outliers are generally much smaller than $N/2$. Eps is set by learning the statistical variance of the ranging errors, so that it guaranteed that the normal positions form a core cluster w.r.t $MinPts$ and Eps. Another parameter $Threshold$ indicates the tolerance to the doubting weights, which is set to 1 in COFFEE to restrictedly exclude the effects of all the position outliers.

Using above parameter settings, COFFEE is guaranteed to converge in m iterations only if the normal potential positions can form the largest core cluster. This is generally true because the number of outliers is smaller than the number of normal distances. In each iteration, the computing complexity of COFFEE is $O(N^4 logN)$, because DBSCAN is the most computational intensive step in an iteration of COFFEE. DBSCAN has complexity $O(nlog(n))$ [4], where $n = C(N, 4)$ in our problem. Therefore DBSCAN has complexity $O(N^4)log(N)$ in COFFEE. After DBSCAN, each position outlier will be checked to assign doubting weights to the linked distances, which needs at most $4*C(N, 4)$ computations. Therefore, the overall complexity of COFFEE algorithm is $O(N^4)log(N)$ in each iteration, which has polynomial time complexity. COFFEE is also robust to the number of outliers, which will
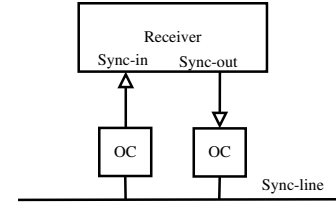

Fig. 2. Design of the Sync-line and receiver

be introduced in the evaluation section.

IV. ROBUST SYNCHRONIZATION AND DRAGON SYSTEM

Except the NLOS outliers, failure of time synchronization is another reason which may affect the positioning robustness. If the RF signal from sender is not received by some receivers, the receivers will not be correctly synchronized and cannot provide correct TOA measurements. Interferences are the main reason of synchronization failures. In some location systems, such as in Cricket [12] and AUITS [15] systems, the RF channel is used not only for time synchronization but also for transmitting data, the synchronization signal may collide with these data traffic signals. Interferences by same-band background signals are also inevitable, such as WiFi, microwave oven.

A. First-Falling-Edge Time Synchronization

Since the interference are generally inevitable, we propose to use sync-line to connect the US receivers to enable a robust, first-falling edge time synchronization technique as shown in Fig.2.

1) Sync-line and synchronizing logic: In each receiver, two IO ports of MCU are connected to the sync-line. The output port for setting electric level of sync-line is called "sync-out", while the input port for capturing falling edge is called "sync-in". Between MCU and sync-line, optocouplers for electric isolation are set as in Fig.2.

Assuming I is the state of sync-in port and O_i is the state of sync-out port of the ith receiver, the logical relationship for first falling edge time synchronization is in (3).

$$I = O_1 \cap O_2 \cap O_3 \cap \cdots \cap O_N \tag{3}$$

where N is the number of receivers. The states of sync-in pins of all receivers are always same and the states of sync-out pins of receivers are independent. When a receiver hears the sync packet from a tag, it sets its sync-out to 0 immediately and resume it to 1 after a fixed interval. As a result, the sync-in port of all the receivers is set to 0 when the first receiver hears sync packet.

B. Development of Dragon System

Based on above methodologies, a ultrasound TOA-based locating system called Dragon is developed which is composed by following components.

- **Tag**: each Tag is an active transmitter attaching on the mobile devices. It is composed of MCU, ultrasound drive circuit, wireless communication chip and battery.

- **Receivers** have ultrasound detection circuit, sync-line driving & detecting module and RF module. They are connected together with a cable to form a chain structure, therefore, we call the system "Dragon". Each receiver can also work in independent mode by plugging off the syncline. In independent mode, it is synchronized and communicate using own RF, which is used as counterpart for performance comparison in our study.

- **Host** contains powerful processor and Ethernet interface, which carries out time-slot scheduling for tracking multiple tag and runs COFFEE algorithm for position calculation. Fig.3 shows the pictures of the prototype of Dragon components.

C. Collision Avoidance for Multiple Target Localization

In Dragon, collision avoidance is a critical requirement for localizing multiple targets because single frequency ultrasound is used. Within the communication range of ultrasound, only one tag is allowed to send ultrasound for successful TOA measurement. We developed a time-slot scheduling scheme in Dragon for tracking multiple targets which is carried out by the host. The host broadcasts slot allocation by a mask packet of N bits, where N is the maximum number of Tags. The mask packet is denoted as B_{map}. Each bit in it stands for a Tag. If the mth bit is "1", tag m is allowed to send RF+US in time-slot s_m, where

$$s_m = \sum_{i=1}^{m} (B_{map} >> (i-1)) \& 1 \tag{4}$$

For example, B_{map} 10001101 means that Tag1 sends at at slot 1; Tag3 sends at slot 2; Tag4 sends at slot 3; and Tag8 sends at slot 4. The other tags have to keep silent in this cycle. Therefore, Tags transmit RF+US in successive slots, which not only avoids collision of ultrasound, but also improves the position updating rate than simple TDMA scheme [15].

D. Dragon Prototype

Based on above methodologies, we developed a prototype of Dragon, which is composed by a number of Tags and fifteen receivers. Each receiver can work independently without sync-line or connected by a sync-line. In sync-line connected mode, receivers will be synchronized by first-falling-edge-synchronization technique. In independent mode, receivers are synchronized and transmit data by RF module on the node. A host component using an ARM9 core at 400MHz is developed to collect TOA measurement from receivers. It calculate target positions by COFFEE algorithms. The specification of the Dragon prototype is shown in Table I. The photos of the prototypes are shown in Fig. 3.

V. NUMERICAL EVALUATIONS

Extensive evaluations were carried out to evaluate COFFEE algorithms and the positioning performance of the Dragon prototype. We firstly evaluate COFFEE by simulation. Then report the experimental results in a Dragon prototype.

TABLE I
SPECIFICATION OF DRAGON PROTOTYPE

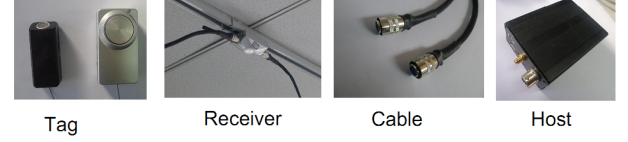|	MCU	Communication Unit	Sensor
Tag	Atmel Mega 128	CC1000 as RF unit.	Ultrasound transmitter at 40Khz
Receiver	Atmel Mega 128	CC1000 as RF unit. MCP2515 as CanBus unit.	40Khz Ultrasound transducer. Digital thermal sensor.
Host	Samsung S3C3440 with ARM9 core running at 400Mhz	MCP2515 as CanBus unit. 802.11 a/b WiFi unit.	NULL.

| Tag | Receiver | Cable | Host |

Fig. 3. Prototype of Dragon Components

A. *Simulation-based Evaluations of COFFEE*

1) Simulation Settings: In simulation, we assume N receivers are randomly distributed on the roof of a room of height $600cm$, and size 1000cm × 1000cm. At each positioning instance, a mobile tag appears at a random place inside this room. We assume N distance measurements are generated, among which, m of them are distance outliers. We assume the distance outliers have large ranging errors which are uniformly distributed in $[30cm, 200cm]$. The other $N-m$ normal distances have smaller ranging errors, which are uniformly distributed in $[-10cm, 10cm]$. The positioning error is evaluated by the Euclidean distance from the positioning result to the real position of the target.

2) Convergence Properties of COFFEE: We firstly evaluate the convergence property of COFFEE to verify how the positioning outliers are filtered in each iteration. An example is visually shown in Fig.4, which shows the outlier detection and rejection process of COFFEE when $N = 15$ and $m = 3$. The three NLOS outlier distances are detected and removed successfully in three iterations. In the figures, points indicate *potential positions*; circles indicate *position outliers*. The sub-
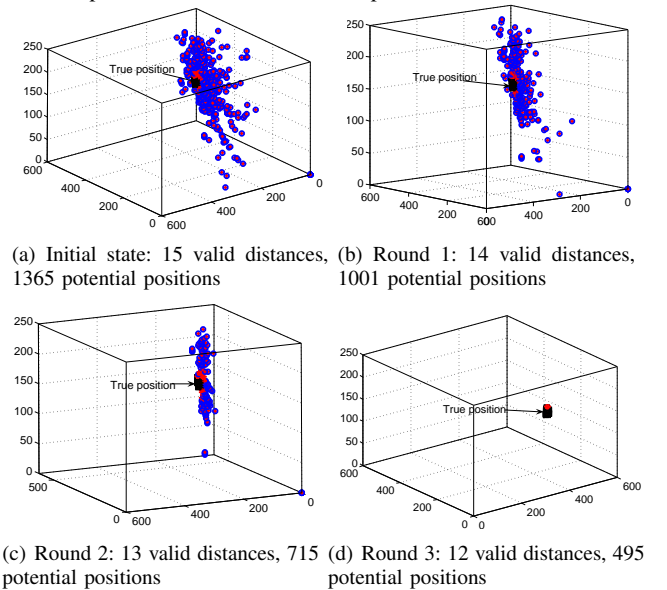

(a) Initial state: 15 valid distances, 1365 potential positions

(b) Round 1: 14 valid distances, 1001 potential positions

(c) Round 2: 13 valid distances, 715 potential positions

(d) Round 3: 12 valid distances, 495 potential positions

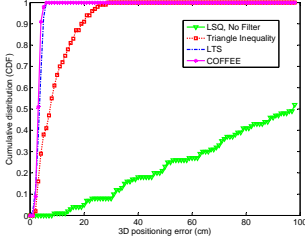Fig. 4. Outlier distances detection and rejection process in COFFEE
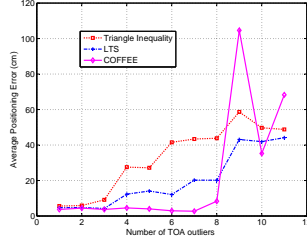
Fig. 5. CDFs of positioning errors

Fig. 6. Robustness COFFEE to the number of outliers.

titles of the figures show the number of valid distances and the number of potential positions in each iteration.

In all simulations, we found COFFEE converge quickly, which needs m iterations to output the robust position estimation. In comparison, if the position outliers are only filtered by DBSCAN, the convergence speed will be slow and cannot be guaranteed. In DBSCAN, although a core cluster can be found in the first iteration, it is hard for DBSCAN to further refine the core cluster in the next iteration. COFFEE uses voting scheme to remove the most doubted distance and corresponding *potential positions* in each iteration, which makes the algorithm converge very quickly.

3) Positioning Accuracy and Robustness Performances: The positioning accuracy and robustness performances are evaluated and are compared with three existing positioning and outlier filtering algorithms:

1) *LSQ, No Filter*, which calculates the target position by Least Sqare Estimation without filtering outliers.
2) *Triangle Inequality* filter [15], which iteratively applies triangle inequality conditions to filter outliers. The target position is calculated by LSQ using the valid distances.
3) *Least Trimmed Squares (LTS)* [9], which finds $N - m$ distances that minimizes the sum of the square residuals.

The cumulative distributions of the 3D positioning errors of different positioning algorithms are shown in Fig. 5. In the simulations, we set $N = 15$ and set the number of distance outliers $m = 3$. 100 identical experiments were run for each algorithm and the CDFs of the 3D positioning errors of these algorithms are compared. The results show that LSQ-based positioning without outlier filtering has large positioning errors in case of distance outliers. *Triangle Inequality* based outlier filtering performs much better than *No Filter*, but is not as good as the other two filtering algorithms. *COFFEE* and *LTS* both improve the positioning accuracy remarkably. Under the simulation settings, their positioning errors all have almost 100% probabilities to be less than 10cm. Among these three algorithms, COFFEE performs the best, which is a little better than LTS. Since COFFEE need less computation costs than LTS, it outperforms the state-of-the-art in terms of positioning accuracy, robustness and efficiency.

4) Robustness to the Number of outliers: Another important issue is the sensitivity of positioning accuracy to the number (portion) of outliers. We conduct simulations to evaluate how the positing accuracies are affected by the number of the distance outliers. For $N = 15$, we increase the number of distance outliers, i.e., m from 1 to 11. The positioning

accuracy performances of *No Filter*, *LTS*, and *COFFEE* are simulated and compared in Fig. 6. In the figure, every point is calculated by the average positioning errors of 100 random experiments. From the simulation results, we find the robust feature of the COFFEE algorithms.

- In COFFEE, the positioning error keeps small when $m < N/2$. Since the number of distance outliers is generally much smaller than the number of normal distances, the result indicates COFFEE is robust to the number of outliers in most application scenarios.
- For Triangle Inequality and LTS based outlier filtering algorithms, the positioning error increases with the number of distance outliers, which is not robust but sensitive to the number of outliers.

B. Practical Experiments using Dragon Prototype

Above simulation results have shown the good accuracy and robustness of the proposed Densest-Ball algorithms. We have also conducted hardware experiments in a prototype of Dragon system to test the real positioning performances in real environments.

1) Deployment of Dragon Prototype: A prototype of Dragon system is deployed in an indoor environment to evaluate the proposed COFFEE and Dentiest-ball algorithms and the robust time synchronization scheme. The deployed Dragon prototype contains 15 RF+US receivers which are connected by cables and are arranged in the roof of a room. The photos of the deployment scenario is shown in Fig.11. The positions of 15 receivers are manually calibrated and their positions on the room ceiling are shown in Fig.7, which forms a receiver array.

The room where the Dragon prototype is deployed is a meeting room of our laboratory , whose dimension is $600cm \times 400cm \times 260cm$. During experiments, 100 positions in the room are chosen as test points. The positions of these
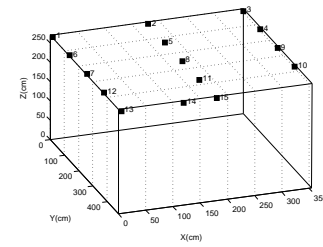

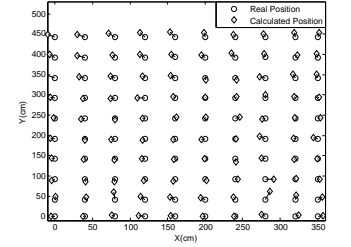
Fig. 7. Topology of RF+US receivers in Dragon

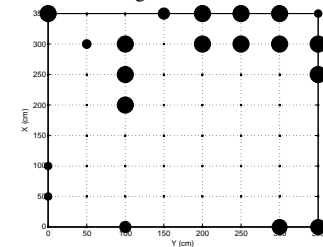Fig. 8. Test points and positioning error from top view

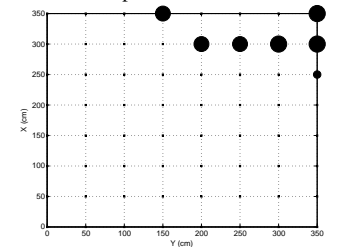Fig. 9. Positioning failure possibility without sync-line

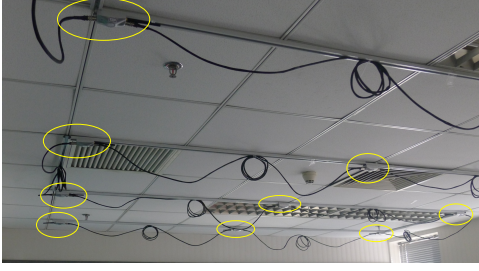Fig. 10. Positioning failure possibility with sync-line

Fig. 11. Deployment of Dragon prototype on the ceiling of a meeting room

Fig. 12. Indoor positioning error: 3D view

Fig. 13. Cumulative distribution function of positioning error

a) test settings

b) Angle effects to ranging errors

Fig. 14. Test the relative angle effects to ranging errors

points and 15 receivers are measured with EDMD (Electronic Distance Measuring Device) as ground truth. These test points are shown in Fig.8.

2) Performances of First-Falling-Edge Synchronization: We firstly conducted experiments to evaluate the performance of first-falling-edge synchronization. In the first setting, the cable between the receivers are plugged off. Receivers become independent. In the second setting, the receivers are connected by sync-line to be synchronized by first-falling-edge scheme. In these experiments, a Tag is localized for 30 times at each test point. In each time, if less than 4 distances are measured, a localization failure is counted. For each test point, its failure probability is evaluated by dividing its failure count by times of experiments (30). The positioning failure possibilities with and without sync-line are shown in Fig.9, Fig.10. The diameter of the circle is proportional to the failure probability. The results show that the First-Falling-Edge synchronization efficiently reduces the positioning failure probability.

3) Positioning accuracy in real-experiments: Experiments were carried out to evaluate the positioning accuracy performances using the Dragon prototype. In the experiments, a student puts a Tag at each point to wait about 10 seconds. Since the host allocates 100 ms time slot for each positioning routine, the Dragon system gives around 100 position estimations to each test point. During experiments, the student intentionally blocked the direct paths from Tag to receivers to to manually generate some NLOS outliers. The positioning error of each test point is evaluated by comparing the averaging position result to the ground truth.

The localization errors in 2D and 3D space are plotted in Fig.8 and Fig. 12 respectively. It can be seen that the positioning error in x and y dimension is smaller than that of z dimension. The reason is that the receivers are deployed in a x-y plane, so the positioning resolution in z dimension is lower than that of x and y dimensions. We also find the location accuracy is better in the center of the room than in the corners. This is mainly because the sender-receiver angle effects to the ranging errors, which will be discussed in the last subsection. The CDF(Cumulative distribution function) of positioning error is plotted in Fig.13. COFFEE outperform the no filtration case without uprising. Using COFFEE, the positioning error of Dragon system is < 10 cm with more than 80% probabilities.

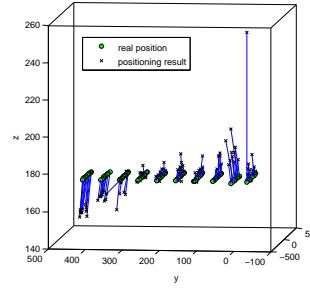*4) Analyzing to positioning errors in real-experiments:* When comparing to the simulation-based evaluations, we found that the positioning accuracy in rea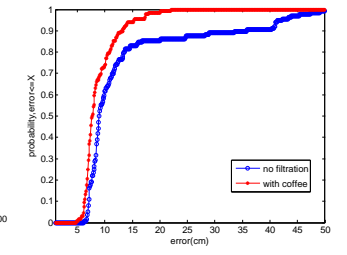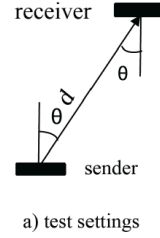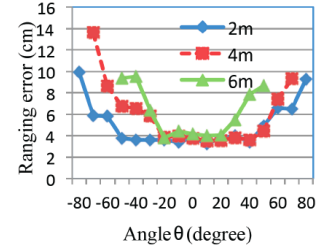l-experiments were much worse than the simulation results. We investigated the reason and found that the positioning errors in real-experiments were mainly caused by the ranging errors affected by the *relative angle* between the sender and the receivers.

We conducted a separated experiment to investigate how the ranging errors are sensitive to the *relative angles*. The settings of the experiments are shown in Fig.14 a). A receiver is controlled to move along a circle around a sender while keeping constance distance to change the relative angles. Three settings were tested, in which the sender-receiver distances are 2m, 4m and 6m respectively. Ranging errors were measured 50 times at each point. The average ranging errors versus the relative angles θ at different sender-receiver distances are shown in Fig14 b). The results show that in real-experiments, the ranging errors are sensitive to the relative angles, which can be as large as 14cm when the relative angles are large. This experiment verified that that test points at room corners that have large positioning errors are mainly affected by the angle effects .

VI. RELATED WORK

Various methods are proposed to improve the robustness of TOA-based position, which can be classified into geometric methods, statistical methods and systematic methods. *1) Geometric methods:* Some researchers studied to filter out the outliers by exploiting the geometry conditions among the measured distances. In [15], *Triangle Inequality* conditions are utilized to filter distance outliers. If a distance triple $\{d_{S,R_a}, d_{S,R_b}, d_{R_a,R_b}\}$ cannot satisfy the triangle inequality conditions, the longer distance between d_{S,R_a} and d_{S,R_b} is judged as an outlier. In the distance triple, S is a sender to be located and R_a and R_b are two arbitrary receivers. The algorithm repeats until all the remained distances pass the

conditions. This filter uses the heuristic that the NLOS paths are mainly caused by reflections, which are generally longer than the direct path. In [8], graph embeddability and rigidity are utilized to carry out the geometry based outlier detection. Only the edges (measured distances) that form embeddable redundantly rigid graphs are marked *normal* and the other edges are marked as *outliers*. But when ranging noises are considered, these geometric filters provide only loose restrictions. Some outliers may still pass these geometric filters. In [10], a bilateration based robust positioning algorithm is proposed. Using bilateration, two *candidate positions* can be estimated from two distance measurements in 2D space. After calculating all candidate positions via enumerating all bilateration pairs, the position of the densest candidate points is calculated as the target position. Although bilateration is robust, it is hard to be calculated in 3D space, because the intersections of balls are difficult to describe and store.

2) Statistical methods: Univariate statistical methods [1] are widely used methods for general outlier detection problems by assuming normal data obey some underlying distributions. But such methods are not suitable for detecting distance outliers, because the distance measurements are dynamic and don't obey a common distribution. Therefore, statistical methods for outlier detection should be applied on the position set instead of the distance set. Using this idea, Least Trimmed Squares (LTS) estimator is proposed in [9] to find m NLOS outliers from N distances by seeking $N - m$ Line-of-Sight distances that minimizes the sum of the square residuals. The lower bounds for NLOS positioning and more related works for least square and maximum likelihood based NLOS mitigation methods are surveyed in [5]. Different from these statistical methods, we present an efficient unsupervised learning approach, which improves both positioning accuracy and calculation efficiency.

3) Systematic methods: Other solutions improved robustness of TOA-based localization via system-type approaches. Because of the positioning variances will reduce when more TOA measurements are used in multilateration [2], [14] [7]proposed receiver array to obtain more TOA measurements for robust position calculation. In ATLINTIDA [3], cross correlation technique is used to detect TOA robustly by transmitting pseudorandom sequences of pulses and use wide-baud US signal. A robust wide-baud ultrasound positioning system was developed in [6]. In this paper, we focus on robust positioning by widely-used, low-cost, single frequency ultrasound.

VII. Conclusion

This paper studied robustness algorithm and system for ultrasound TOA-based indoor localization. By a bipartite graph model which maps the distance set to the potential positions, COFFEE algorithms is developed, which conduct unsupervised classification on the position set to eliminate the NLOS distances. The fast convergence and accuracy of these algorithms have been verified and analyzed, showing it outperforms the state-of-the-art algorithms. COFFEE also shows robustness to the number of distance outliers, which implies

its robust performances in various positioning scenarios. In system aspect, a first-falling-edge synchronization technique for robust time synchronization is developed, based on which we have developed a ultrasound positioning system, called Dragon. The positioning performances of Dragon prototype were verified in deliberately interfering environments. The results showed remarkable positioning robustness improvements by using the proposed methodologies. The angle effects to the positioning error have also been investigated. In future work, we will generalize the outlier detection algorithms to other positioning algorithms, such as radio-map based positioning methods.

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