Sharing Electricity Storage

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***Abstract*— The emerging sharing economy has disrupted the housing and transportation sectors. The underlining business model exploits underutilized infrastructure through sharing. In this paper, we explore sharing economy opportunities in electricity sector. There are considerable obstacles to sharing electricity. First, the flow of electricity is governed by Kirchoff's Laws and we cannot prescribe a point to point path for its flow. Second, regulatory and policy obstacles may impede sharing opportunities. As a result, early adopters will be in the context of behind-the-meter sharing opportunities. In this paper, we study one of these opportunities. Specifically, we consider a collection of firms that invest in storage to arbitrage against the time of use pricing they face. We show that the investment decision of the firms form a Nash equilibrium which supports the social welfare. We offer explicit expressions for optimal storage investments and equilibrium prices for shared storage in a spot market. Finally, we use field data to assess the performance of our proposed sharing scheme.**

***Index Terms*— Sharing economy, electricity storage, time-of-use pricing, Nash equilibrium**

I. Introduction

The sharing economy has witnessed dramatic recent growth. Its worldwide market size has reached $113 billion in 2015, up $40 billion from 2014, as shown in Fig. 1. While sharing better utilizes underused resources in transportation, housing, and many other sectors, can similar business models realize better asset utilization in the power grid? In this paper, we explore and analyze the specific opportunity of sharing electricity storage.

A. Opportunities and Challenges for the Electricity Sector

Thus far, sharing economy successes in the electricity sector have been confined to crowd-funding [1]. We can imagine more lucrative examples of the sharing economy in energy systems, such as a) sharing flexible demand recruited by an aggregator or utility; b) sharing unused capacity in the installed electricity storage. A principal difficulty with sharing economy business models for electricity is in tracing power flow point-to-point [2]. Electricity injected at various nodes and extracted at others flows according to Kirchoff's Laws, and we cannot assert that a kWh of electricity was sold by one party to another. As a result, supporting peer-to-peer shared electricity services requires coordination in the hardware that transfers power [3]. An alternative is to devise pooled markets which is possible because electricity is an undifferentiated good. Regulatory and policy obstacles may impede wider adoption of sharing in the electricity sector [4].

This work is supported in part by CERTS under sub-award 09-206, NSF under Grants ECCS-0925337, 1129061, CNS-1239178.

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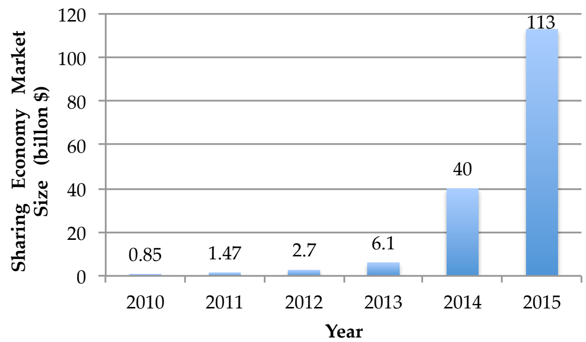

Fig. 1: Trend of sharing economy's market size worldwide, data source: [5], [6].

The early adopters of sharing in the electricity sector will be in behind-the-meter opportunities such as industrial parks or campuses. Here, sharing can be conducted privately without utility interference.

B. Related Work

There is recent literature on estimating the arbitrage value and welfare effects of storage in power systems. Graves *et al.* observe electricity storage's opportunity in arbitrage in the deregulated electricity market in [7]. In [8], Sioshansi *et al.* explore the role of storage in wholesale electricity markets. Bradbury *et al.* examine the economic viability of the storage systems through price arbitrage in [9]. Van de Ven *et al.* propose an optimal control framework for end user energy storage devices in [10]. Zheng *et al.* introduce agent-based model to understand tariff arbitrage opportunities for storage systems in the residential sector in [11]. While these previous works illuminate the economic value of storage, to the best of our knowledge, the analysis of *shared electricity services* has not been addressed in the literature.

C. Our Research Contributions

We study the specific problem of a collection of firms sharing their electricity storage. The principal contributions of this paper are:

- *Under or Over Investment*. Intuitively, one might imagine that sharing reduces the total investment in storage capacity. However, in Section III-B, we show that sharing may at times increase the total investment because firms may choose to monetize larger revenue opportunities that arise from sharing.

- *Equilibrium Characterization*: We show that the investment decisions of the firms form a Nash equilibrium which supports the social welfare. We offer explicit expressions for optimal storage investments and equilibrium prices for sharing in Section IV.
- *Coalitional Stability*: We prove that at this Nash equilibrium, no firm or subset of firms is better off defecting to form their own coalition. This is a much stronger stability guarantee than that offered in general Nash equilibrium theory, where only individual rationality is assured.

The remainder of this paper is organized as follows. In Section II, we describe our problem formulation. Then, we treat the optimal storage investment problem when there is no sharing between firms in Section III. We investigate the case when the firms have the opportunity of sharing their unused capacity in Section IV. We conduct simulations to verify our proposed theory to illuminate our results in Section V. Concluding remarks and future directions are given in Section VI.

II. System Model

Consider a scenario where an aggregator serves electricity to a set $\mathcal{N}$ of firms. The aggregator acts as the interface between these firms and the power grid, and itself does not consume any electricity. In other words, the aggregator purchases the total electricity needed by the firms from the grid and then resells it to the firms. We imagine the firms can trade electricity with each other, or purchase from the grid through the intermediary aggregator. These physical delivery of electricity for these transactions are conducted over a private distribution system within the aggregators purview. Prices imposed by the grid are passed through to the firms. The aggregator does not have the opportunity to sell excess electricity back to the grid (that is, there is no net metering).

Remark: Examples of this situation includes industrial park, university campus, and residential complex. The aggregator might be the owner of the industrial park, the university campus, or the housing complex community, respectively. The common feature of these examples is that it is possible to develop behind the meter transactions, which could be outside the regulatory jurisdiction of the utility. Also, in these examples, there is a single metered connection point with the utility company, and the underlining distribution grid could be private.

In our system, each day is divided into two fixed, contiguous periods – peak hours and off-peak hours. The firms face the common time-of-use (ToU) prices, as illustrated in Figure 2. During peak hours, they face a price π_h, while during off-peak hours, they face a discounted price π_ℓ. These prices are usually fixed and known.

We denote the consumption of firm k during peak and off-peak periods by the random variables X_k and Y_k respectively. Let $F_{X_k}(\cdot)$ and $F_{Y_k}(\cdot)$ be the known cumulative distribution functions (*cdf*s) of these random variables. These

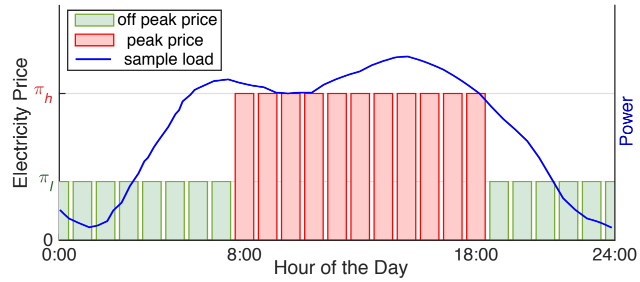

Fig. 2: Time-of-Use Pricing.

can be estimated from historical data using standard methods [12].

If storage is sufficiently cheap, firms will invest in storage to arbitrage ToU pricing. They could charge their storage during off-peak hours when electricity is cheap, and discharge it during peak hours when it becomes expensive. Note that the energy that held in storage is always acquired at price π_ℓ.

Let π_s be the daily capital cost of storage amortized over its lifespan. Define the arbitrage price

$$\pi_\delta = \pi_h - \pi_\ell > 0. \tag{1}$$

Clearly, we require $\pi_s < \pi_\delta$ for storage to offer a viable arbitrage opportunity.

Remark: Electricity storage is expensive. Tesla's Powerwall, for example, offers the amortized cycle cost around 25¢/kWh [13].

At current storage prices, ToU pricing rarely offers arbitrage opportunities. An exception is PG&E A6 program where the electricity price for peak hours (from 12:00 pm to 6:00 pm) is around 54¢/kWh; the price for partial peak hours (from 8:30 am to 12:00 pm, and from 6:00 pm to 9:30 pm) is around 25¢/kWh; and that for off peak hours (the rest of the day) is around 18¢/kWh [14]. Our results offer a framework for the analysis of shared storage in the future when storage prices are even lower. Storage prices are projected to reduce by 30% by 2020 [13].

To simplify our analysis we assume that the electricity storage is lossless, and is perfectly efficient in charging and discharging. Also, the storage investments by the firms are made simultaneously. We will briefly discuss how to dispense with these assumptions in the concluding remarks.

III. Main Results: No Sharing

In this section, we treat the single firm case. The analysis will help us better understand the structure of the problem and cast light on solving the more complicated case - firms which can share energy between each other.

A. Optimal Investment Decisions

Let X, Y be the random peak and off-peak consumption of a firm. When there is no storage system, its daily expected cost, denoted by J^0, is composed of two parts: the energy payment during peak hours, and the energy payment during off-peak hours:

$$J^0 = \mathbb{E}\left[\pi_h X + \pi_\ell Y\right]. \quad (2)$$

When the firm elects to invest in storage capacity C, its daily cost, denoted by $J(C)$, has a new component - the amortized storage cost. In addition, the other two parts - the energy payment during peak and off peak hours - *no longer* remain the same. The firm can always fully charge the battery during off-peak hours to avoid purchasing energy from the aggregator during peak hours. Thus, the firm will purchase $(X - C)^+$ from the aggregator during peak hours, where $(x)^+ = \max\{x, 0\}$, and then purchase $Y + \min\{C, X\}$ during off-peak hours to support its own off-peak consumption and recharge the battery. Hence, its daily expected cost is

$$\begin{aligned} J(C) = \pi_s C + \mathbb{E}\left[\pi_h (X - C)^+\right] \\ + \mathbb{E}\left[\pi_\ell Y + \pi_\ell \min\{C, X\}\right]. \end{aligned} \quad (3)$$

Theorem 1. *The optimal decision of a firm under no sharing is to purchase C^o kWh of storage where*

$$F(C^o) = \frac{\pi_\delta - \pi_s}{\pi_\delta} = \gamma. \quad (4)$$

$F(\cdot)$ is the cdf *of random variable X.*

Remark: It is straightforward to observe that the optimal storage investment C^o is monotone decreasing in the amortized storage price π_s and monotone increasing in the arbitrage price π_δ.

B. An Example: Merge Firms

To better understand the sharing economy in smart grid, we consider a simple scenario: two firms want to merge together. Before merging, their optimal decisions are to purchase C_1 and C_2 of storage, respectively. After merging, the optimal decision for the merged firm is to purchase C_m of storage. Intuitively, C_m should be smaller than the sum of C_1 and C_2 since without sharing, firms might over-invest in storage because they are going it alone and do not have the opportunity to buy stored electricity from other firms. However, we use the following example to highlight that they might also under-invest because they forgo revenue opportunities that arise from selling their stored electricity to other firms. The particular nature of this sub-optimality depends on the statistical character of their consumption relative to other firms.

Consider two firms, indexed by $k = 1, 2$, whose peak period demands are the discrete random variables X_1, X_2 respectively. Suppose X_1, X_2 are *i.i.d.*, and for $k = 1, 2$,

$$X_k = \begin{cases} 1 & \text{with probability } 0.5, \\ 2 & \text{with probability } 0.5. \end{cases}$$

Then, we have

$$F_{X_k}(x) = \begin{cases} 0 & \text{if } x < 1, \\ 0.5 & \text{if } x \in [1, 2), \\ 1 & \text{if } x > 1. \end{cases}$$

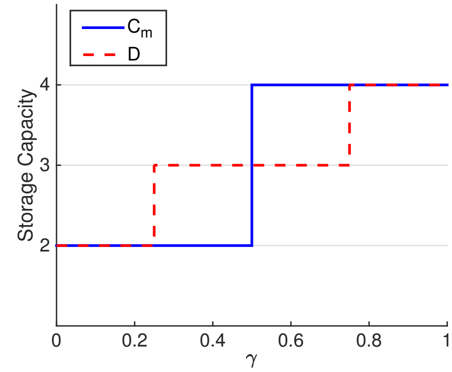

Fig. 3: Under- and over-investment.

For fixed $\gamma \in [0, 1]$, the optimal storage investment of both firms is identical, and using Theorem 1, we know

$$C^o = F_{X_k}^{-1}(\gamma) = \begin{cases} 1 & \text{if } \gamma \in [0, 0.5], \\ 2 & \text{if } \gamma \in (0.5, 1]. \end{cases}$$

Their combined storage investment is $C_m = 2C^o$.

On the other hand, for the merged entity, we can calculate the *cdf* $F_X(x)$ for the combined peak period demand $X = X_1 + X_2$:

$$F_X(x) = \begin{cases} 0 & \text{if } x < 2, \\ 0.25 & \text{if } x \in [2, 3), \\ 0.75 & \text{if } x \in [3, 4) \\ 1 & \text{if } x \geq 4. \end{cases}$$

Using Theorem 1 again, we can show the optimal storage investment D for the merged entity, which is

$$D = F_X^{-1}(\gamma) = \begin{cases} 2 & \text{if } \gamma \in (0, 0.25], \\ 3 & \text{if } \gamma \in (0.25, 0.75], \\ 4 & \text{if } \gamma \in (0.75, 1]. \end{cases}$$

We compare C_m and D in Figure 3. We note that $D < C_m$ when $\gamma \in (0.25, 0.5)$ and $D > C_m$ when $\gamma \in (0.5, 0.75)$. ■

IV. Main Results: With Sharing

In the situation where firms can share their unused storage capacity, we must explore the market structure that supports this sharing. As electricity is an undifferentiated good, it is natural to consider a spot market for stored energy in this context. We derive the equilibrium price for shared electricity in this spot market. This allows us to analyze the competitive behavior and the storage investment decisions of these firms.

Consider again the set $\mathcal{N}$ of firms. Firm $k \in \mathcal{N}$ has chosen to invest in C_k kWh of storage to arbitrage against the ToU pricing it faces. On a given day, suppose the total energy consumption of firm k is X_k during peak hours, and Y_k during off-peak hours. Similar to the analysis in Section III, the firm will choose to service X_k first using its cheaper stored energy. This may result in a surplus of stored energy or deficit of demand. The excess energy available to firm k in its storage is $(C_k - X_k)^+$. The deficit of energy that firm k must acquire during peak hours is $(X_k - C_k)^+$. The collated surplus from the firms can be sold to other firms that face a deficit, which enables the sharing between firms.

A. The Spot Market for Stored Energy

Let S be the total supply of energy available from storage from the collective of firms after they service their own peak period demand. Let D be the total deficit of energy that must be acquired by the collective of firms (after they service their own peak period demand). It is clear that

$$S = \sum_{k\in\mathcal{N}} (C_k - X_k)^+, \quad D = \sum_{k\in\mathcal{N}} (X_k - C_k)^+.$$

Consider a competitive spot market for trading energy. If $S > D$, the suppliers compete against each other and drive the price down to their (common) acquisition cost of π_ℓ. Note that unsold supply is simply held. Since all supply was acquired at price π_ℓ, this is equivalent to selling unsold supply at π_ℓ to an imaginary buyer, and buying it back during the next off-peak period at price π_ℓ. Note that the storage is completely discharged during the peak period, and fully recharged during the subsequent off-peak period. So the entire supply S is sold at π_ℓ if $S > D$.

If $S < D$, consumers compete and drive up the price to that offered by the aggregator π_h. Note that all unmet demand can be supplied by the aggregator (through the grid). This is also at price π_h. Thus, the entire demand is supplied at price π_h if $D > S$.

The market clearing price is therefore

$$\pi_{eq} = \begin{cases} \pi_\ell & \text{if } S > D, \\ \pi_h & \text{if } S < D. \end{cases} \tag{5}$$

Note that π_{eq} can be defined arbitrarily between π_ℓ and π_h when $S = D$, and it won't affect our subsequent analysis because the clearing price π_{eq} is a continuous random variable. In fact, S and D are also random variables due to the randomness in X_k's.

B. Optimal Investment Decisions

Let us first examine the expected cost function, denoted by J_k, for firm k:

$$\begin{aligned} J_k(C_k) = \pi_s C_k + \pi_\ell C_k + \pi_\ell \mathbb{E}\left[Y_k\right] \\ + \mathbb{E}\left[\pi_{eq}(X_k - C_k)^+ - \pi_{eq}(C_k - X_k)^+\right]. \end{aligned} \tag{6}$$

The five terms above are (in sequence) amortized cost of storage system, recharging of storage, supporting the off-peak demand, buying deficits, and selling surpluses.

Note that

$$(X_k - C_k)^+ - (C_k - X_k)^+ = X_k - C_k.$$

Thus, combining the last two terms in (6) yields

$$J_k(C_k) = \pi_s C_k + \pi_\ell C_k + \pi_\ell \mathbb{E}\left[Y_k\right] + \mathbb{E}\left[\pi_{eq}(X_k - C_k)\right]. \tag{7}$$

Note that, this cost function does not solely depend on C_k, but also all the other C_i's, $i \in \mathcal{N}, i \neq k$. They are coupled through π_{eq}. This naturally forms a game between all the firms.

Energy Sharing Game

- *Players*: the set $\mathcal{N}$ of all the firms;
- *Strategies*: the optimal energy storage investment C_k;
- *Payoffs*: for each firm $k \in \mathcal{N}$, its expected payoff function $g_k(C_k) = -J_k(C_k)$.

Thus, the optimal investment decision for firm k is simply firm k's best response to all the other firms' choices.

Denote the total demand during peak hours of all firms by

$$X^c = \sum_{k\in\mathcal{N}} X_k.$$

We further denote the probability density function of X^c by $f_{X^c}(\cdot)$. Suppose firm $i, i \neq k$, has invested in C_i kWh of storage. Denote

$$C_{-i} = \sum_{i\neq k} C_i.$$

To avoid degenerate cases, we make the following statistical assumption:

Assumption 2. *The probability density function $f_k(\cdot)$ is continuously differentiable and $f_{X^c}(x) > 0$ for $x \geq 0$.*

Then, we can characterize the best response as follows.

Lemma 3. *The first order optimality condition of firm k under sharing is as follows:*

$$\begin{aligned} 0 =& \pi_s - \mathit{Prob}(S < D)\pi_\delta + \pi_\delta C_k^0 f_{X^c}(C_k^0 + C_{-i}) \\ &- \pi_\delta f_{X^c}(C_k^0 + C_{-i}) \mathbb{E}\left[X_k | X^c = C_k^0 + C_{-i}\right]. \end{aligned} \tag{8}$$

C. The Competitive Equilibrium

Based on Lemma 3, we can further characterize the competitive Nash equilibrium and analyze its properties.

Theorem 4. *If there exists a Nash equilibrium, then the energy sharing game admits a unique Nash equilibrium, i.e. all firms make optimal storage investments. At this equilibrium, the optimal storage investments of the forms are given by*

$$C_k^o = \mathbb{E}[X_k | X^c = C^c], \ k \in \mathcal{N}. \tag{9}$$

where $C^c = \sum_k C_k^o$ solves

$$F(C^c) = \frac{\pi_\delta - \pi_s}{\pi_\delta} = \gamma. \tag{10}$$

These optimal storage investments support the social welfare.

Remark: Theorem 4 shows that given the existence of equilibrium, the optimal decisions, i.e., C_k^o forms a Nash equilibrium, and the cumulative investments coincide with the optimal investment of the combined entity. Furthermore, according to Theorem 4, the only information needed by each firm is the aggregate decisions and the aggregate statistics. This warrants protecting the private information of other firms. The uniqueness proof relies on Assumption 2. If this assumption is violated, we can construct examples where infinitely many equilibria exist. For example, when the total energy consumption X^c is a constant, it is straightforward to verify that any storage investment satisfying $\sum_k C_k = X^c$ is a Nash equilibrium.

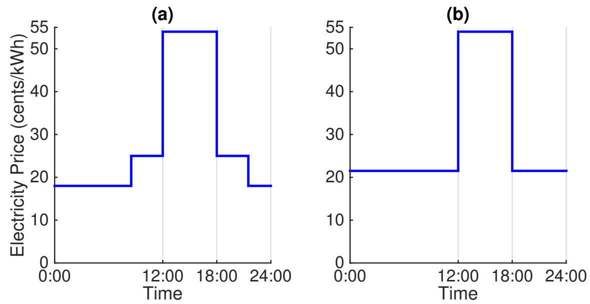

Fig. 4: ToU pricing: (a) real three-period pricing, (b) simplified two-period pricing.

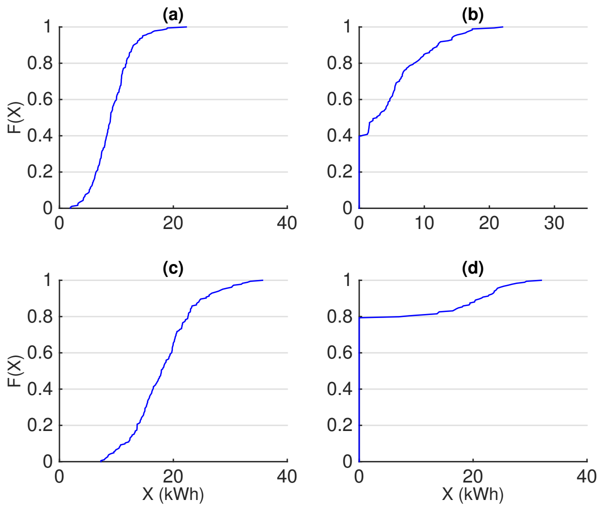

Fig. 5: Sample cumulative distribution functions of X_k's.

Theorem 5. *The Nash equilibrium of Theorem 4 has the following properties:*

(a) If firm k has peak-period consumption $X_k = 0$, it will not invest in storage, i.e. $C_k^o = 0$.

(b) The investment decisions of the firms are individually rational.

(c) No firm or subset of firms is better off defecting to form their own coalition.

Remark: As the aggregator does not itself consume electricity, it has no profit incentive to invest in storage. An important consequence of the above result is that the aggregator is in a position of neutrality with respect to the firms. It can therefore act to supply the information necessary for firm k to make its optimal investment choice. This information consists of (a) the joint statistics of X_k and X^c, and (b) the cumulative optimal investment C^c of all the firms. With this information, firm k can compute its share of the optimal storage investment C_k^o as in (9). As a result, the private information $X_i, i \neq k$ of the other firms is protected. The neutrality of the aggregator affords it a position to operate the market and determine the market clearing price of shared storage.

V. Simulation Studies

Having proved the theoretical properties of our scheme, in this section, we seek to use numerical examples to illuminate its performance. For instance, in real world, when users choose not to share with each other, do they often over invest or under invest? How much the end users may benefit from sharing? To answer these questions, we need to setup the ToU pricing scheme and utilize real data to model users' peak hour consumption - X_k's.

Figure 4(a) shows the three-period pricing scheme (peak, partial peak, and off peak), employed by PG&E during the summer. In the simulation, for simplicity, we use the simplified two-period pricing scheme, as shown in Figure 4(b). We set $\pi_h = 54$¢/kWh, and $\pi_\ell = 21.5$¢/kWh, which is the average of the partial peak and off peak prices. We use the Pecan Street data set [15] to validate the performance of our results. Figure 5 demonstrates some sample *cdf*s of the X_k's. In particular, Figure 5(a) shows the general *cdf* of a user. Figure 5(b) and (c) illustrates that some of the users may be constantly away from home during peak hours, while others have a constant background load. In addition, some user may even have a step function as the *cdf*, as shown in Figure 5(d). This means that this user has a rather stable demand during peak hours.

It is straightforward that we do not want most of the correlation coefficients between firms close to one, in which case there will be little potential for sharing to improve the social welfare. We plot the histogram of the pairwise correlation coefficients in the Pecan Street data set in Figure 6. The average pairwise correlation coefficient between the users is around 0.5, and there are also many valuable users creating negative correlations. In this situation, we conduct the simulation to compare two cases:

- *without sharing*: the users choose not to share their storage systems with others;
- *with sharing*: we employ the peer-to-peer sharing scheme to enable the energy sharing between the users.

We have already showed that in these two cases, in principle, it is not clear which case will lead to a smaller total storage capacity. Figure 7 demonstrates that, for the Pecan Street data set, sharing will likely reduce the average storage investment, and hence reduce the total storage investments. In other words, without sharing, the users are likely to over invest the storage systems. Also, with the number of participants in the aggregator increases, the average storage capacity under no sharing stays the same (after 50 participants) while the average storage capacity under sharing is increasing. Hence, we believe sharing will further speed up the deployment of end user storage systems in the near future.

Finally, we investigate the average net profit for the participants. Recall that we have defined the expected costs for each user without storage system, J^0 in (2), and without storage system, $J(C)$ in (3). Thus, we can define the difference between these two costs as the net profit for each user. More

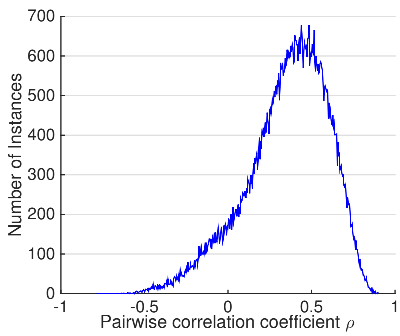

Fig. 6: Pairwise correlation coefficients histogram in the data set.

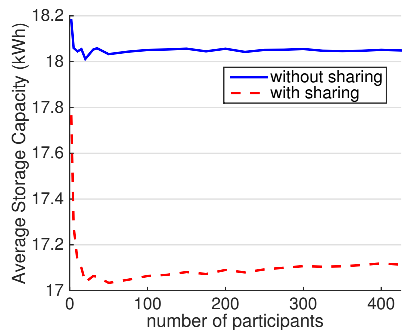

Fig. 7: Average optimal investment in storage systems.

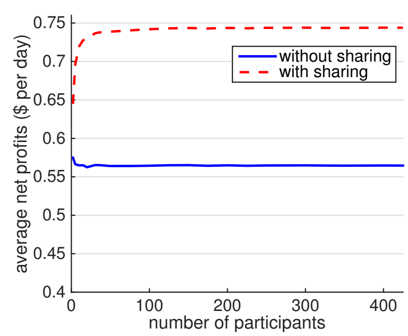

Fig. 8: Average savings through arbitrage.

precisely, the net profit $L(C)$ is:

$$\begin{aligned} L(C) &= J^0 - J(C) \\ &= (\pi_h - \pi_\ell)\mathbb{E}\left[\min\{C, X\}\right] - \pi_s C. \end{aligned} \tag{11}$$

Based on this definition, Figure 8 shows that although the users earn more than 55¢ per day without sharing on average, sharing can improve the average net profit by almost 50%. We imagine for industrial loads, sharing will also be able to improve the average net profit by a comparable percentage. Unfortunately, without the access to proper industrial load data sets, we are not able to verify this conjecture.

VI. Conclusions and Future Work

In this paper, we investigate the sharing economy for smart grid. In particular, we analyze the peer-to-peer energy transactions between a collection of firms. We characterize the competitive equilibrium and demonstrate its properties. Finally, we use numerical studies to illustrate the real world performance of proposed scheme. Besides the degenerate case that we discuss in Section IV-C, we are aware of examples where a Nash equilibrium does not exist. We plan to characterize the sufficient conditions for the Nash equilibrium existence in our subsequent work.

Our work is built upon two assumptions. The first one assumes the storage is lossless and perfectly efficient. Most of our analysis can accommodate the power losses and charging or discharging inefficiencies. However, the competitive equilibrium analysis could be challenging, especially when every storage system has its own characteristics. The second assumption is related to the equilibrium realization. When the decisions are made sequentially, instead of simultaneously, we need to introduce the notion of membership fee from club theory [16]. In essence, every new firm who wants to join the aggregator needs to pay a membership fee to accommodate the change in the equilibrium. And this fee could be positive or negative if including the new firm will make everyone better off (e.g., the new firm's peak hour energy consumption has negative correlations with most of the existing firms). A rigorous treatment to dispense these assumptions is one of the most interesting future research directions for us.

Appendix: Proofs

A. *Proof of Theorem 1*

The cost function for a single firm under no sharing is

$$\begin{aligned} J(C) =& \pi_s C + \mathbb{E}\left[\pi_h (X-C)^+ + \pi_\ell Y + \pi_\ell \min\{C, X\}\right] \\ =& \pi_s C + \pi_h \int_C^\infty (x - C) f_X(x) dx + \mathbb{E}\left[\pi_\ell Y\right] \\ & + \pi_\ell C \text{ Prob}\left(X \geq C\right) + \pi_\ell \int_0^C x f_X(x) dx. \end{aligned}$$

One can verify that this is strictly convex in C. The optimal investment C^o is the unique solution of the first-order optimality condition:

$$\begin{aligned} 0 &= \frac{dJ}{dC} = \pi_s - \pi_h \int_C^\infty f_X(x) dx + \pi_\ell \text{Prob}\left(X \geq C\right) \\ &= \pi_s + (\pi_\ell - \pi_h)\left(1 - F(C)\right). \end{aligned}$$

Rearranging this expression yields the claim. □

B. *Proof of Lemma 3*

Note that the clearing price π_{eq} is random. Let

$$p = \text{Prob}\left(S > D\right). \tag{12}$$

Denote the total storage in the system by C^t, where

$$C^t = \sum_{k \in \mathcal{N}} C_k.$$

Then, we have

$$\begin{aligned} \mathbb{E}\left[\pi_{eq}\right] &= p\pi_\ell + (1-p)\pi_h = \pi_h - p\pi_\delta, \\ \frac{\partial \mathbb{E}\left[\pi_{eq}\right]}{\partial C_k} &= -f_{X^c}(C^t)\pi_\delta. \end{aligned}$$

Recall that the expected cost function for firm k:

$$J_k(C_k) = \pi_s C_k + \pi_\ell C_k + \mathbb{E}\left[\pi_\ell Y_k + \pi_{eq}(X_k - C_k)\right] \tag{13}$$

The random variables π_{eq} and X_k are possibly dependent. Taking derivatives with respect to C_k:

$$\begin{aligned} 0 =& \pi_s - (1-p)\pi_\delta + \pi_\delta f_{X^c}(C^t) C_k \\ & - \pi_\delta f_X^c(C^t) \int_{x_k=0}^\infty x_k f_{X_k|X^c}(x_k|C^t) dx_k. \end{aligned} \tag{14}$$

Note that

$$\int_{x_k=0}^{\infty} x_k f_{X_k|X^c}(x_k|C^t)dx_k = \mathbb{E}\left[X_k|X^c = C^t\right]. \quad (15)$$

Combining (15) into (14) yields the lemma. □

C. Proof of Theorem 4

Let $D_k, k = 1, \cdots, n$ be any Nash equilibrium. We show that $D = C^*$ where

$$C_k^* = \mathbb{E}\left[X_k \mid X_c = Q\right], \quad F_c(Q) = \gamma = \frac{\pi_\delta - \pi_s}{\pi_\delta}$$

Under Assumption 2, simple algebra reveals that Q is the unique solution of

$$\pi_s - \pi_\delta + \pi_\delta F_c(Q) = 0$$

Let $\beta = \sum_k D_k$, and define the constants

$$\begin{aligned} K_1 &= \pi_s - \pi_\delta + \pi_\delta F_c(\beta) \\ K_2 &= \pi_\delta f_{X_c}(\beta) > 0 \end{aligned}$$

Define the index sets

$$\mathbb{M} = \{i : D_i > 0\}, \quad \mathbb{N} = \{j : D_j = 0\}$$

Since D is a Nash equilibrium, it follows that D_i satisfies the first-order optimality conditions. For $i \in \mathbb{M}$:

$$\begin{aligned} 0 &= \left.\frac{dJ_i(C_i \mid D_{-i})}{dC_i}\right|_D \\ &= K_1 - K_2 \cdot \mathbb{E}\left[X_i - D_i \mid X_c = \beta\right] \quad (16) \end{aligned}$$

The first-order optimality conditions for $j \in \mathbb{N}$ are:

$$\begin{aligned} 0 &\leq \left.\frac{dJ_j(C_j \mid D_{-j})}{dC_j}\right|_D \\ &= K_1 - K_2 \cdot \mathbb{E}\left[X_j - D_j \mid X_c = \beta\right] \quad (17) \end{aligned}$$

Summing these conditions, we get

$$\begin{aligned} 0 &\leq nK_1 - K_2 \cdot \sum_{k=1}^{n} \mathbb{E}\left[X_k - D_k \mid X_c = \beta\right] \\ &= nK_1 - K_2 \cdot \mathbb{E}\left[X_c - \beta \mid X_c = \beta\right] \\ &= nK_1 \end{aligned}$$

Thus, $K_1 \geq 0$. Using (16), for any $i \in \mathbb{M}$ we write

$$\mathbb{E}\left[X_i - D_i \mid X_c = \beta\right] = \frac{K_1}{K_2} \geq 0 \quad (18)$$

Next, we have

$$\begin{aligned} 0 &= \sum_{k=1}^{n} \mathbb{E}\left[X_k - D_k \mid X_c = \beta\right] \\ &= \sum_{i \in \mathbb{M}} \mathbb{E}\left[X_i - D_i \mid X_c = \beta\right] + \sum_{j \in \mathbb{N}} \mathbb{E}\left[X_j \mid X_c = \beta\right] \\ &\geq \sum_{i \in \mathbb{M}} \mathbb{E}\left[X_i - D_i \mid X_c = \beta\right] \\ &\geq 0 \end{aligned}$$

Here, we have used (18) and the fact that the random variables X_j are non-negative. As a result, we have

for $i \in \mathbb{M}$: $\quad \mathbb{E}\left[X_i - D_i \mid X_c = \beta\right] = 0$

for $j \in \mathbb{N}$: $\quad \mathbb{E}\left[X_j - D_j \mid X_c = \beta\right] = \mathbb{E}\left[X_j \mid X_c = \beta\right] = 0$

So for $k = 1, \cdots, n$,

$$0 = \mathbb{E}\left[X_k - D_k \mid X_c = \beta\right] \iff D_k = \mathbb{E}\left[X_k \mid X_c = \beta\right]$$

Using this in (16) yields

$$0 = K_1 = \pi_s - \pi_\delta + \pi_\delta F_c(\beta)$$

This implies $\beta = Q$, and it follows that $D_k = C_k^*$ for all k, proving the claim. □

D. Proof of Theorem 5

It is straightforward to verify part (a). The proof techniques for part (b) and part (c) are quite similar. Hence, we only show the proof of part (b).

Let us consider the costs for firm k in three situations.

- The minimal cost under *no sharing*: We denote the optimal purchased storage capacity by $C_k^{i,*}$, and the associated cost by $J_k^i(C_k^{i,*})$.
- The cost under *sharing* but firm k chooses to install a storage system with capacity of $C_k^{i,*}$ (defecting from the equilibrium). We denote the associated cost by $J_k(C_k^{i,*})$.
- The minimal cost under *sharing*: Firm k invests the optimal capacity C_k^o. We denote the associated cost by $J_k(C_k^o)$.

By the definition of C_k^o, it is obvious that

$$J_k(C_k^o) \leq J_k(C_k^{i,*}). \quad (19)$$

Therefore, to prove the individual rationality, it suffices to show that $J_k(C_k^{i,*}) \leq J_k^i(C_k^{i,*})$ for any given parameters (i.e., $C_j, \forall j \in \mathcal{N}, j \neq k, X_j, \forall j \in \mathcal{N}$).

In fact, there are only four cases to justify and two of them are trivial.

When $S > D$ and $C_k^{i,*} > X_k$,

$$J_k(C_k^{i,*}) = J_k^i(C_k^{i,*}).$$

Similarly, when $S \leq D$ and $C_k^{i,*} \leq X_k$, the above equality still holds.

When $S > D$ and $C_k^{i,*} \leq X_k$, we know $\pi_{eq} = \pi_\ell$. Thus,

$$\begin{aligned} J_k(C_k^{i,*}) &= \pi_s C_k^{i,*} + \pi_\ell C_k^{i,*}, \\ J_k^i(C_k^{i,*}) &= \pi_s C_k^{i,*} + \pi_\ell C_k^{i,*} + \pi_j(X_k - C_k^{i,*}). \end{aligned}$$

These imply that $J_k(C_k^{i,*}) \leq J_k^i(C_k^{i,*})$.

For the last case, when $S \leq D$ and $C_k^{i,*} > X_k$, we know $\pi_{eq} = \pi_h$. Thus,

$$\begin{aligned} J_k(C_k^{i,*}) &= \pi_s C_k^{i,*} + \pi_\ell X_k - \pi_h(C_k^{i,*} - X_k) \\ &\quad + \pi_\ell(C_k^{i,*} - X_k), \\ J_k^i(C_k^{i,*}) &= \pi_s C_k^{i,*} + \pi_\ell X_k. \end{aligned}$$

Again, $J_k(C_k^{i,*}) \leq J_k^i(C_k^{i,*})$.

Above all, we know $J_k(C_k^{i,*}) \leq J_k^i(C_k^{i,*})$. Together with (19), we prove part (b). □

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