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OPTICAL PHYSICS

Coupling librational and translational motion of a levitated nanoparticle in an optical cavity

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An optically levitated nonspherical nanoparticle can exhibit both librational and translational vibrations due to orientational and translational confinements of the optical tweezer, respectively. Usually, the frequency of its librational mode in a linearly polarized optical tweezer is much larger than the frequency of its translational mode. Because of the frequency mismatch, the intrinsic coupling between librational and translational modes is very weak in vacuum. Here we propose a scheme to couple its librational and center-of-mass modes with an optical cavity mode. By adiabatically eliminating the cavity mode, the beam splitter Hamiltonian between librational and center-of-mass modes can be realized. We find that high-fidelity quantum state transfer between the librational and translational modes can be achieved with practical parameters. Our work may find applications in sympathetic cooling of multiple modes and quantum information processing.

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1. INTRODUCTION

Quantum optomechanics is a rapidly developing field that deals with the interaction between an optical field and the mechanical motion of an object [1,2]. In the last decade, there were many studies on the interaction between the light and the center-of-mass motion of a mechanical oscillator. Quantum ground cooling of mechanical oscillators has been realized [3,4]. The study of optomechanics has many applications in macroscopic quantum mechanics [5,6], precise measurements [7], and quantum information processing [8,9].

An optically levitated dielectric nanoparticle in vacuum can have an ultrahigh mechanical $Q > 10^9$ [10–13]. Therefore, it can be used for ultrasensitive force detection [14], searching for hypothetical millicharged particles and dark energy interactions [15,16], and testing the boundary between quantum and classical mechanics [17,18]. A levitated nanoparticle has 6 degrees of freedom: three translational modes and three rotational modes [19]. If its orientation is confined by the optical tweezer, it will exhibit libration. (Such motion was called "torsional vibration" in Ref. [20,21], and "rotation" in Ref. [22,23]. Several recent papers called it "libration" [24,25], which may be a better term, as it is similar to the libration of a molecule in an external field.) The librational mode of an optically levitated nonspherical nanoparticle has been observed recently [21,23]. Both translational motion and libration of a nanoparticle could be coupled with light and cooled towards quantum ground state by a cavity mode [19]. The librational mode frequency could be 1 order of magnitude higher than the frequency of a translational mode [21]. The coupling between the librational mode and the cavity mode can also be larger than the coupling between the translational mode and the cavity [21]. Therefore, it requires less cooling laser power to cool the librational mode to the quantum regime than to cool the translational mode [21,22,26,27].

In an optical trap in vacuum, the six motional degrees of freedom of a nanoparticle are uncoupled from each other when they are near ground state. It would be interesting to study how to induce strong coupling between them. Such coupling will have several applications. For example, we may use one of these modes to synthetically cool other modes. It is also useful for quantum information, as we may use all six motional modes to store quantum bits, and realize quantum processes such as controlled gates. By dynamically tuning the polarization orientation of a trapping laser, it was found that two different translational modes could be coupled with each other [28]. In this way, one translational mode was synthetically cooled by coupling it to another translational mode, which was feedback cooled. It has been proposed to couple the translational and rotational motion of a sphere with a spot painted on its surface by a continuous joint measurement of two motional modes [29]. However, a coherent way to couple the rotational and translational motion of a nanoparticle is still lacking.

In this paper, we propose a scheme to realize strong coupling between librational and translational modes of a levitated nanoparticle. We consider an optically trapped nanoparticle that resides in an optical cavity. Both its translational and librational modes couple with the cavity mode. We discuss the effects of cavity decay, and find that high-fidelity quantum state transfer could be realized under realistic experimental conditions. We also find that two-mode-squeezing Hamiltonian between librational and translational modes could be realized by adjusting the detunings of driving lasers.

2. MODEL

As shown in Fig. 1, we consider a system that contains an optical cavity and an ellipsoidal nanoparticle levitated by a trapping laser. The trapping laser is linearly polarized. Therefore, both location and direction of the nanoparticle are fixed [21]. The nanoparticle has translational mode b with frequency ω_m and librational mode c with frequency ω_φ. They are both coupled to the cavity mode a. The frequency of the mode c is usually much larger than the frequency of the mode b. The optical mode is driven by two lasers of frequencies ω_{L_1} and ω_{L_2}. The Hamiltonian of the system can be divided into three parts, H_E, H_I, and H_D, such that

$$H = H_E + H_I + H_D, \tag{1}$$

where

$$H_E = \hbar\omega_0 a^\dagger a + \hbar\omega_m b^\dagger b + \hbar\omega_\varphi c^\dagger c, \tag{2}$$

$$H_I = \hbar g_{ab} a^\dagger a(b^\dagger + b) + \hbar g_{ac} a^\dagger a(c^\dagger + c), \tag{3}$$

$$H_D = \frac{\hbar\Omega_1}{2}(a^\dagger e^{-i\omega_{L_1}t} + ae^{i\omega_{L_1}t}) + \frac{\hbar\Omega_2}{2}(a^\dagger e^{-i\omega_{L_2}t} + ae^{i\omega_{L_2}t}). \tag{4}$$

Here H_E is the energy term of translational mode b, librational mode c, and cavity mode a. H_I describes the couplings between the cavity mode a and two mechanical modes b and c. The coupling rates g_{ab} and g_{ac} are small, but they can be amplified by the driving lasers H_D. We will discuss how to derive the effective Hamiltonian between a and b, c modes in the next section.

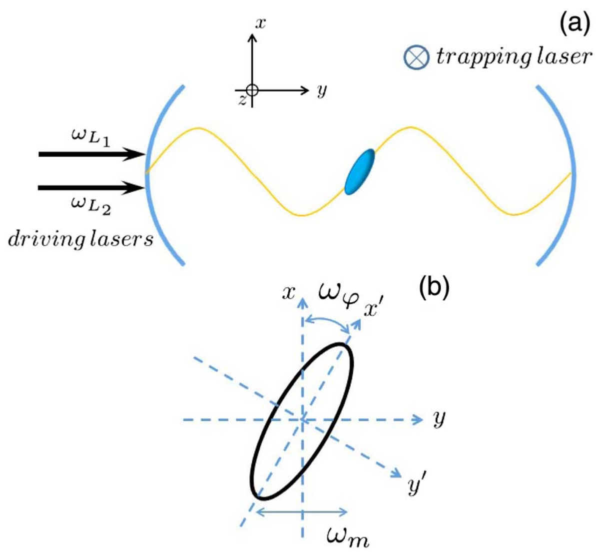

Fig. 1. (a) A nanoparticle is levitated by the trapping laser and is placed in a cavity. The trapping laser propagates along the z axis. The cavity is driven by two lasers of frequencies ω_{L_1} and ω_{L_2}. (b) Details of the nanoparticle: (x, y, z) is the coordinate system of the cavity. The x axis aligns with the polarization direction of the trapping laser. (x', y', z') is the coordinate fixed on the nanoparticle. The x' axis aligns with the long axis of the nanoparticle.

3. EFFECTIVE HAMILTONIAN

We first consider an ideal system without decay. In order to get the effective Hamiltonian between the cavity mode a and mechanical modes b and c, we first give the Heisenberg equation corresponding to Eq. (1):

$$\begin{aligned}\dot{a} = &-i\omega_0 a - ig_{ab}a(b^\dagger + b) - ig_{ac}a(c^\dagger + c)\\ &- i\frac{\Omega_1}{2}e^{-i\omega_{L_1}t} - i\frac{\Omega_2}{2}e^{-i\omega_{L_2}t}.\end{aligned} \tag{5}$$

To deal with it, we make a semiclassical ansatz:

$$a = a_0(t) + \alpha_1(t)e^{-i\omega_{L_1}t} + \alpha_2(t)e^{-i\omega_{L_2}t}, \tag{6}$$

where α_1 and α_2 are the classical amplitudes of mode a with frequencies ω_{L1} and ω_{L2}, and a_0 is the quantum fluctuation operator.

Inserting Eq. (6) into Eq. (5), we get the equation for the classical amplitudes α_1 and α_2:

$$\begin{aligned}&-i\omega_{L_1}\alpha_1 e^{-i\omega_{L_1}t} - i\omega_{L_2}\alpha_2 e^{-i\omega_{L_2}t} + \dot{\alpha}_1 e^{-i\omega_{L_1}t} + \dot{\alpha}_2 e^{-i\omega_{L_2}t}\\ &= -i\omega_0\alpha_1 e^{-i\omega_{L_1}t} - i\omega_0\alpha_2 e^{-i\omega_{L_2}t} - i\frac{\Omega_1}{2}e^{-i\omega_{L_1}t} - i\frac{\Omega_2}{2}e^{-i\omega_{L_2}t}.\end{aligned} \tag{7}$$

As α_1 and α_2 have different frequencies, we have equations for each of them:

$$\begin{aligned}\dot{\alpha}_1 &= -i\omega_0\alpha_1 + i\omega_{L_1}\alpha_1 - i\frac{\Omega_1}{2}\\ \dot{\alpha}_2 &= -i\omega_0\alpha_2 + i\omega_{L_2}\alpha_2 - i\frac{\Omega_2}{2}.\end{aligned} \tag{8}$$

So we can get their classical steady-state amplitude ($\dot{\alpha}_1 = \dot{\alpha}_2 = 0$): $\alpha_1 = \frac{\Omega_1}{2\Delta_1}$ and $\alpha_2 = \frac{\Omega_2}{2\Delta_2}$, where $\Delta_1 = \omega_{L_1} - \omega_0$, $\Delta_2 = \omega_{L_2} - \omega_0$. So, we get

$$a = a_0 e^{-i\omega_0 t} + \frac{\Omega_1}{2\Delta_1}e^{-i\omega_{L_1}t} + \frac{\Omega_2}{2\Delta_2}e^{-i\omega_{L_2}t}. \tag{9}$$

We can derive steady-state displacements β and γ for b and c in the same way:

$$b = b_0 + \beta$$
$$c = c_0 + \gamma, \quad (10)$$

where $\beta = -g_{ab}(\alpha_1^2 + \alpha_2^2)/\omega_m$, $\gamma = -g_{ac}(\alpha_1^2 + \alpha_2^2)/\omega_\varphi$. We substitute Eqs. (9) and (10) into Hamiltonian and in the rotating frame with $U = e^{-iH_0 t/\hbar}$, where $H_0 = \hbar\omega_0 a_0^\dagger a_0 + \hbar\omega_m b_0^\dagger b_0 + \hbar\omega_\varphi c_0^\dagger c_0$. The Hamiltonian $H_{RW} = U^\dagger(H - H_0)U$. By tuning the lasers' detunings, we can get different Hamiltonians between mechanical modes and the cavity mode. The different tasks, such as quantum state transfer and entanglement generating, can be realized. For example, if the driving lasers fulfill $\Delta_1 = -\omega_m$, $\Delta_2 = -\omega_\varphi$, we can neglect fast oscillation terms. The effective Hamiltonian reads

$$H_{RW} = \hbar g_{ab}\alpha_1(a_0^\dagger b_0 + a_0 b_0^\dagger) + \hbar g_{ac}\alpha_2(a_0^\dagger c_0 + a_0 c_0^\dagger). \quad (11)$$

The perfect quantum state transfer between the translational and the librational modes requires $|g_{ab}\alpha_1| = |g_{ac}\alpha_2| = G$. If we initialize the system as $|\psi_a(t=0)\rangle|\psi_b(t=0)\rangle|\psi_c(t=0)\rangle = |0\rangle|0\rangle|1\rangle$, we can get

$$|\psi_a\psi_b\psi_c(t)\rangle = \frac{1}{2}\left(1 + \cos\sqrt{2}Gt\right)|001\rangle - \frac{1}{2}\left(1 - \cos\sqrt{2}Gt\right)|010\rangle - \frac{i\sqrt{2}}{2}\sin\sqrt{2}Gt|100\rangle. \quad (12)$$

If we let $t = \frac{\pi}{\sqrt{2}G}$, we can transfer a state from librational mode to translation mode and vice versa.

If we set $\Delta_1 + \omega_m = \Delta_2 + \omega_\varphi = \delta$, and in the large detuning limit $\delta \gg |g_{ab}\alpha_1|, |g_{ac}\alpha_2|$, the cavity mode can be adiabatically eliminated [30]. Here we include all fast rotating terms, both rotating wave and antirotating wave. If the cavity mode a_0 is initially in the vacuum state, the effective Hamiltonian is

$$H_{\text{eff}} = \hbar G_1 b_0^\dagger b_0 + \hbar G_2 c_0^\dagger c_0 + \hbar G_3(b_0^\dagger c_0 + b_0 c_0^\dagger), \quad (13)$$

where

$$G_1 = \frac{\alpha_1^2 g_{ab}^2}{\Delta_1 + \omega_m} + \frac{\alpha_1^2 g_{ab}^2}{\Delta_1 - \omega_m} + \frac{\alpha_2^2 g_{ab}^2}{\Delta_2 + \omega_m} + \frac{\alpha_2^2 g_{ab}^2}{\Delta_2 - \omega_m}, \quad (14)$$

$$G_2 = \frac{\alpha_2^2 g_{ac}^2}{\Delta_2 + \omega_\varphi} + \frac{\alpha_2^2 g_{ac}^2}{\Delta_2 - \omega_\varphi} + \frac{\alpha_1^2 g_{ac}^2}{\Delta_1 + \omega_\varphi} + \frac{\alpha_1^2 g_{ac}^2}{\Delta_1 - \omega_\varphi}, \quad (15)$$

$$G_3 = \left(\frac{\alpha_1\alpha_2 g_{ab}g_{ac}}{\Delta_1 + \omega_m} + \frac{\alpha_1\alpha_2 g_{ab}g_{ac}}{\Delta_1 - \omega_\varphi}\right). \quad (16)$$

If $G_1 = G_2$ (we will provide workable parameters in the next section), and we take the initial state as $|\psi_b(t=0)\rangle|\psi_c(t=0)\rangle = |0\rangle|1\rangle$, we can get

$$|\psi_b(t)\rangle|\psi_c(t)\rangle = \frac{1}{2}(e^{-i(G_1+G_3)t} + e^{-i(G_1-G_3)t})|0\rangle|1\rangle + \frac{1}{2}(e^{-i(G_1+G_3)t} - e^{-i(G_1-G_3)t})|1\rangle|0\rangle. \quad (17)$$

In the lab reference frame, we have

$$|\psi_b(t)\rangle|\psi_c(t)\rangle = \frac{1}{2}e^{-i\omega_\varphi t}(e^{-i(G_1+G_3)t} + e^{-i(G_1-G_3)t})|0\rangle|1\rangle + \frac{1}{2}e^{-i\omega_m t}(e^{-i(G_1+G_3)t} - e^{-i(G_1-G_3)t})|1\rangle|0\rangle. \quad (18)$$

If we let $t = \frac{\pi}{2G_3}$, we can transfer a state from librational mode to translational mode and vice versa.

We can also choose $\Delta_1 - \omega_m = \Delta_2 - \omega_\varphi = \delta$. In the limit $\delta \gg G$, we can adiabatically eliminate the cavity mode, and get a two-mode-squeezing effective Hamiltonian [9,31]:

$$H_{RW} = \hbar G_1'(b_0^\dagger b_0 + c_0^\dagger c_0) + \hbar G_3'(b_0^\dagger c_0^\dagger + b_0 c_0), \quad (19)$$

which could be used for generating entanglement between modes b_0 and c_0.

4. EXPERIMENTAL FEASIBILITY AND DISSIPATION EFFECTS

In this section, we will provide the feasible parameters in experiment and consider the effect of dissipations. In our scheme, the steady-state amplitudes α_1 and α_2 are on the order of 10^4 to 10^5. Therefore, the strengths of linear couplings between the cavity mode and the mechanical modes are enhanced by 10^4 to 10^5 times. The photon number fluctuation is on the order of $\sqrt{\alpha_{1,2}} \sim 10^2$, which is related to nonlinear coupling between the cavity and the mechanical modes. Therefore, the linear coupling strength is 10^2 times larger than the nonlinear coupling strength. The effect of the photon number fluctuation is negligible in our scheme.

In experiments, the dissipation by the cavity mode and mechanical modes decay is inevitable. However, in high vacuum, the mechanical decay rates are much less than the cavity decay rate [21,32,33]. Therefore, we only need to consider the cavity decay effect. Considering the dissipation, the steady-amplitudes will change and we can derive them by adding a term of $-i\hbar\frac{\kappa}{2}a^\dagger a$ into Hamiltonian Eq. (1). Following the same procedure mentioned above, we can get

$$\alpha_1 = \frac{\Omega_1}{2\left(\Delta_1 + i\frac{\kappa}{2}\right)},$$
$$\alpha_2 = \frac{\Omega_2}{2\left(\Delta_2 + i\frac{\kappa}{2}\right)}. \quad (20)$$

And in order to maintain the form of the Hamiltonian, we should do the transformation $b_0 \to \alpha_1 b_0/|\alpha_1|$, $b_0^\dagger \to \alpha_1^* b_0^\dagger/|\alpha_1|$ and $c_0 \to \alpha_2 c_0/|\alpha_2|$, $c_0^\dagger \to \alpha_2^* c_0^\dagger/|\alpha_2|$. Using perturbation theory [13,34], we can obtain the coupling constants in the same way as Ref. [21]. If we restrict the librational motion of the long axis of the nanoparticle in the plane xOy, we get

$$g_{ab} = \sqrt{\frac{\hbar}{2M\omega_m}}\frac{32\pi^2 c e^{-\frac{4\pi(x^2+z^2)}{\lambda L}}\cos ky \sin ky}{\epsilon_0\lambda^3 L^2} \cdot (s_2 + \cos^2\varphi(s_1 - s_2)), \quad (21)$$

$$g_{ac} = \sqrt{\frac{\hbar}{2I\omega_\varphi}} \frac{8\pi c e^{-\frac{4\pi(x^2+z^2)}{\lambda L}} \cos^2 ky}{\epsilon_0 \lambda^2 L^2} (s_1 - s_2) \sin 2\varphi. \tag{22}$$

Here L is the length of the cavity; λ and k are the wavelength and wavenumber of the cavity mode. M and I are the mass and the moment of inertia of the nanoparticle. s_1 and s_2 are the diagonal elements of the susceptibility matrix. (x, y, z, φ) are the parameters describing the position of the nanoparticle: (x, y, z) are the coordinates of the center of mass (origin is the center of the cavity), and φ is the angle between the long axis of the nanoparticle and the x axis. x, y, z, and φ can be changed by adjusting the trapping laser. For example, we choose the angle between the polarization direction of the trapping laser and the y axis (φ) as 45°, the equilibrium position of the center of mass is $(0, \pi/4k, 0)$. We can get $g_{ab}/2\pi = 0.3056$ Hz and $g_{ac}/2\pi = 0.2189$ Hz. (The parameters of the nanoparticle we choose are as follows: $\rho = 3500\ \mathrm{kg/m^3}$, long axis $a = 50$ nm, short axis $b = 25$ nm, $\epsilon_r = 5.7$, waist of the trapping laser $W_t = 600$ nm, power of the trapping laser is 100 mW, wavelength $\lambda_{\mathrm{cav}} = 1540$ nm, length of the cavity $L = 10$ nm.) In this situation, $\omega_m/2\pi = 247.7$ kHz, $\omega_\varphi/2\pi = 2.6$ MHz. If the finesse of our cavity $\mathcal{F} = 10^5$, and we can get $\kappa/2\pi = 75.2$ kHz. For example, we let $\delta/2\pi = 200$ kHz, $\Omega_1/2\pi = 2.66 \times 10^9$ Hz, $\Omega_2/2\pi = 5.0 \times 10^{10}$ Hz. We can get $G_3/2\pi = 25$ kHz and time of state transfer $t = 1 \times 10^{-5}$ s; thus it is not difficult to realize.

A. Large Detuning Scheme

Under the large detuning condition that $\Delta_1 + \omega_m = \Delta_2 + \omega_\varphi = \delta \gg G$, we change the system Hamiltonian to the rotating wave frame, and neglect the fast rotating terms in H_{RW}. In order to deal with the cavity loss effects, here we adopt the conditional Hamiltonian [35,36]. We assume that the cavity decay rate is weak. Therefore, we can only consider the situation that the system evolves without photon leakage. Under the condition that no photon is leaking out, we get the conditional Hamiltonian from the quantum trajectory method [35]:

$$H = \hbar G(a_0^\dagger b_0 e^{-i\delta t} + a_0 b_0^\dagger e^{i\delta t}) + \hbar G(a_0^\dagger c_0 e^{-i\delta t} + a_0 c_0^\dagger e^{i\delta t}) - i\hbar \frac{\kappa}{2} a_0^\dagger a_0, \tag{23}$$

where κ is the decay rate of the cavity mode a. We can use the above conditional Hamiltonian to calculate the possibility P of the system evolving without photon leakage. Because we suppose that the initial state of the system is $|0\rangle_a|01\rangle_{bc}$, so the subspace only includes three basis states: $|0\rangle_a|01\rangle_{bc}$, $|0\rangle_a|10\rangle_{bc}$, and $|1\rangle_a|00\rangle_{bc}$. And at any time t, the state of the system is

$$|\psi_d(t)\rangle = C_{d1}(t)|0\rangle_a|01\rangle_{bc} + C_{d2}(t)|0\rangle_a|10\rangle_{bc} + C_{d3}(t)|1\rangle_a|00\rangle_{bc}, \tag{24}$$

where

$$C_{d1}(t) = \frac{1}{2} + \frac{(2\delta + i\kappa + \chi)}{4\chi} e^{-iE_3 t/\hbar} - \frac{(2\delta + i\kappa - \chi)}{4\chi} e^{-iE_2 t/\hbar}, \tag{25}$$

$$C_{d2}(t) = -\frac{1}{2} + \frac{(2\delta + i\kappa + \chi)}{4\chi} e^{-iE_3 t/\hbar} - \frac{(2\delta + i\kappa - \chi)}{4\chi} e^{-iE_2 t/\hbar}, \tag{26}$$

$$C_{d3}(t) = -\frac{e^{-i\delta t}(2\delta + i\kappa + \chi)(2\delta + i\kappa - \chi)}{16G\chi} \cdot (e^{-iE_3 t/\hbar} - e^{-iE_2 t/\hbar}); \tag{27}$$

here $\chi = \sqrt{4\delta^2 + 32\ G^2 + 4i\delta\kappa - \kappa^2}$, $E_2 = \frac{1}{4}(-2\delta - i\kappa - \chi)$, $E_3 = \frac{1}{4}(-2\delta - i\kappa + \chi)$.

We first normalize the state $|\psi_d\rangle$ to calculate the fidelity; we can get $|\psi_{dn}\rangle = |\psi_d\rangle/\sqrt{|C_{d1}|^2 + |C_{d2}|^2 + |C_{d3}|^2}$. As shown in Fig. 2(a), we plot the fidelity $F = |\langle\psi_{dn}(t)|010\rangle|$ at the time $t = \frac{\pi\delta}{2\ G^2 - \kappa^2/16}$ which can be directly derived from the strict solution of the Schrödinger equation and $\kappa/2\pi = 75.2$ kHz as a function of δ and G. The possibility of the system evolving without photon leakage is $P = |C_{d1}|^2 + |C_{d2}|^2 + |C_{d3}|^2$. It is found that the fidelity could approach 1 when G is small and δ is large. However, at this regime, the effective coupling between two mechanical modes is also pretty small. In Fig. 2(b), we plot P as a function of δ and G as well. When we choose $\delta = 200$ kHz and $G = 50$ kHz, the fidelity $F = 0.95$ and the successful possibility $P = 0.68$.

B. Resonant Scheme

In the resonance case, the Hamiltonian reads

$$H = \hbar G(a_0^\dagger b_0 + a_0 b_0^\dagger) + \hbar G(a_0^\dagger c_0 + a_0 c_0^\dagger) - i\hbar\frac{\kappa}{2} a_0^\dagger a_0. \tag{28}$$

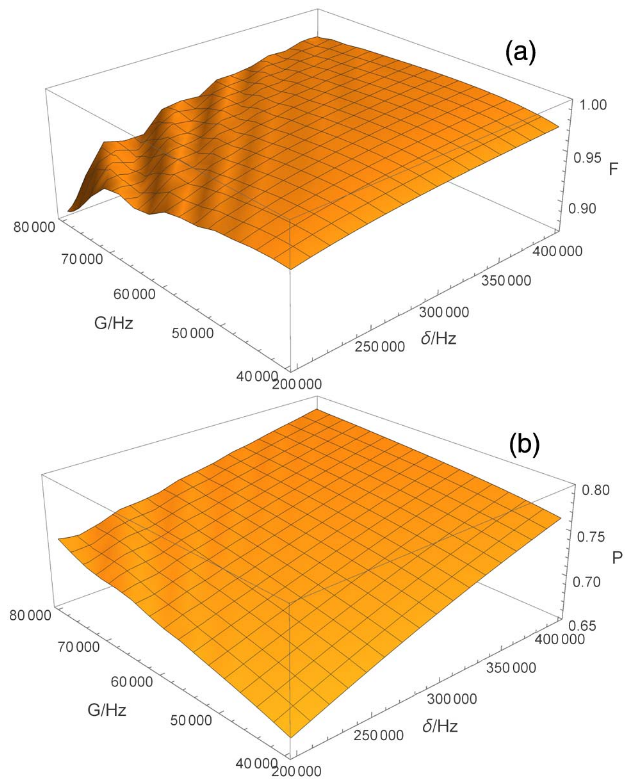

Fig. 2. (a) Fidelity of state transfer as a function of G and δ in the situation of large detuning. (b) Probability of the system being in this state as a function of G and δ.

Because we suppose that the initial state of the system is $|0\rangle_a|01\rangle_{bc}$, the subspace only includes three basis states: $|0\rangle_a|01\rangle_{bc}$, $|0\rangle_a|10\rangle_{bc}$, and $|1\rangle_a|00\rangle_{bc}$ as well. And at any time t, the state of the system is

$$|\psi_r(t)\rangle = C_{r1}(t)|0\rangle_a|01\rangle_{bc} + C_{r2}(t)|0\rangle_a|10\rangle_{bc} + C_{r3}(t)|1\rangle_a|00\rangle_{bc}, \tag{29}$$

and

$$C_{r1}(t) = \frac{1}{2} + \frac{1}{2}e^{-\kappa t/4}\cos\frac{\sqrt{32\ G^2 - \kappa^2}}{4}t + \frac{\kappa}{\sqrt{32\ G^2-\kappa^2}}e^{-\kappa t/4}\sin\frac{\sqrt{32\ G^2 - \kappa^2}}{4}t, \tag{30}$$

$$C_{r2}(t) = -\frac{1}{2} + \frac{1}{2}e^{-\kappa t/4}\cos\frac{\sqrt{32\ G^2 - \kappa^2}}{4}t + \frac{\kappa}{\sqrt{32\ G^2-\kappa^2}}e^{-\kappa t/4}\sin\frac{\sqrt{32\ G^2 - \kappa^2}}{4}t, \tag{31}$$

$$C_{r3}(t) = -i\frac{4G}{\sqrt{32\ G^2-\kappa^2}}e^{-\kappa t/4}\sin\frac{\sqrt{32\ G^2 - \kappa^2}}{4}t. \tag{32}$$

As with the large detuning case, we plot the fidelity $F = |\langle\psi_{nr}(t)|010\rangle|$ in Fig. 3(a) and possibility $P = |C_{r1}|^2 + |C_{r2}|^2 + |C_{r3}|^2$ at $t = \frac{4\pi}{\sqrt{32\ G^2-\kappa^2}}$ in Fig. 3(b) as a function of G and κ. Here $|\psi_{nr}(t)\rangle$ is the normalized state of $|\psi_r(t)\rangle$. It is found that both P and F are in favor of larger G and less κ. When we choose $\kappa = 75.2$ kHz and $G = 50$ kHz, the fidelity $F = 0.926$ and the successful possibility $P = 0.59$. As we can see, for both large detuning and resonant schemes, the quantum state transfer could be realized with pretty high fidelity and successful possibility. In experiment, we can choose either of them for convenience.

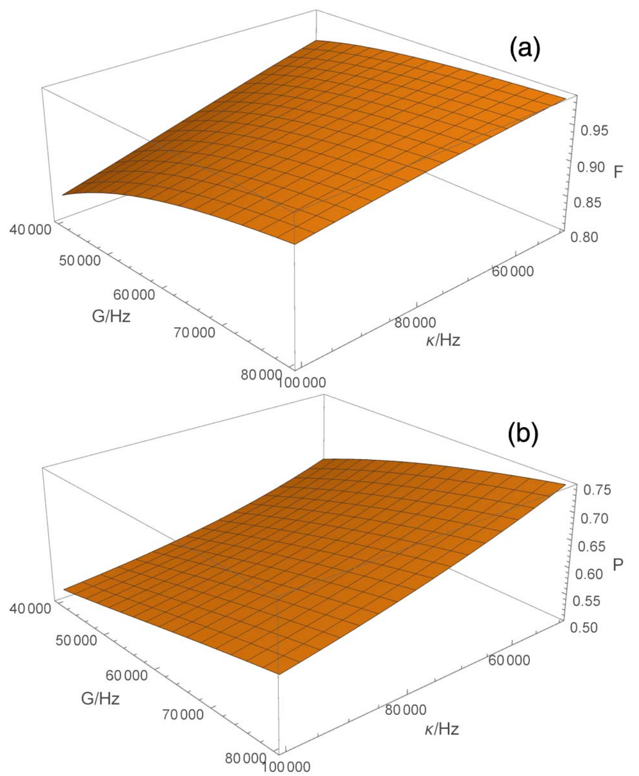

Fig. 3. (a) Fidelity of state transfer as a function of G and κ in the situation of resonance. (b) Probability of the system being in this state as a function of G and κ.

5. CONCLUSION

In this paper, we propose a scheme to couple librational and translational modes of a levitated nanoparticle with an optical cavity mode. We discuss how to realize quantum state transfer from a librational mode to a translational mode, and vice versa. We also discuss the effects of cavity decay on the fidelity of state transfer. We find that the high-fidelity state transfer could be realized under practical experimental conditions.

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