

Digital Good Exchange

(Extended Abstract)

Wenyi Fang, Pingzhong Tang, and Song Zuo
Institute for Interdisciplinary Information Sciences
Tsinghua University, Beijing, China
{fwy13@mails., kenshin@, zuos13@mails.}@tsinghua.edu.cn

ABSTRACT

Over the past decade, computer-automated barter exchange has become one of the most successful applications at the intersection of AI and economics. Standard exchange models, such as house allocation and kidney exchange cannot be applied to an emerging industrial application, coined *digital good exchange*, where an agent still possesses her initial endowment after exchanging with others. However, her valuation toward her endowment decreases as it is possessed by more agents.

We put forward game theoretical models tailored for digital good exchange. In the first part of the paper, we first consider a natural class of games where agents can choose either a subset of other participants' items or no participation at all. It turns out that this class of games can be modeled as a variant of congestion games. We prove that it is in general NP-complete to determine whether there exists a non-trivial pure Nash equilibrium where at least some agent chooses a nonempty subset of items. However, we show that there exist non-trivial Pure Nash equilibria for subsets of games and put forward efficient algorithms to find such equilibria.

In the second part of the paper, we investigate digital good exchange from a mechanism design perspective. We ask if there is a truthful mechanism in this setting that can achieve good social welfare guarantee. To this end, we design a randomized fixed-price-exchange mechanism that is individually rational and truthful, and for two-player case yields a tight log-approximation with respect to any individually rational allocation.

Keywords

Barter Exchange; Digital Good Exchange; Congestion Games; Mechanism Design

1. INTRODUCTION

In the stylized model of barter exchanges, agents enter the exchange with some endowments and exchange their endowments for better allocations without monetary transfers. However, standard models of barter exchanges [5, 1, 3, 4] have certain limitations. For one, agents lose ownerships of their endowments once the exchange takes place. This is not

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the case with exchanges of digital goods, in which case agents still possess one copy of their endowments even though the exchange has taken place. In addition, agents have negative externality for other agents to own their items, namely, an agent's valuation towards her endowment decreases as more agents possess her item. A representative type of digital good is data. The owner can produce as many copies of data as possible, however, it is commonly believed that her value of data decreases as it is owned by more agents, due to the fact that her market power on possessing data decreases as it is shared with more agents.

Over the past few years, digital good exchange has become a common industrial practice, as a part of the *sharing economy tsunami*. As Bloomberg reports, "A market for data swaps is rapidly emerging. Factual, a Los Angeles-based startup, has put together a database that houses location data and details on retailers and restaurants. Access to the database costs companies money, but they can accrue discounts by agreeing to contribute some of their own information."¹ Another successful example of data exchange is www.datatang.com, which encourages individual data owners to share their data in a centralized database and in return awards them a discount, or limited-time free access for obtaining other data sets in their wish lists.

To our best knowledge, there has been no theoretical model tailored for this domain. In this paper, we put forward several game-theoretical models for digital good exchange and investigate their properties. Our goal is to model and analyze existing digital exchange mechanisms, as well as to provide theoretical and practical guidelines for designing new mechanisms in this domain.

2. MODEL

Let $A = \{a_1, \dots, a_n\}$ be a set of n agents, and $D = \{d_1, \dots, d_n\}$ be the corresponding set of digital goods (or simply goods), where d_i is initially owned by agent a_i .

Let $\mathbf{x}_j = (x_{1j}, \dots, x_{nj}) \in \{0, 1\}^n$ be a deterministic allocation of good d_j over all agents and $v_{ij}(\mathbf{x}_j)$ be the valuation of agent a_i over allocation \mathbf{x}_j , in this way, we let agent a_i 's valuation over good d_j explicitly depend on the allocation of d_j . All agents have *additive, quasi-linear utility* functions,

$$u_i = \sum_{j \in [n]} v_{ij}(\mathbf{x}_j) - p_i, \quad \forall i \in [n],$$

where p_i is the monetary transfer from agent a_i to anybody else. In this paper, we consider barter exchanges, so $p_i = 0$.

¹<http://www.bloomberg.com/bw/articles/2012-11-15/data-bartering-is-everywhere>

As mentioned, agents never lose ownerships of their endowments, so the feasibility constraint, $x_{ii} = 1$, is imposed on each deterministic allocation.

Besides, an agent's valuation toward a digital good decreases as it is owned by more agents. Let $\text{Supp}(\mathbf{x}_j) = \{i | x_{ij} = 1\}$. The valuation functions satisfy the following

$$\begin{cases} v_{ij}(\mathbf{x}_j) = 0, & i \notin \text{Supp}(\mathbf{x}_j) \\ v_{ij}(\mathbf{x}_j) \geq v_{ij}(\mathbf{x}'_j), & i \in \text{Supp}(\mathbf{x}_j) \subseteq \text{Supp}(\mathbf{x}'_j) \end{cases}$$

3. DIGITAL EXCHANGE GAME

The following game captures the interesting feature of the data exchange websites mentioned in the introduction: agents share their own goods to exchange for the rights to download others' data.

DEFINITION 1 (DIGITAL EXCHANGE GAME). *The game $G = (A, \mathcal{S}, u)$ is defined as follows,*

- *The set of actions \mathcal{S}_i of agent a_i is to either choose a subset S_i (must include d_i) of the goods, or exit this game (denoted by $S_i = \perp$). The goods allocated to a_i , is either the intersection of S_i and the goods whose owners do not exit this game, if $S_i \neq \perp$; or $\{d_j\}$, otherwise.*
- *The utility function u_i of agent a_i is the sum of the values from the allocation of each item, $u_i(\mathbf{x}) = \sum_j v_{ij}(\mathbf{x}_j)$, where \mathbf{x} is the allocation induced by the game play.*

The above game has an interesting congestion-game interpretation, so we coin this game as "player-specific congestion game with endowments".

3.1 The Pure Nash Equilibria

First of all, non-participation for all agents forms a PNE. However, determining whether there is a non-trivial PNE, where at least one agent participates, is hard.

CLAIM 1. *The game always has a trivial PNE, where each $S_i = \perp$. Meanwhile, $S_i = A$ always weakly dominates any other subsets (except for \perp).*

THEOREM 1. *Given an instance of the digital exchange game, determining if there is a non-trivial PNE is NP-complete.*

COROLLARY 1. *Even if each agent is limited to choosing at most two goods, or each agent has strictly positive valuation over goods, the determination problem is still NP-hard.*

3.2 Two Cases for Efficiently Computable PNEs

We derive positive results for two special cases: ordered agents and single-minded agents with unit demand.

DEFINITION 2 (ORDERED AGENTS). *The agents in A are ordered under game G , if there is an order a_1, \dots, a_n , such that for $i < j$, and any PNE S where $S_i = \perp$ and $S_j \neq \perp$, then $S^{(i,j)}$ is also a PNE, where $S^{(i,j)}$ is obtained by switching each appearance of i and j .*

THEOREM 2. *If the agents are ordered under game G , then we can find a non-trivial PNE in poly-time. In particular, if all the agents have identical valuation functions, we can find a non-trivial PNE in poly-time.*

For single-minded agents with unit demand, where each agent has positive valuation towards exactly one item besides its own endowment, we consider *efficient* PNEs.

We use $\delta(i) = j$ for agent a_i desires good d_j . Then *efficient* means that in the PNE, $S = (S_1, \dots, S_n)$,

$$\forall a_i \in A, \delta(i) = j, S_i \in \{\{a_i, a_j\}, \{a_i\}, \perp\},$$

and at least one exchange is performed.

THEOREM 3. *For single-minded agents with unit demand case, there is a dynamic program based algorithm to find or prove non-existence of efficient PNEs in poly-time.*

4. MECHANISM DESIGN

We put forward a direct mechanism coined "Randomized FPE Mechanism" (RFPE), which is the expectation of parametric *fixed-price-exchange* (FPE) mechanisms with parameters drawn from some pre-specified distribution.

The FPE mechanism is a simplified version of *fixed-price-trading* [2]. In our case, each agent only has one type of endowment, and the exchange is conducted according to one single fixed exchange rate (fixed-price). The exchange amount is determined in an incentive compatible (utility maximizing) way.

MECHANISM 1 (RANDOMIZED FPE MECHANISM). *Given distribution \mathcal{F} of fixed-price (\mathcal{F} specified prior to the mechanism) the randomized FPE mechanism is defined as follows,*

$$\text{RFPE}^{\mathcal{F}}(\{\nu_{ij}\}_{i,j=1}^n) = \mathbf{E}_{\Pi \sim \mathcal{F}} \text{FPE}^{\Pi}(\{\nu_{ij}\}_{i,j=1}^n).$$

THEOREM 4. *Any RFPE mechanism is IC and IR.*

Particularly, for two-agent case, there is a tight bound of the social welfare approximation ratio against the optimal IR allocation.

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REFERENCES

- [1] D. J. Abraham, A. Blum, and T. Sandholm. Clearing algorithms for barter exchange markets: Enabling nationwide kidney exchanges. In *Proceedings of the 8th ACM conference on Electronic commerce*, pages 295–304. ACM, 2007.
- [2] S. Barberà and M. O. Jackson. Strategy-proof exchange. *Econometrica: Journal of the Econometric Society*, pages 51–87, 1995.
- [3] Y. Liu, P. Tang, and W. Fang. Internally stable matchings and exchanges. In *AAAI Conference on Artificial Intelligence (AAAI)*, pages 1433–1439, 2014.
- [4] S. Luo and P. Tang. Mechanism design and implementation for lung exchange. In *Proceedings of the International Joint Conference on Artificial Intelligence (IJCAI)*, pages 209–215, 2015.
- [5] A. E. Roth, T. Sönmez, and M. Ünver. Kidney exchange. *The Quarterly Journal of Economics*, 119(2):457–488, 2004.