

# How Does Fairness Matter in Group Recommendation

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**Abstract.** Group recommendation has attracted significant research efforts for its importance in benefiting a group of users. In contrast to personalized recommendation, group recommendation tries to recommend same set of items to a group of users. Therefore a gap exists between the group recommendation and individual recommendation in terms of individual satisfaction. We aim to explore the possibility of narrowing this gap by introducing the concept of fairness in group recommendation.

In this work, we propose the concept of fairness in group recommendation and try to accommodate it into the recommendation algorithm so that the satisfaction of users in group recommendation can get close to that of individual recommendation. We utilize the concept of Ordered Weighted Average from fuzzy logic to evaluate the individual satisfaction of users and use min-max fairness metrics to accommodate the fairness into group recommendation process. We formulate the problem of group recommendation with fairness as an integer programming problem and propose efficient algorithms for three different OWA scenarios. Extensive experiments have been conducted on the real-world datasets and the results corroborate our analyses.

**Keywords:** Group recommendation · Fairness · Individual recommendation · Optimization

## 1 Introduction

Group recommendation is an interesting research topic where a same set of items is recommended to groups of users whose preferences are distinct from each other. Some real-life scenarios can be found when group recommendation is applied: friends may go to a restaurant for dinner and they need to decide which food to order; or a group of friends go to the cinema and need to decide which movie to watch so that everyone can enjoy it. Another important reason is that personalized recommendation may face the challenge when the number of users is too large and making recommendation to a set of users can help relieve the challenge caused by data volume.

The study on group recommendation can be first found in [5] where a group recommender system of movies is proposed. More studies on group recommendation emerge since then. Most of studies focus on designing semantics of group recommendation which evaluates how much a group of users are satisfied with the recommendation. It is inevitable for group members to compromise with each other since their preferences are usually different. Therefore, the user in group recommendation is less satisfied with the recommendation compared with that in individual recommendation. Some of the existing semantics can be found in [1, 12].

In order to narrow the gap of satisfaction between individual and group recommendation, two requirements need to be met: first, the recommendation should be of interest to users; second, the users should be more or less equally satisfied. To meet these two requirements, we need to model how an individual user is satisfied with the set of recommended items in group recommendation. We borrow the concept of Ordered Weighted Average to model the individual satisfaction for its capability of linguistically expressed aggregation instructions. Meanwhile, we adopt the min-max fairness principle to guarantee that each group member can be satisfied with the recommendation at a reasonable level.

We formulate the group recommendation problem with OWA and Max-min fairness as an integer programming. We further prove its computational complexity in some cases and show that it is NP-Hard in some scenarios. Extensive experiments are conducted on the real-world dataset and the results indicate that with a proper selection of semantic from OWA and fairness threshold, the user satisfaction in group recommendation can get closer to that in individual recommendation.

The remaining of the work is organized as follows: we introduce some important related work in the next section; the formal semantics of OWA and fairness metrics with model complexity analyses are proposed in Sect. 3; a general optimization framework is proposed in Sect. 4; the experimental results are presented in Sect. 5 and the conclusion is summarized in Sect. 6.

## 2 Related Work

In this section, we review some of the important studies about group recommendation. The study of group recommendation first originates from extending a personalized recommender system to a system making recommendations to a group of users in [5]. The system recommends same movies to a group of users and ask for the rating from the group as a whole. The recommendation strategy of group recommender systems is to compute a rating of a candidate item given by a group as the whole. Therefore, the group is seen as a virtual user and the preference of the group is an aggregation of individual preferences of group members. Thus the group recommendation can be performed in two ways: the first method specifies the individual preferences of group members on candidate items beforehand and then aggregates the preferences of individual users into group preferences and rank the candidate items [2, 4]; the second method first

aggregates the individual preferences on purchased items into group preferences and then predicts the group preference on candidate items by treating the group as a user [7, 8].

Some studies attempt to propose proper aggregation semantics for evaluating the group preference on a specific item. Some important preference aggregation techniques are summarized in [9]. The typical semantics for preference aggregation include Average Voting, Least misery and Most pleasure. The semantics are widely used in group recommendation studies [1, 15].

Ordered Weighted Average is a mean operator that assigns order-related weights to the elements of a vector and generates the linear weighted sum of the elements as output [16]. This operator is first adopted in group recommendation in [9]. The satisfaction of each user is modeled as an OWA function of the relevance of the items to the user and the aim of group recommendation is to maximize the total derived satisfaction of all the users. The powerful expression capability makes it possible to model the satisfaction of the user with a set of recommended items in different ways. In this paper, we use the OWA function for individual satisfaction modeling as well.

An interesting topic in group recommendation is the concept of fairness. Since the preferences of users are distinct, it is inevitable for users to compromise to reach an agreement on the recommendation. Therefore it is usually appreciated if the group recommendation is fair to users. There are few studies on the fairness issue in group recommendation. A recent study is to maximize the fairness in group recommendation [10]. The recommendation is thought to be fair if an enough number of users find a favourite item among them. Some studies look at the group recommendation problem from the perspective of game theory. [14] attempt to model the group decision process considering the power balance between group members; [6] introduces the phenomenon of multi-party into the user modeling of group recommendation; the group recommendation process is modeled as a non-cooperative game in [3] and the final recommendation is generated as the equilibria of the game.

### 3 Problem Setting

In this section, we formulate the Group Recommendation problem considering individual satisfaction and fairness. There are typical semantics that are widely adopted in group recommendation works [1]. We use these semantics to evaluate how much the group is satisfied with the items. Meanwhile, we consider the satisfaction of each individual user given recommended items. Therefore, we consider the problem of group recommendation that maximizes the group satisfaction while the individual satisfaction is guaranteed with a max-min fairness. First we introduce the group recommendation semantics and individual satisfaction semantics. Then we formulate the problem of Group Recommendation with Individual Satisfaction Guarantee as fairness.

### 3.1 Individual Satisfaction in Group Recommendation

In the context of group recommendation, the set of users and items are denoted as  $U$  and  $I$ , for each pair of user  $u \in U$  and item  $i \in I$ , a real-valued relevance score  $rel(u, i)$  denotes how much user  $u$  is satisfied with item  $i$ . In most cases, the relevance scores of some user-item pairs are observed, and various approaches have been proposed to estimate the relevance of the unobserved pairs [11, 13]. In this paper, we restrict the scale of  $rel(u, i)$  into  $[0, 1]$ .

When a package of items are recommended to users, each user may have different preferences towards the items, which leads to different individual satisfaction. The former studies about group recommendation seldom consider the individual satisfaction and the balance between individuals in the group recommendation context. As introduced before, Ordered Weighted Average provides a general and powerful function that assigns a vector to a real value. Following the study of [9], we use similar methodology to model the satisfaction of each individual given a package of recommended items with OWA.

First, we provide a formal definition of Ordered Weighted Average (OWA):

**Definition 1.** Denote a  $K$  dimensional vector as  $X \in R^K$ , a OWA function  $f_{OWA} : R^K \rightarrow R$  is an operator that maps a vector into a real value.  $f_{OWA}(X) = \sum_{i=1}^K \alpha_i \tilde{X}(i)$ , where  $\tilde{X}(i)$  is the  $i$ -th element of a rearranged vector  $X$  (Ordered in Descending Order),  $\alpha_i \geq 0, \forall i \in \{1, 2, \dots, K\}$  is the pre-specified weight for  $\tilde{X}(i)$  and  $\sum_{i=1}^K \alpha_i = 1$ .

**Definition 2.** Denote  $IS(u, I)$  as the individual satisfaction of user  $u$  on the set of items  $I$ ,  $IS(u, I)$  is an OWA function of the relevance of items:  $IS(u, I) = f_{OWA}(X), X = \{rel(u, i), \forall i \in I\}$ .

We can use OWA to reformulate and extend the semantics with a proper modeling of  $\alpha$  and  $X$  from the relevances.

- Average:  $X(u, i) = rel(u, i), \forall i, u \in G, \alpha_j = \frac{1}{k}, \forall j \in \{1, 2, \dots, k\}$
- Least Misery:  $X(u, i) = rel(u, i), \forall i, u \in G, \alpha_j = 0, \forall j \in \{1, 2, \dots, k - 1\}$  and  $\alpha_k = 1$
- Most Pleasure:  $X(u, i) = rel(u, i), \forall i, u \in G, \alpha_1 = 1$  and  $\alpha_j = 0, \forall j \in \{2, 3, \dots, k\}$
- Median:  $X(u, i) = rel(u, i), \forall i, u \in G, \alpha_j = 0, \forall j \in \{1, 2, \dots, k\} \setminus \lceil \frac{k}{2} \rceil$  and  $\alpha_{\lceil \frac{k}{2} \rceil} = 1$

We use OWA function to model the satisfaction of users on the recommended items. The intuition behind this is that the satisfaction of users is related to both the relevance of recommended items and their positions in the recommendation list.

### 3.2 Fairness in Group Recommendation

There are several semantics for evaluating the fairness in division, including proportional division and envy freeness. However they do not quite fit our problem

due to the difference between the nature of division and recommendation. We use max-min fairness to evaluate the fairness between individuals in group recommendation. The rationale of this fairness is to provide a worst-case guarantee for all users so that the result is not so bad for the less satisfied users.

**Definition 3. max-min fairness:** *The max-min fairness of individual satisfaction is to provide a worst-case guarantee for the users inside the group: given a semantic for individual satisfaction and a threshold  $T$ , the max-min fairness means:*

$$\begin{aligned} & \max . \min_u \{T_u\} \\ & \text{s.t. } IS(u, I) \geq T_u, \forall u \end{aligned} \tag{1}$$

In this study, we aim to recommend a set of  $K$  items to the group, so that the max-min fairness is maximized. The intuition behind this is to guarantee that each user is satisfied with the recommendation at a certain degree.

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**Algorithm 1.** ALGORITHM FOR LM SEMANTIC

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**Input:** Relevance matrix  $R$ , the set of users  $U$  and items  $I$

**Output:** Recommendation List  $\hat{I}$

- 1: **for** Each item  $i \in I$  **do**
  - 2:   Compute the score as  $Sc(i) = \min_{u \in G} rel(u, i)$ ;
  - 3: **end for**
  - 4: Select Top-K items with highest  $Sc(i)$  as recommendation list  $\hat{I}$ ;
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### 3.3 Problem Formulation

We formulate the fairness maximization group recommendation problem as an integer programming problem:

$$\begin{aligned} & \max . \min_u T_u \\ & \text{s.t. } \sum_{i \in I_g} X_i = k, i \subseteq I \\ & IS(u, \hat{I}) \geq T_u, \forall u, \hat{I} = \{i | X_i = 1, \forall i \in I\} \\ & X_i \in \{0, 1\} \end{aligned} \tag{2}$$

The objective function is to maximize the lowest satisfaction of users inside the group; the first constraint requires that the recommended items are exactly  $K$  which is specified beforehand; the second constraint requires that the satisfaction of each user is at least  $T$ ; Meanwhile,  $X_i$  is a binary indicator meaning whether item  $i$  is recommended to the group.  $IS(u, \hat{I})$  denotes the individual satisfaction of user  $u$  with the recommendation, which is specified by the OWA semantic in previous section.

## 4 Optimization Framework

In this section, we formally introduce the optimization framework for the problem. As shown in previous section, the individual satisfaction is a OWA function. The semantics of OWA function are related to the hardness of the problem. In this paper, we consider three typical semantics: the most pleasure semantic; the least misery semantic and the average semantic.

### 4.1 Least Misery and Most Pleasure Semantics

Consider the two ordering related semantics: Least Misery and Most Pleasure semantic, we design two effective algorithms for the Fairness Maximization Group Recommendation problem.

For the Least Misery and Most Pleasure semantics, we select the items greedily: For the Most Pleasure semantic, we also use the greedy algorithm for recommendation:

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**Algorithm 2.** ALGORITHM FOR MP SEMANTIC

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**Input:** Relevance matrix  $R$ , the set of users  $U$  and items  $I$

**Output:** Recommendation List  $\hat{I}$

- 1: Initialize the Recommendation List  $\hat{I} = \emptyset$ ;
  - 2: **while**  $|\hat{I}| < K$  **do**
  - 3:   Select an item  $i \in I \setminus \hat{I}$  so that the objective function of  $i \cup \hat{I}$  is maximized;
  - 4:    $\hat{I} = \hat{I} \cup i$
  - 5: **end while**
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**Algorithm 3.** ALGORITHM FOR AVERAGE SEMANTIC

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**Input:** Relevance matrix  $R$ , the set of users  $U$  and items  $I$

**Output:** Recommendation List  $\hat{I}$

- 1: Relax the integer program into a linear program as Eqn. 4;
  - 2: Solve the linear program with the fractional solution  $X$ ;
  - 3: Round the solution  $X$  into integers by setting top-K  $X$  to 1;
  - 4: Recommend the items with  $X_i = 1, \forall i \in I$ ;
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### 4.2 Average Semantic

Consider the Average semantic for individual satisfaction, when  $IS(u, \hat{I})$  is a linear function of  $rel(u, i), \forall i \in I$ , this programming is a linear integer programming:

$$\begin{aligned}
 & \max T \\
 & s.t. \sum_{i \in I} X_i = k \\
 & \sum_{i \in I} rel(u, i)X_i \geq T, \forall u \in G \\
 & X_i \in \{0, 1\}, \forall i \in I
 \end{aligned} \tag{3}$$

We relax the constraint  $X_i \in \{0, 1\}$  into a fractional constraint:  $X_i \in [0, 1]$ . The program can be transformed into a linear program and a fractional solution can be achieved. Items with highest  $X_i$  are selected for group recommendation.

$$\begin{aligned}
 & \max .T \\
 & s.t. \sum_{i \in I} X_i = k \\
 & \sum_{i \in I} rel(u, i)X_i \geq T, \forall u \in G \\
 & 0 \leq X_i \leq 1, \forall i \in I
 \end{aligned} \tag{4}$$

Besides the rounding techniques, we can also apply Primal-Dual approaches to solve the problem.

## 5 Experiment

We conduct extensive experiments on real-world datasets to evaluate our algorithms.

### 5.1 Experiment Settings

The Movielens dataset is a Movie Rating dataset which contains the ratings of movies given by the users. The dataset contains only the ratings of individual users with no real-world group structures. We randomly divide users into several groups and try to make recommendation to these groups.

The Movielens-100K datasets contain 100,000 ratings from 943 users and 1,682 items. We split users into 100 groups randomly where each group contains 10 users and adopt three different semantics for fairness maximization group recommendation.

We select some typical group recommendation algorithms as baselines:

- LM Ranking Algorithm [2]: this algorithm considers the relevance of each item to the group following the Least Misery relevances and recommend the Top-K items with highest relevances;
- Ave Ranking Algorithm [2]: this algorithm considers the relevance of each item to the group following the Average relevances and recommend the Top-K items with highest relevances;
- SPGreedy Algorithm [10]: this algorithm proposes a fairness metric called proportionality and greedily selects items to maximize the fairness;
- EFGreedy Algorithm [10]: this algorithm proposes a fairness metric called envy-freeness and greedily selects items to maximize the fairness;

## 5.2 Performance Evaluation

In this section, we present the results of group recommendation with typical recommendation metrics, including Precision, Recall and NDCG:

$$Rec@K = \frac{\sum_{i=1}^K rel_i}{|y_u^{test}|};$$

$$Prec@K = \frac{\sum_{i=1}^K rel_i}{K};$$

$$DCG@K = \sum_{i=1}^K \frac{2^{rel_i} - 1}{\log_2(i + 1)}; NDCG@K = \frac{DCG@K}{IDCG@K}$$

We consider the cases when 10 items are recommended to the groups, the results are listed in Table 1.

**Table 1.** Performances comparisons on Movielens-100K with binary relevance,  $K = 10$

Algorithms	LM ranking	Ave ranking	SPGreedy	EFGreedy	IPAlg-Ave
Prec@K	0.0431	0.0540	0.0003	0.0010	<b>0.0522</b>
Rec@K	0.0889	0.1072	0.0003	0.0015	<b>0.1039</b>
NDCG@K	0.2395	0.2567	0.0007	0.0065	<b>0.2541</b>

Judging from the results, maximizing fairness in group recommendation can achieve comparable performances with those typical recommendation approaches in typical recommendation metrics.

## 6 Conclusion

In this paper, we propose the concept of individual satisfaction and fairness in group recommendation. The concept of individual satisfaction is formulated with OWA (Ordered Weighted Average) function and the fairness is modeled as max-min function. We design heuristic algorithms for the fairness maximization group recommendation problem in three typical semantics. Extensive experiments have been conducted on real-world datasets and the results corroborate our analyses.

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