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Polynomial-time approximation scheme for minimum connected dominating set under routing cost constraint in wireless sensor networks

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ABSTRACT

To reduce routing cost in wireless sensor networks, we study a problem of minimizing the size of connected dominating set D under constraint that for any two nodes u and v, $m_D(u, v) \leq \alpha \cdot m(u, v)$ where α is a constant, $m_D(u, v)$ is the number of intermediate nodes on a shortest path connecting u and v through D and $m(u, v)$ is the number of intermediate nodes in a shortest path between u and v in a given unit disk graph. We show that for $\alpha \geq 5$, this problem has a polynomial-time approximation scheme, that is, for any $\varepsilon > 0$, there is a polynomial-time $(1 + \varepsilon)$-approximation.

1. Introduction

Given a graph $G = (V, E)$, a node subset $D \subseteq V$ is called a *dominating set* if every node is either in D or adjacent to a node in D. A *connected dominating set* (CDS) is a dominating set which induces a connected subgraph.

Due to wide applications in wireless networks, the MCDS problem, i.e., computing the minimum connected dominating set (MCDS) for a given graph, has been studied extensively since 1998 [9,16,1,11–13,15,2,3,10,8].

Recently, motivated from reducing routing cost [4] and from improving road load balancing [14], the following problem has been proposed:

> MOC-CDS: given a connected graph $G = (V, E)$, compute a connected dominating set D with minimum cardinality under the condition that for every two nodes $u, v \in V$, there exists a shortest path between u and v such that all intermediate nodes belong to D.

Ding et al. [4] showed that MOC-CDS has no polynomial time approximation with performance ratio $\rho \ln \delta$ for $0 < \rho < 1$ unless $NP \subseteq DTIME(n^{O(\log \log n)})$ where δ is the maximum node degree of input graph G. They also gave a polynomial time distributed approximation algorithm with performance ratio $H(\frac{\delta(\delta-1)}{2})$ where H is the harmonic function, i.e., $H(k) = \sum_{i=1}^{k} \frac{1}{i}$.

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However, some examples show that the solution of MOC-CDS may be much bigger than the solution of MCDS. Thus, to reach the minimum routing cost, the size of CDS may be increased too much. Motivated from this situation, Ding et al. [5] and Du et al. [7] proposed the following problem for any constant $\alpha \geq 1$.

> αMOC-CDS: given a graph, compute the minimum CDS D such that for any two nodes u and v, $m_D(u, v) \leq \alpha \cdot m(u, v)$ where $m_D(u, v)$ is the number of intermediate nodes on a shortest path connecting u and v through D and $m(u, v) = m_G(u, v)$.

1MOC-CDS is exactly MOC-CDS. For $\alpha > 1$, the constraint on routing cost is relaxed and hence the CDS size becomes smaller. Actually, Ding et al. [6] indicated through computer simulation that αMOC-CDS for $\alpha \geq 5$ is much smaller than MOC-CDS. For $\alpha \geq 5$, Du et al. [7] presented a polynomial-time constant-approximation for αMOC-CDS in unit disk graphs.

In this paper, we further show that for $\alpha \geq 5$, αMOC-CDS in unit disk graphs has PTAS, that is, for any $\varepsilon > 0$. αMOC-CDS has polynomial-time $(1 + \varepsilon)$-approximation.

2. Main result

First, let us quote a lemma in [5], which simplifies the routing cost constraint. For convenience of the reader, we also give a proof here.

Lemma 1. *Let G be a connected graph and D a dominating set D of G. Then, for any two nodes u and v,*

$$m_D(u, v) \leq \alpha m(u, v),$$

if and only if for any two nodes u and v with $m(u, v) = 1$,

$$m_D(u, v) \leq \alpha. \tag{1}$$

Proof. It is trivial to show the "only if" part. Next, we show the "if" part. Consider two nodes u and v. If $m(u, v) = 0$, it is clear that $m_D(u, v) = 0 = \alpha m(u, v)$. Next, assume $m(u, v) \geq 1$. Consider a shortest path $(u, w_1, \ldots, w_k, v)$ where $k = m(u, v) \geq 1$. Let us assume that k is even. For odd k, the proof is similar.

Note that $m(u, w_2) = m(w_2, w_4) = \cdots = m(w_{k-2}, w_k) = 1$. By (1), there exist paths $(u, s_{1,1}, s_{1,2}, \ldots, s_{1,h_1}, w_2)$, $(w_2, s_{3,1}, s_{3,2}, \ldots, s_{3,h_3}, w_4)$, ..., $(w_{k-2}, s_{k-1,1}, s_{k-1,2}, \ldots, s_{k-1,h_{k-1}}, w_k)$ such that $1 \leq h_i \leq \alpha$ for all $i = 1, 3, \ldots, k-1$ and $s_{i,j} \in D$ for all $i = 1, 3, \ldots, k-1$ and $j = 1, 2, \ldots, h_i$. Now, note that $m(s_{1,h_1}, s_{3,1}) = \cdots = m(s_{k-1,h_{k-1}}, v) = 1$. By (1), there exist paths $(s_{1,h_1}, s_{2,1}, s_{2,2}, \ldots, s_{2,h_2}, s_{3,1})$, ..., $(s_{k-1,h_{k-1}}, s_{k,1}, s_{k,2}, \ldots, s_{k,h_k}, v)$ such that $1 \leq h_i \leq \alpha$ for $i = 2, 4, \ldots, k$ and $s_{i,j} \in D$ for $i = 2, 4, \ldots, k$ and $j = 1, 2, \ldots, h_i$. Therefore, there is a path $(u, s_{1,1}, \ldots, s_{1,h_1}, s_{2,1}, \ldots, s_{k,h_k}, v)$ with $h_1 + h_2 + \cdots + h_k$ $(\leq \alpha k)$ intermediate nodes all in D. Thus, $m_D(u, v) \leq \alpha \cdot m(u, v)$. □

The following lemma indicates that a dominating set satisfying condition (1) must be feasible for αMOC-CDS.

Lemma 2. *In a connected graph, a dominating set D satisfying condition* (1) *must be a connected dominating set.*

Proof. If $|D| = 1$, then the subgraph induced by D consists of only single node, which is clearly a connected subgraph.

Next, assume $|D| \geq 2$. For any two nodes $u, v \in D$, if u and v are not adjacent, then by Lemma 1, there is a path between u and v with all intermediate nodes in D. Therefore, D induces a connected subgraph. □

Next, we start to construct a PTAS for unit disk graphs.

First, we put input unit disk graph $G = (V, E)$ in the interior of the square $[0, q] \times [0, q]$. Then construct a grid $P(0)$ as shown in Fig. 1. $P(0)$ divides the square $[0, pa] \times [0, pa]$ into p^2 cells where $a = 2(\alpha + 2)k$ for a positive integer k and $p = 1 + \lceil q/a \rceil$. Each cell e is a $a \times a$ square, including its left boundary and its lower boundary, so that all cells are disjoint and their union covers the interior of the square $[0, q] \times [0, q]$.

For each cell e, construct a $(a+4) \times (a+4)$ square and a $(a+2\alpha+4) \times (a+2\alpha+4)$ square with the same center as that of e (Fig. 2). The closed area bounded by the first square is called the *central area* of cell e, denoted by e^c. The area between the second square and cell e, including the boundary of e and excluding the boundary of the second square, is called the *boundary area* of cell e, denoted by e^b. The union of the boundary area and the central area is the open area bounded by the second square, denoted by e^{cb}.

Now, for each cell e, we study the following problem.

> LOCAL(e): find the minimum subset D of nodes in $V \cap e^{cb}$ such that (a) D dominates all nodes in $V \cap e^c$, and (b) for any two nodes $u, v \in V \cap e^c$ with $m(u, v) = 1$ and $\{u, v\} \cap e \neq \emptyset$, $m_D(u, v) \leq \alpha$.

Lemma 3. *Suppose* $\alpha \geq 5$ *and* $|V \cap e^{cb}| = n_e$. *Then the minimum solution of* LOCAL(e) *problem can be computed in time* $n_e^{O(\alpha^4)}$.

Proof. Cut e^c into $\lceil (a+4)\sqrt{2} \rceil^2$ small squares with edge length at most $\sqrt{2}/2$ (Fig. 3).

Then for each (closed) small square s, if $V \cap s \neq \emptyset$, then choose one which would dominates all nodes in $V \cap s$. Those nodes form a set D dominating $V \cap e^c$ and $|D| \leq \lceil (a+4)\sqrt{2} \rceil^2$.

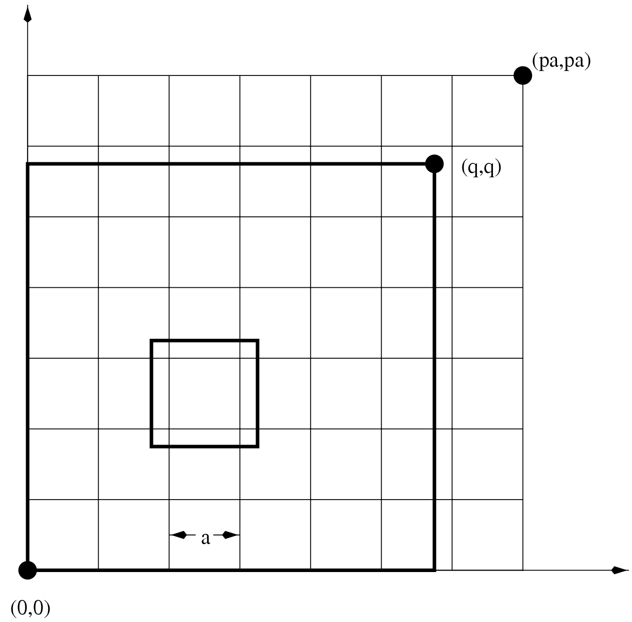

Fig. 1. Grid $P(0)$.

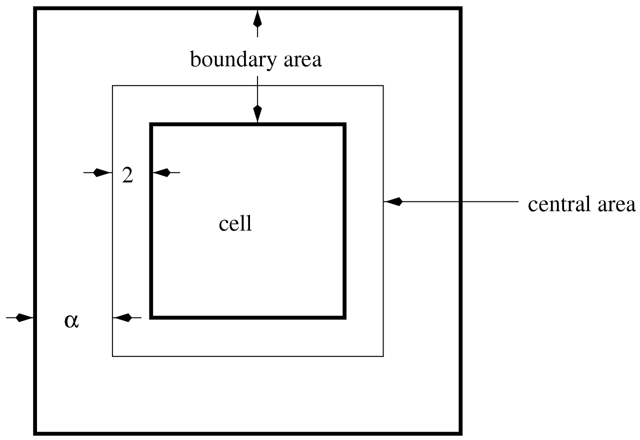

Fig. 2. Central area e^c and boundary area e^b.

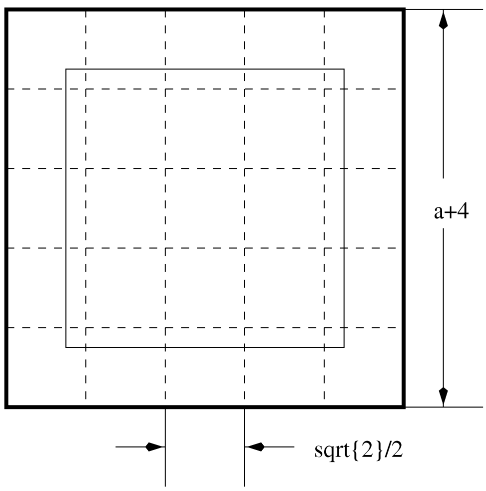

Fig. 3. Decomposition of central area e^c.

For any two nodes $u, v \in D$ with $m(u, v) \leq 3$, connect them with a shortest path between u and v. Namely, let $M(u, v)$ denote the set of all intermediate nodes on a shortest path between u and v. Define

$$C = D \cup \left(\bigcup_{u,v \in D: m(u,v) \leq 3} M(u, v) \right).$$

We show that C is a feasible solution of Local(e) problem. For any two nodes $u, v \in V \cap e^c$ with $m(u, v) = 1$ and $\{u, v\} \neq \emptyset$, since D dominates $V \cap e^c$, there are $u', v' \in D$ such that u is adjacent to u' and v is adjacent to v'. Thus, $m(u', v') \leq 3$. This implies that $M(u, v) \subseteq C$ and hence $m_C(u, v) \leq 5$. Therefore, C is a feasible solution of αMOC-CDS. Moreover,

$$|C| \leq |D| + 3 \cdot \frac{|D|(|D| - 1)}{2} \leq 1.5|D|^2 \leq 1.5 \cdot \lceil (a + 4)\sqrt{2} \rceil^4.$$

This means that the minimum solution of Local(e) has size at most $1.5 \cdot \lceil (a + 4)\sqrt{2} \rceil^4$. Therefore, by an exhausting search, we can compute the minimum solution of Local(e) in time $n_e^{O(a^4)}$. □

Let D_e denote the minimum solution for the Local(e) problem. Define $D(0) = \cup_{e \in P(0)} D_e$ where $e \in P(0)$ means that e is over all cells in partition $P(0)$.

Lemma 4. *$D(0)$ is a feasible solution of αMOC-CDS and $D(0)$ can be computed in time $n^{O(a^4)}$ where $n = |V|$.*

Proof. Since every node in V belongs to some e^c, $D(0)$ is a dominating set. Moreover, for every two nodes $u, v \in V$ with $m(u, v) = 1$, we have $u \in e$ for some cell e, which implies that $u, v \in e^c$. Hence, $m_{D_e}(u, v) \leq \alpha$. If follows that $m_{D(0)}(u, v) \leq \alpha$. By Lemma 2, $D(0)$ is feasible for αMOC-CDS.

Note that each node may appear in e^{cb} for at most four cells e. Therefore, by Lemma 3, $D(0)$ can be computed in time

$$\sum_{e \in P(0)} n_e^{O(a^4)} \leq (4n)^{O(a^4)} = n^{O(a^4)}$$

where $n = |V|$. □

To estimate $|D(0)|$, we consider a minimum solution D^* of αMOC-CDS. Let $P(0)^b = \cup_{e \in P(0)} e^b$.

Lemma 5. $|D(0)| \leq |D^*| + 4|D^* \cap P(0)^b|$.

Proof. We claim that $D^* \cap e^{cb}$ is feasible for Local(e) problem. In fact, it is clear that $D^* \cap e^{cb}$ dominates $V \cap e^c$. For any two nodes $u, v \in e^c$ with $m(u, v) = 1$ and $\{u, v\} \cap e \neq \emptyset$, the path between u and v with at most α intermediate nodes must lie inside of e^{cb} and $m_{D^*}(u, v) \leq \alpha$ implies $m_{D^* \cap e^{cb}}(u, v) \leq \alpha$. This completes the proof of our claim.

Our claim implies that $|D_e| \leq |D^* \cap e^{cb}|$. Thus

$$\begin{aligned}
|D(0)| &\leq \sum_{e \in P(0)} |D_e| \\
&\leq \sum_{e \in P(0)} |D^* \cap e^{cb}| \\
&\leq \sum_{e \in P(0)} |D^* \cap e| + \sum_{e \in P(0)} |D^* \cap e^b| \\
&\leq |D^*| + 4|D^* \cap P(0)^b|. \quad \square
\end{aligned}$$

Now, we shift partition $P(0)$ to $P(i)$ as shown in Fig. 4 such that the left and lower corner of the grid is moved to point $(-2(\alpha + 2)i, -2(\alpha + 2)i)$.

For each $P(i)$, we can compute a feasible solution $D(i)$ in the same way as $D(0)$ for $P(0)$. Then we have the following.

(a) $D(i)$ is a feasible solution of αMOC-CDS.

(b) $D(i)$ can be computed in time $n^{O(a^4)}$.

(c) $|D(i)| \leq |D^*| + 4|D^* \cap P(i)^b|$.

In addition, we have the following.

Lemma 6. $|D(0) + |D(1)| + \cdots + |D(k - 1)| \leq (k + 8)|D^*|$.

Proof. Note that $P(i)^b$ consists of a group of horizontal strips and a group of vertical strips (Fig. 5). All horizontal strips in $P(0)^b \cup P(1)^b \cup \cdots \cup P(k - 1)^b$ are disjoint and all vertical strips in $P(0)^b \cup P(1)^b \cup \cdots \cup P(k - 1)^b$ are also disjoint.

Therefore,

$$\sum_{i=0}^{k-1} |D^* \cap P(i)^b| \leq 2|D^*|.$$

Hence,

$$\sum_{i=0}^{k-1} |D(i)| \leq (k + 8)|D^*|. \quad \square$$

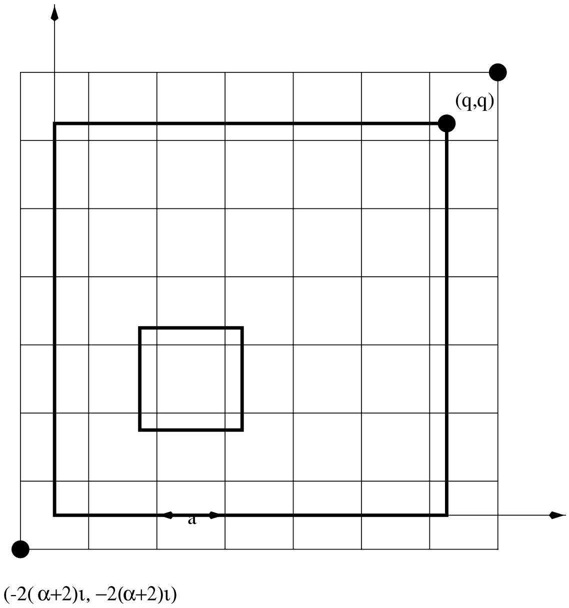

Fig. 4. Grid $P(i)$.

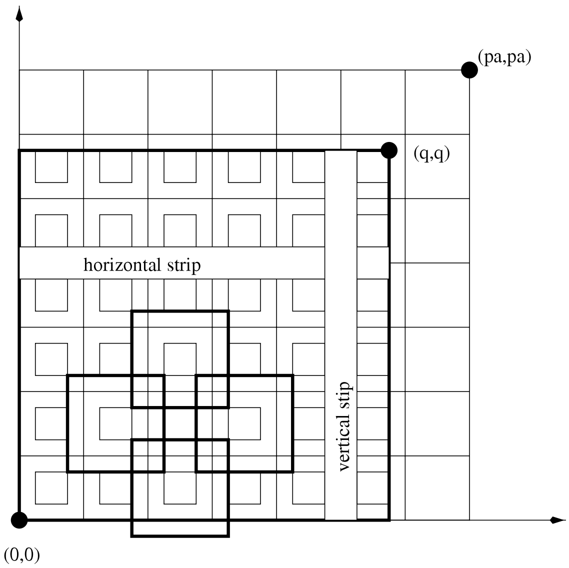

Fig. 5. Horizontal and vertical strips.

Set $k = \lceil 1/(8\varepsilon) \rceil$ and run the following algorithm.

Algorithm PTAS

Compute $D(0), D(1), \ldots, D(k-1)$;
Choose i^*, $0 \le i^* \le k-1$ such that
$|D(i^*)| = \min(|D(0)|, |D(1)|, \ldots, |D(k-1)|)$;
Output $D(i^*)$.

Theorem 7. *Algorithm PTAS produces an approximation solution for αMOC-CDS with size*

$$|D(i^*)| \le (1+\varepsilon)|D^*|$$

and runs in time $n^{O(1/\varepsilon^4)}$.

Proof. It follows from Lemmas 4 and 6. □

3. Discussion

We showed that for $\alpha \geq 5$, αMOC-CDS has PTAS and leave the problem open for $1 \leq \alpha < 5$. Actually, how to connect a dominating set into a feasible solution for αMOC-CDS is the main difficulty for $1 \leq \alpha < 5$. So far, no good method has been found without increasing too much number of nodes.

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