

Two-mode squeezing of distant nitrogen-vacancy-center ensembles by manipulating the reservoirW. L. Yang,^{1,*} Z. Q. Yin,^{2,3,†} Q. Chen,¹ C. Y. Chen,⁴ and M. Feng^{1,‡}¹*State Key Laboratory of Magnetic Resonance and Atomic and Molecular Physics, Wuhan Institute of Physics and Mathematics, Chinese Academy of Sciences, and Wuhan National Laboratory for Optoelectronics, Wuhan 430071, China*²*Key Laboratory of Quantum Information, University of Science and Technology of China, Chinese Academy of Sciences, Hefei 230026, China*³*Center for Quantum Information, IIIS, Tsinghua University, Beijing, China*⁴*Department of Physics and Information Engineering, Hunan Institute of Humanities, Science and Technology, Loudi 417000, China*

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We present a scheme to engineer a two-mode squeezed state of effective bosonic modes realized by collective excitations of two distant nitrogen-vacancy-center ensembles (NVEs) coupled to separated transmission line resonators (TLRs), which are interconnected by a current-biased Josephson-junction superconducting qubit. By making use of the engineered NVE-TLR magnetic coupling with Raman transition between the ground sublevels of the NVEs, we may manipulate the artificial reservoir by tuning the external driving fields. The TLR decay induces an artificial reservoir, which can drive the system to the desired entangled squeezed states. Our idea provides a scalable way to a NVE-based continuous-variable quantum-information processing, which is close to being achievable with currently available technology.

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Recently, many efforts have been devoted to the hybrid system that combines exceptional spin properties of solid-state qubits with the technological advances in cavity quantum electrodynamics (QED) [1] or in circuit QED systems [2,3]. Among various hybrid systems, the composite system consisting of a nitrogen-vacancy-center ensemble (NVE) [4] and superconducting resonators [5,6] or superconducting qubits [7] has emerged as one of the most promising candidates for quantum-information applications. These systems often resort to collectively enhanced magnetic dipole interaction for obtaining appreciable coupling strengths through the collective excitation of the NVE in the microwave domain. In the low-excitation limit, these collective variables behave as bosonic modes or harmonic oscillators. Very recently, a series of experimental achievements has also provided experimental evidence for strong coupling between the NVE and the modes of superconducting resonators [8–10].

On the other hand, using dissipative effects as powerful resources, the steady-state entanglement engineering has recently been paid particular attention in quantum computation [11–13]. Different from traditional coherent unitary dynamics, these methods do not require accurate control of the evolution time, regardless of the initial state of the system. So the dissipative quantum dynamical process can be rather robust against parameter fluctuations or certain stochastic errors.

Motivated by these works, we propose a scheme in this paper to engineer a two-mode squeezed state [14–17] of two distant NVEs coupled to separate transmission line resonators (TLRs) [2], which are interconnected by a current-biased Josephson-junction (CJJ) superconducting qubit [18–22]. In our scheme, the decay of two TLRs will drive the system into the desired two-mode squeezed state of two effective bosonic modes realized by collective excitations of NVEs,

which is actually a steady-state entanglement through an engineered reservoir for the NVEs [23]. We further show that the degree of squeezing can be manipulated by tuning the external driving fields, based on the NVE-TLR magnetic coupling with Raman transition between the ground sublevels of the NVEs. Here we propose to prepare continuous-variable (CV) entanglement for nitrogen-vacancy (NV) centers using dissipative state preparation. In addition, our idea provides a scalable way to a NVE-based CV quantum-information processing, which is the prerequisite for realization of large-scale spin-based quantum networks [24,25]. We further note that matter-light state mapping schemes [26] could be applied to transfer the entanglement from the spin ensembles to TLRs. This in turn might allow one to realize the squeezed states or entanglement between distant superconducting cavities.

Before proceeding we note that a number of proposals have been made for generating entanglement between distant NV centers. Some are based on the magnetic dipolar coupling [24], whereas others involve spin-dependent optical transitions [25], the giant optical Faraday rotation [27], and unitary dynamical evolution [6,28,29]. In contrast to these mechanisms the present approach has the following merits: (i) The two delocalized TLR-decay channels play positive roles in actively driving the system to the desired entangled squeezed state. In other words, our model works well in the bad-cavity limit, which makes it more applicable to current laboratory techniques and is advantageous over the idea in Ref. [6] to suppress the TLR decay for realization of high-fidelity entangled states. (ii) The entanglement between two NVEs is produced in a CV way, which can be manipulated by tuning the external microwave pulses, and the quadrature-variance can be measured by cavity output fields. (iii) The precise control of the evolution time is not required, which makes our scheme robust against parameter fluctuations.

As illustrated in Fig. 1, we consider that two NVEs are coupled to two separate TLRs connected by a CJJJ, which serves as a quantum transducer to create and transfer photonic

*ywl@wipm.ac.cn

†yinzhangqi@gmail.com

‡mangfeng@wipm.ac.cn

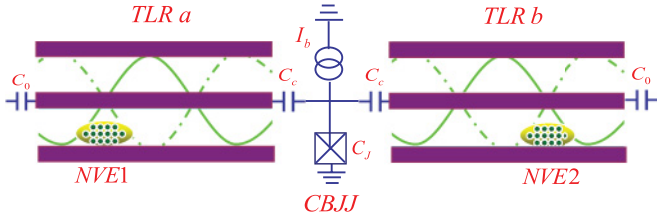


FIG. 1. (Color online) The coupled system of a CBBJ and two TLRs, where TLR a and TLR b are interconnected by a CBBJ from the left and right by the coupling capacitors C_c , where C_J is the junction capacitance and I_b is the bias current. CBBJ acts as a tunable coupler, which provides the one-way quantum channels between spatially distant spin ensembles.

states between the TLRs. The TLR with length L , inductance F_t , and capacitance C_t consists of a narrow central conductor and two nearby lateral ground planes. In our case, TLR a and TLR b can be capacitively coupled through the CBBJ by tuning the coupling capacitors C_c and the junction capacitance C_J . By quantizing the electromagnetic modes, TLR a and TLR b could be simply modeled as $H_a = \frac{\hbar}{2} \omega_a a^\dagger a$ and $H_b = \frac{\hbar}{2} \omega_b b^\dagger b$, where a (a^\dagger) and b (b^\dagger) are the annihilation (creation) operators of the full-wave modes of TLR a and TLR b, respectively, and $\omega_{a(b)} = 2\pi/(\sqrt{F_t C_t})$ is the corresponding eigenfrequency. The NV center number in the j th spin ensemble is denoted by N_j , with $j = 1, 2$.

The energy level configuration of the NV center is shown in Fig. 2. Each NV center is negatively charged with two unpaired electrons located at the vacancy, which can be modeled as a two-level system in the ground subspaces. However, the degeneracy of the levels $m_S = \pm 1$ in spin-triplet ground state 3A can be lifted by an external magnetic field B_{ex} along the quantized symmetric axis of the NV centers, which induces an energy splitting $D_B = \chi_e |B_{ex}|$ between the sublevels $m_S = \pm 1$, with χ_e being the charge-mass ratio of an electron. The transition $|{}^3A, m_S = 0\rangle \rightarrow |{}^3A, m_S = -1\rangle$ is coupled to the corresponding TLR modes via collectively magnetic dipole interaction with the single NV center's vacuum Rabi frequencies g_a and g_b and detunings Δ_1 and Δ_2 . Meanwhile, they are driven by two classical fields with Rabi frequencies Ω_1 and Ω_2 and detunings Δ_1 and Δ_2 . The two NVEs are initially prepared via separate microwave fields in

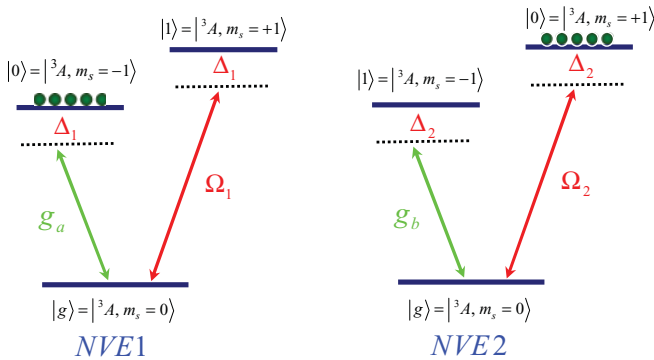


FIG. 2. (Color online) Level structure of the ground state of a single NV center under an appropriate external magnetic field, which is employed to remove the degeneracy between $m_S = \pm 1$.

different ground states, but for convenience we relabel the ground states in NVE 2 so that all NV centers are initially in state $|0\rangle$ in our theoretical treatment, as shown in Fig. 2. In our proposal, we adopt different qubit definitions for the two NVEs [23] so that the logic state $|0\rangle$ ($|1\rangle$) of NVE 1 is denoted by the state $|{}^3A, m_S = -1\rangle$ ($|{}^3A, m_S = 1\rangle$), but the logic state $|0\rangle$ ($|1\rangle$) of NVE 2 is represented by the state $|{}^3A, m_S = 1\rangle$ ($|{}^3A, m_S = -1\rangle$).

With these qubit definitions and large detunings Δ_1 and Δ_2 from the transition frequencies of the ground states, the Hamiltonian in the interaction picture describing the NVE-TLR interaction is (in units of $\hbar = 1$)

$$H_I = \sum_{j=1}^{N_1} \left[\frac{\Omega_1^2}{4\Delta_1} |1\rangle_{1,1}^j \langle 1| + \frac{g_a^2}{\Delta_1} a^\dagger a |0\rangle_{1,1}^j \langle 0| + \left(\frac{\Omega_1 g_a a^\dagger}{2\Delta_1} |0\rangle_{1,1}^j \langle 1| + \text{H.c.} \right) \right] + \sum_{j=1}^{N_2} \left[\frac{\Omega_2^2}{4\Delta_2} |0\rangle_{2,2}^j \langle 0| + \frac{g_b^2}{\Delta_2} b^\dagger b |1\rangle_{2,2}^j \langle 1| + \left(\frac{\Omega_2 g_b b^\dagger}{2\Delta_2} |1\rangle_{2,2}^j \langle 0| + \text{H.c.} \right) \right]. \quad (1)$$

Using the Holstein-Primakoff transformation [30], we set the collective spin operators $S_i^\dagger = \sum_{j=1}^{N_i} |1\rangle_{i,i}^j \langle 0|$ for the i th NVE, with $i = 1, 2$, and map them into the boson operators c^\dagger (c), namely,

$$\begin{aligned} S_i^\dagger &= c_i^\dagger \sqrt{N - c_i^\dagger c_i} \simeq \sqrt{N_i} c_i^\dagger, \\ S_i^- &= c_i \sqrt{N - c_i^\dagger c_i} \simeq \sqrt{N_i} c_i, \\ S_i^z &= \left(c_i^\dagger c_i - \frac{N_i}{2} \right). \end{aligned} \quad (2)$$

Neglecting the constant-energy terms in Hamiltonian H_I , we rewrite H_I , based on Eq. (2), as

$$\begin{aligned} H_{\text{eff}} &= H_p + H_m, \\ H_p &= \sqrt{N_1} \frac{\Omega_1 g_a}{2\Delta_1} (a^\dagger c_1 + a c_1^\dagger), \\ H_m &= \sqrt{N_2} \frac{\Omega_2 g_b}{2\Delta_2} (b^\dagger c_2^\dagger + b c_2), \end{aligned} \quad (3)$$

where the Hamiltonians H_p and H_m denote the parametric amplification interaction and linear mixing interaction, respectively. In our scheme, the two-mode squeezed states of the separate NVEs can be realized by following three key steps: (i) Bosonic mode c_1 in NVE 1 is entangled with the TLR a mode through the parametric amplification process H_p . (ii) Due to the cavity decay, this entangled light will exit from TLR a with the cavity-decay rate κ_a and then will be coupled into TLR b through the CBBJ, which acts as a tunable coupler to mediate the single-excitation transfer. (iii) The entanglement of the light field with bosonic mode c_1 in NVE 1 may be transferred to bosonic mode c_2 in NVE 2 via the linear mixing mechanism H_m . As a result, two-mode squeezed states of the separate NVE 1 and NVE 2 can be realized.

Taking into account the decoherence effects, we simulate the dynamics of the transfer process by integrating the full phenomenological quantum master equation.

$$\dot{\rho} = -i[H_{\text{eff}}, \rho] + \frac{\kappa_a}{2} D[a]\rho + \frac{\kappa_b}{2} D[b]\rho - 2\sqrt{\lambda\kappa_a\kappa_b}([b^\dagger, a\rho] + [\rho a^\dagger, b]), \quad (4)$$

where H_{eff} [Eq. (3)] describes the effective coupling between the bosonic modes of the NVEs and the TLR light field as derived above. $D[A]\rho = 2A\rho A^\dagger - A^\dagger A\rho - \rho A^\dagger A$, and κ_a and κ_b are the decay rates of TLR a and TLR b, respectively. The parameter $\lambda \in [0, 1]$ represents the losses in transmission and for the inefficiency of coupling, where $\lambda = 1$ corresponds to the ideal transmission and coupling. The master equation [Eq. (4)] for our system can be derived in the form

$$\dot{\rho} = (\hat{L}_0 + \hat{L}_c)\rho, \quad (5)$$

where $\hat{L}_0\rho = -i[H_{\text{eff}}, \rho]$ and

$$\hat{L}_c\rho = \frac{\kappa_a}{2} D[a]\rho + \frac{\kappa_b}{2} D[b]\rho - 2\sqrt{\lambda\kappa_a\kappa_b}([b^\dagger, a\rho] + [\rho a^\dagger, b]). \quad (6)$$

We assume that the decay rates (κ_a and κ_b) are large enough compared to other coupling rates, and thereby the TLR fields can be adiabatically eliminated from the system dynamics. This leads to a reduced master equation for the density operator ρ_{nv} of the two NVEs, given formally by [31]

$$\dot{\rho}_{nv} = \text{Tr}_c \left\{ \hat{L}_0 \int_0^\infty d\tau e^{\hat{L}_c\tau} \hat{L}_0 \rho_c^{ss} \right\} \rho_{nv}, \quad (7)$$

where ρ_c^{ss} is the density matrix of the steady states for the two TLR a and TLR b modes, and can be obtained by evaluating the steady state correlation functions for the two TLR modes, a and b .

Next, we will focus on the calculation of the steady-state correlation functions using the quantum regression theorem [32]. First, based on Eq. (6), we straightforwardly write the equations of motion for the mean values of the amplitudes of the two TLR modes as

$$\begin{aligned} \frac{d}{dt} \langle a \rangle &= -\kappa_a \langle a \rangle, \\ \frac{d}{dt} \langle a^\dagger \rangle &= -\kappa_a \langle a^\dagger \rangle, \\ \frac{d}{dt} \langle b \rangle &= -\kappa_b \langle b \rangle - 2\sqrt{\lambda\kappa_a\kappa_b} \langle a \rangle, \\ \frac{d}{dt} \langle b^\dagger \rangle &= -\kappa_b \langle b^\dagger \rangle - 2\sqrt{\lambda\kappa_a\kappa_b} \langle a^\dagger \rangle, \end{aligned} \quad (8)$$

for which the solutions are of the general form

$$\begin{pmatrix} \langle a(t) \rangle \\ \langle a^\dagger(t) \rangle \\ \langle b(t) \rangle \\ \langle b^\dagger(t) \rangle \end{pmatrix} = M_{ij}(t) \begin{pmatrix} \langle a(0) \rangle \\ \langle a^\dagger(0) \rangle \\ \langle b(0) \rangle \\ \langle b^\dagger(0) \rangle \end{pmatrix}, \quad (9)$$

where

$$M_{ij}(t) = \begin{pmatrix} e^{-\kappa_a t} & 0 & 0 & 0 \\ 0 & e^{-\kappa_a t} & 0 & 0 \\ \xi & 0 & e^{-\kappa_b t} & 0 \\ 0 & \xi & 0 & e^{-\kappa_b t} \end{pmatrix},$$

with $\xi = \frac{2\sqrt{\lambda\kappa_a\kappa_b}}{\kappa_a - \kappa_b} (e^{-\kappa_a t} - e^{-\kappa_b t})$.

Using the quantum regression theorem [32], we have the two-time correlation functions as

$$\langle A_i(\tau) A_k(0) \rangle = \sum_j M_{ij}(\tau) \langle A_i(0) A_k(0) \rangle. \quad (10)$$

In the steady state, the only nonzero equal-time correlation functions in our case are

$$\langle aa^\dagger \rangle_s = \langle bb^\dagger \rangle_s = 1. \quad (11)$$

This leads to the only nonzero two-time correlation functions as

$$\begin{aligned} \langle a(\tau) a^\dagger(0) \rangle &= \langle a(0) a^\dagger(\tau) \rangle = e^{-\kappa_a \tau}, \\ \langle b(\tau) b^\dagger(0) \rangle &= \langle b(0) b^\dagger(\tau) \rangle = e^{-\kappa_b \tau}, \\ \langle a(0) b^\dagger(\tau) \rangle &= \langle b(\tau) a^\dagger(0) \rangle = \frac{2\sqrt{\lambda\kappa_a\kappa_b}}{\kappa_a - \kappa_b} (e^{-\kappa_a \tau} - e^{-\kappa_b \tau}). \end{aligned} \quad (12)$$

After using these nonzero correlation functions in Eq. (12), the reduced master equation for the density operator of the two NVEs is given by

$$\begin{aligned} \dot{\rho}_{nv} &= \frac{\tilde{\Omega}_1^2}{\kappa_a} D'[c_1]\rho_{nv} + \frac{\tilde{\Omega}_2^2}{\kappa_b} D[c_2]\rho_{nv} \\ &\quad - \chi((c_2^\dagger, c_1^\dagger)\rho_{nv} + [\rho_{nv} c_1, c_2]), \end{aligned} \quad (13)$$

where $D'[A] = 2A^\dagger \rho A - AA^\dagger \rho - \rho AA^\dagger$ and $\chi = 2\tilde{\Omega}_1\tilde{\Omega}_2\sqrt{\lambda/\kappa_a\kappa_b}$, with $\tilde{\Omega}_1 = \Omega_1 g_a / 2\Delta_1$ and $\tilde{\Omega}_2 = \Omega_2 g_b / 2\Delta_2$.

Next, we calculate the bosonic mode correlation between the two separate NVEs. Based on the reduced master equation [Eq. (13)], we can obtain a set of differential equations for the correlation functions,

$$\begin{aligned} \frac{d}{dt} \langle c_1 c_2 \rangle &= (\tilde{\Omega}_1^2/\kappa_a - \tilde{\Omega}_2^2/\kappa_b) \langle c_1 c_2 \rangle + \chi(\langle c_1^\dagger c_1 \rangle + 1), \\ \frac{d}{dt} \langle c_1^\dagger c_2^\dagger \rangle &= (\tilde{\Omega}_1^2/\kappa_a - \tilde{\Omega}_2^2/\kappa_b) \langle c_1^\dagger c_2^\dagger \rangle + \chi(\langle c_1^\dagger c_1 \rangle + 1), \\ \frac{d}{dt} \langle c_1^\dagger c_1 \rangle &= (2\tilde{\Omega}_1^2/\kappa_a)(\langle c_1^\dagger c_1 \rangle + 1), \\ \frac{d}{dt} \langle c_2^\dagger c_2 \rangle &= (-2\tilde{\Omega}_2^2/\kappa_b) \langle c_2^\dagger c_2 \rangle + \chi(\langle c_1^\dagger c_2^\dagger \rangle + \langle c_1 c_2 \rangle). \end{aligned} \quad (14)$$

The solutions are

$$\begin{aligned} \langle c_1 c_2 \rangle &= \langle c_1^\dagger c_2^\dagger \rangle = \frac{\chi (e^{2\tilde{\Omega}_1^2 t/\kappa_a} + e^{\tilde{\Omega}_1^2 t/\kappa_a - \tilde{\Omega}_2^2 t/\kappa_b})}{\tilde{\Omega}_1^2/\kappa_a + \tilde{\Omega}_2^2/\kappa_b}, \\ \langle c_1^\dagger c_1 \rangle &= e^{2\tilde{\Omega}_1^2 t/\kappa_a} - 1, \\ \langle c_2^\dagger c_2 \rangle &= \frac{\chi^2 (e^{\tilde{\Omega}_1^2 t/\kappa_a} - e^{-\tilde{\Omega}_2^2 t/\kappa_b})^2}{(\tilde{\Omega}_1^2/\kappa_a + \tilde{\Omega}_2^2/\kappa_b)^2}. \end{aligned} \quad (15)$$

Defining position and moment operators for the bosonic modes of the NVEs as $X_j = c_j + c_j^\dagger$ and $P_j = (c_j - c_j^\dagger)/i$,

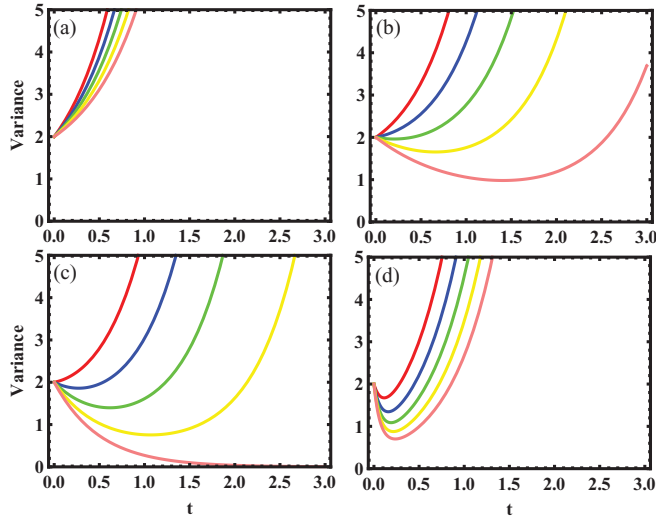


FIG. 3. (Color online) The variance $\langle (X_1 - X_2)^2 \rangle$ as a function of dimensionless time, where the curves from top to bottom denote $\lambda = 0.2, 0.4, 0.6, 0.8, 1$, respectively. (a) $\Lambda_1 = 1$ and $\Lambda_2 = 0.1$; (b) $\Lambda_1 = 1$ and $\Lambda_2 = 0.5$; (c) $\Lambda_1 = 1$ and $\Lambda_2 = 1$; (d) $\Lambda_1 = 1$ and $\Lambda_2 = 10$.

with $j = 1, 2$, we may calculate the variances in the sum and difference operators as

$$\begin{aligned} \langle (X_1 + X_2)^2 \rangle &= 2(1 + \langle c_1^\dagger c_1 \rangle) + \langle c_2^\dagger c_2 \rangle + \langle c_1 c_2 \rangle + \langle c_1^\dagger c_2^\dagger \rangle \\ &= 2e^{2\tilde{\Omega}_1^2 t / \kappa_a} \left[1 + \frac{\chi(1 - e^{-\tilde{\Omega}_2^2 \kappa_a t / \tilde{\Omega}_1^2 \kappa_b})}{\tilde{\Omega}_1^2 / \kappa_a + \tilde{\Omega}_2^2 / \kappa_b} \right]^2, \end{aligned} \quad (16)$$

$$\begin{aligned} \langle (X_1 - X_2)^2 \rangle &= 2(1 + \langle c_1^\dagger c_1 \rangle) + \langle c_2^\dagger c_2 \rangle - \langle c_1 c_2 \rangle - \langle c_1^\dagger c_2^\dagger \rangle \\ &= 2e^{2\tilde{\Omega}_1^2 t / \kappa_a} \left[1 - \frac{\chi(1 - e^{-\tilde{\Omega}_2^2 \kappa_a t / \tilde{\Omega}_1^2 \kappa_b})}{\tilde{\Omega}_1^2 / \kappa_a + \tilde{\Omega}_2^2 / \kappa_b} \right]^2. \end{aligned} \quad (17)$$

We now consider in detail the conditions under which the two-mode squeezing of the separate NVEs in our model can be achieved. As shown in Fig. 3, the variance $\langle (X_1 - X_2)^2 \rangle$ is plotted as a function of time for various values of the ratio Λ_2 / Λ_1 , with $\Lambda_1 = \tilde{\Omega}_1^2 / \kappa_a$ and $\Lambda_2 = \tilde{\Omega}_2^2 / \kappa_b$. From Fig. 3(a), we find that $\langle (X_1 - X_2)^2 \rangle$ is always larger than 2 in the case of $\Lambda_2 / \Lambda_1 \ll 1$. This implies that the entangled squeezed states of NVEs cannot be obtained when the parametric amplification mechanism (tuned by the externally driven field Ω_1) dominates the dissipative dynamics process.

However, once the parameter Λ_2 becomes comparable to Λ_1 so that the linear mixing mechanism (tuned by the externally driven field Ω_2) plays a more and more important role, as shown in Figs. 3(b) and 3(c), we can find that the variance $\langle (X_1 - X_2)^2 \rangle$ is effectively reduced to less than 2, i.e., the entanglement criterion is satisfied [33], and the variance reaches a finite minimum value at a particular time, after which it will exceed 2 and grow indefinitely. Thus, the system evolves into an entangled state and remains entangled only for a limited period of time. But an exception of long-lived entanglement appears in the case of $\Lambda_1 = \Lambda_2$ and $\lambda = 1$, as shown in Fig. 3(c), where $\lambda = 1$ corresponds to the ideal transmission and coupling case and $\Lambda_1 = \Lambda_2$ denotes a balanced competition between the linear mixing mechanism and parametric am-

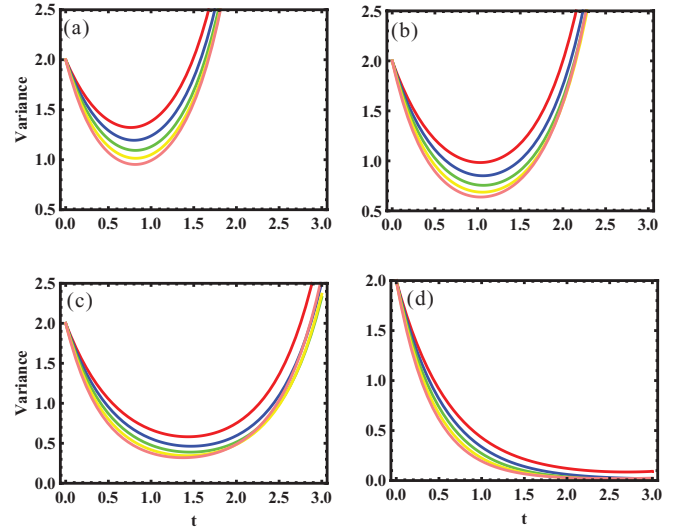


FIG. 4. (Color online) The variance $\langle (X_1 - X_2)^2 \rangle$ as a function of dimensionless time, where the curves from top to bottom denote $\Lambda_2 / \Lambda_1 = 0.8, 0.9, 1.0, 1.1$, and 1.2 , respectively. (a) $\lambda = 0.7$; (b) $\lambda = 0.8$; (c) $\lambda = 0.9$; (d) $\lambda = 1$.

plification mechanism. In this case, the value of variance $\langle (X_1 - X_2)^2 \rangle$ could gradually decrease until it illimitably approaches zero when $t \rightarrow \infty$. Figure 3(d) shows that the degree of squeezing becomes higher when the linear mixing mechanism dominates the system evolution, at the expense of reduction in the duration of entanglement remaining.

To visualize the effect of cavity-coupling parameter λ , which actually plays a crucial role in our scheme, we have plotted in Fig. 4 the time-dependent variance $\langle (X_1 - X_2)^2 \rangle$ in the case of different rates of Λ_2 / Λ_1 . It is clear that, regardless of the values of Λ_2 / Λ_1 , the variance $\langle (X_1 - X_2)^2 \rangle$ always decreases with the growth of λ . A particular case is for $\lambda = 1$, as shown in Fig. 4(d), which yields the biggest squeezing, i.e., 17.37 dB in the case of $t = 2$ and $\Lambda_1 = \Lambda_2$. Additionally, when $\Lambda_1 = \Lambda_2$, the optimal reductions in the variance are 2.62, 4.22, and 7.11 dB, respectively, in Figs. 4(a), 4(b), and 4(c).

The condition of the Holstein-Primakoff transformation for the NVEs requires that the number of rotational excitations is much smaller than the number of NV centers in each NVE. In our case, the mean polariton number in the two NVEs can be calculated by

$$\begin{aligned} \langle \hat{N} \rangle &= \chi^2 (e^{\tilde{\Omega}_1^2 t / \kappa_a} - e^{-\tilde{\Omega}_2^2 t / \kappa_b})^2 / (\tilde{\Omega}_1^2 / \kappa_a + \tilde{\Omega}_2^2 / \kappa_b)^2 \\ &\quad + e^{2\tilde{\Omega}_1^2 t / \kappa_a} - 1. \end{aligned} \quad (18)$$

In order to get some insight, we show the time development of the average polariton number \hat{N} in our system. As shown in Fig. 5, the value of \hat{N} is smaller than 800. So the small value of \hat{N} could ensure that the Holstein-Primakoff transformation in our scheme is reasonable.

We survey the relevant experimental parameters. First, the TLR cavity with an inductance $F_l = 70$ nH and capacitance $C_l = 2$ pF leads to a full-wave frequency $\omega_0 / 2\pi = D_{gs} - \Delta = 2.67$ GHz, with $\Delta / 2\pi = \Delta_1 / 2\pi = \Delta_2 / 2\pi = 200$ MHz and $D_{gs} / 2\pi = 2.87$ GHz being the zero-field splitting between the states $m_S = 0$ and $m_S = \pm 1$ in the absence of the external magnetic field. Second, our scheme requires

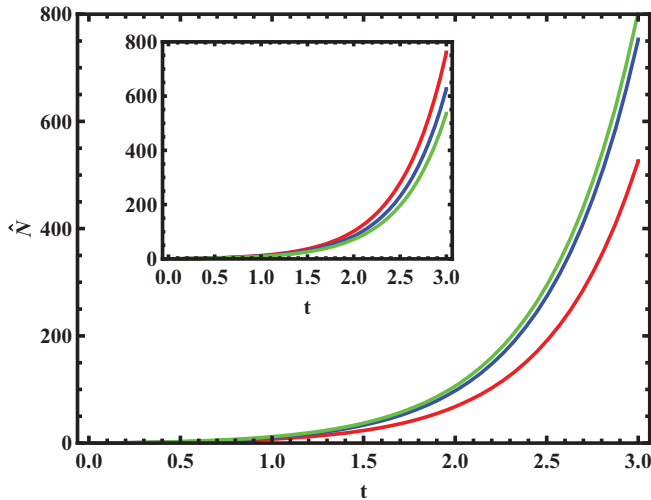


FIG. 5. (Color online) The mean polariton number $\langle \hat{N} \rangle$ as a function of dimensionless time, where $\lambda = 1$ and $\Lambda_1 = 1$. The curves from top to bottom denote $\Lambda_2 = 1$, $\Lambda_2 = 0.5$, and $\Lambda_2 = 0.1$, respectively. The curves from top to bottom in the inset denote $\Lambda_2 = 2$, $\Lambda_2 = 5$, and $\Lambda_2 = 10$, respectively.

the large-detuning condition, namely, the detuning $\Delta \gg \{g_a, g_b, \Omega_1, \Omega_2\}$. So if we take the values of the parameters $g_a/2\pi = g_b/2\pi = 10$ MHz, $\kappa_a/2\pi = \kappa_b/2\pi = 10$ MHz and tune the values of $\Omega_i/2\pi$ from 10 to 40 MHz, which yields $\tilde{\Omega}_i/2\pi \in [0.25, 1]$ MHz and $\Lambda_i/2\pi \in [0.006, 0.1]$ MHz, with $i = 1$ and 2 , the effective coupling strength $\tilde{\Omega}_i$ between each bosonic mode c_i and the corresponding TLR mode is much smaller than the TLR decay rate so that the two TLR modes (a and b) can be eliminated from the dynamics.

On the other hand, the electron spin relaxation time T_1 of NV centers ranges from 6 ms at room temperature [34] to 28–265 s at low temperature [35]. In addition, the dephasing time $T_2 > 600 \mu\text{s}$ for a NVE with a natural abundance of ^{13}C has been reported [36]. A later experimental study [37] with an isotopically pure diamond sample has demonstrated a longer

dephasing time of $T_2 = 1.8$ ms. Therefore, the dissipation rates of the NVE should not be a serious problem in our scheme.

It should be noted that the two-mode squeezed states are of crucial importance for CV teleportation [38], efficient distribution of entanglement and implementation of quantum channels [39], nonlocality tests with CV states [40], and quantum communication [41]. So our work opens perspectives in NVE-based CV quantum-information processing, which is the prerequisite for large-scale spin-based quantum networks. For example, the combined TLR-CBJJ-TLR system we study is potentially scalable, and a recent experimental demonstration of an excellent quantum control over photonic Fock states in three resonators interconnected by two phase qubits [3] gives us hopes to extend our idea to more complicated architectures. So one favorable application of our scheme is the preparation of multiqubit CV entangled states, i.e., cluster states, using a similar approach in this distributed architecture, which is potentially practical for large-scale one-way quantum computation [42].

In summary, we have proposed a scheme to engineer a two-mode squeezed state of effective bosonic modes realized by collective excitations of two distant NVEs coupled to separate TLRs, through a dissipative quantum dynamical process. Our architecture is, in principle, scalable, and the degree of squeezing can be manipulated. More importantly, our idea is close to being reachable with currently available technology.

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